UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering
ECE 417 MULTIMEDIA SIGNAL PROCESSING

Lecture 4 Sample Problems

Problem 4.1

A periodic discrete-time signal with period P, $x[n] = x[n+P]$, and fundamental frequency $\omega_0 = \frac{2\pi}{P}$, has a discrete-time Fourier series given by

$$x[n] = \sum_{k=0}^{P-1} X_k e^{jk\omega_0 n}, \quad X_k = \frac{1}{P}\sum_{n=0}^{P-1} x[n]e^{-jk\omega_0 n} \tag{4.1-1}$$

Use equation (4.1-1) to show that

$$\sum_{p=-\infty}^{\infty} \delta[n-pP] = \frac{1}{P}\sum_{m=0}^{P-1} e^{j\omega_0 nm}$$

Problem 4.2

Suppose turbulence makes white Gaussian noise, $e[n] \sim \mathcal{N}(0,1)$. Suppose $x[n] = h[n] * e[n]$, where $h[n]$ is real-valued. In terms of $h[n]$ and $|H(e^{j\omega})|$, find the statistical autocorrelation $R_{xx}[n]$ and the power spectrum $S_{xx}(\omega)$.

Problem 4.3

Suppose $e[n] = \sum_{p=-\infty}^{\infty} \delta[n-pP] = \frac{1}{P}\sum_{m=0}^{P-1} e^{j\omega_0 nm}$, and $s[n] = h[n] * e[n]$. Note that, in this case, the speech signal $s[n]$ is also periodic with period P, so we can write $s[n] = \sum_{m=0}^{P-1} S_m e^{j\omega_0 nm}$. Find S_m in terms of $H(e^{j\omega})$.

Problem 4.4

Suppose $s[n] = \sum_{p=-\infty}^{\infty} h[n-pP]$. Find $S(e^{j\omega})$ in terms of $H(e^{j\omega})$.