UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering
ECE 417 Multimedia Signal Processing

# Lecture 2 Sample Problems

### Problem 2.1

Suppose $x[n] = \sin\left(\frac{\pi n}{10}\right)$ and $h[n]$ is the local averaging filter,

$$h[n] = \begin{cases} \frac{1}{N} & -\left(\frac{N-1}{2}\right) \le n \le \left(\frac{N-1}{2}\right) \\ 0 & \text{else} \end{cases}$$

$y[n] = h[n] * x[n]$ turns out to be $A\sin\left(\frac{n\pi}{10}\right)$ for some real constant $A$. Find the numerical value of $A$ for averaging lengths of $N = 1, 3, 5, 7, 9, 11$. Usually in this course, an explicit numerical formula like $\sin(4\pi/5)$ is considered equivalent to an actual number, but for this problem ONLY, please plug the formulas into your calculator to find out what the actual numbers are.

### Problem 2.2

Suppose the signal $x[n] = \cos(n/3)$ is windowed with a 256-sample rectangular window, then a 256-sample DFT is computed. Find $X[k]$. Express your answer in terms of the "digital sinc" function, which is defined as

$$\text{dsinc}(\theta, N) = \frac{\sin(\theta N/2)}{N\sin(\theta/2)}$$

### Problem 2.3

Consider a local averaging system, as in problem 1, but suppose it computes a weighted local average instead of an unweighted average:

$$y[n] = \sum_{-\left(\frac{N-1}{2}\right)}^{\left(\frac{N-1}{2}\right)} \cos\left(\frac{\pi m}{N}\right) x[n-m]$$

Express $H(e^{j\omega})$ in terms of the dsinc function, $\text{dsinc}(\theta, N) = \frac{\sin(\theta N/2)}{N\sin(\theta/2)}$.

### Problem 2.4

If you want a zero-phase filter, one way to do it is to filter your signal first with $g[n]$, then with $g[-n]$, thus:

$$y[n] = g[n] * x[n] = \sum_{m=-\infty}^{\infty} g[m]x[n-m] = \sum_{m=-\infty}^{\infty} x[m]g[n-m]$$

$$z[n] = g[-n] * y[n] = \sum_{m=-\infty}^{\infty} g[-m]y[n-m] = \sum_{m=-\infty}^{\infty} y[m]g[m-n]$$

Define $H(e^{j\omega}) = Z(e^{j\omega})/X(e^{j\omega})$. Find $H(e^{j\omega})$ in terms of $G(e^{j\omega})$, and show that, for real-valued $g[n]$, $H(e^{j\omega})$ is real-valued, even if $G(e^{j\omega})$ isn't.