UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering
ECE 417 Multimedia Signal Processing

Lecture 21 Sample Problem Solutions

Problem 21.1

$\vec{u} = A\vec{x}$, where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 20 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\left(-\frac{\pi}{6}\right) & \sin\left(-\frac{\pi}{6}\right) & 0 \\ -\sin\left(-\frac{\pi}{6}\right) & \cos\left(-\frac{\pi}{6}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 20 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{3} & \frac{\sqrt{3}}{3} & \frac{40}{3} \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 21.2

$\vec{u} = A\vec{x}$, where

$$A = \begin{bmatrix} \cos(-\pi/6) & \sin(-\pi/6) & 0 \\ -\sin(-\pi/6) & \cos(-\pi/6) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 20 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 20 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{3} & -10 \\ \frac{1}{2} & \frac{\sqrt{3}}{3} & -10\sqrt{3} \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 21.3

We have a mapping $U \rightarrow X$ where

$$U = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \rightarrow X = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 2 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

The output point is

$$\vec{x} = \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}$$

We need to find $\vec{\lambda}$ so that

$$\vec{x} = X\vec{\lambda}, \quad \vec{\lambda} = X^{-1}\vec{x}$$

If you know how to invert a 3×3 matrix, you can solve the problem that way. You might be required to invert a 2×2 matrix by hand on an exam in this course, but you would not be required to invert a 3×3 matrix, because it's too much work—there will always be some kind of symmetry you can take advantage of. In this case, you can take advantage of symmetry to see that

$$\begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + \frac{1}{4}\begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix}$$

Therefore

$$\vec{u} = \frac{1}{4}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + \frac{1}{4}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.75 \\ 1 \end{bmatrix}$$