UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering
ECE 417 Multimedia Signal Processing

# Lecture 5 Sample Problem Solutions

**Problem 5.1**

$$E\left[\|\vec{y}\|^2\right] = \sum_{d=1}^{N} \lambda_d^M = D\sigma^2 \sum_{d=1}^{M} \left(\frac{1}{2}\right)^d$$

which exceeds $0.95D\sigma^2$ if $M = 5$.

**Problem 5.2**

$$p_Y(\vec{y}) = \prod_{d=1}^{M} \frac{1}{\sqrt{2\pi\lambda_d}} e^{-\frac{y_d^2}{2\lambda_d}}$$

**Problem 5.3**

$$R^\dagger = [\vec{v}_1, \ldots, \vec{v}_M] \begin{bmatrix} \frac{1}{\lambda_1} & 0 & \cdots \\ 0 & \frac{1}{\lambda_2} & \cdots \\ \cdots & \cdots & \frac{1}{\lambda_M} \end{bmatrix} [\vec{v}_1, \ldots, \vec{v}_M]^T$$

**Problem 5.4**

$$\vec{\mu} = \pi_0 \vec{\mu}_0 + (1 - \pi_0)\vec{\mu}_1$$
$$R = \pi_0(1 - \pi_0)(\vec{\mu}_0 - \vec{\mu}_1)(\vec{\mu}_0 - \vec{\mu}_1)^T$$

The first orthonormal eigenvector, $\vec{v}_1$, is

$$\vec{v}_1 = \frac{\vec{\mu}_0 - \vec{\mu}_1}{\|\vec{\mu}_0 - \vec{\mu}_1\|}$$

and the corresponding eigenvalue is

$$\lambda_1 = \pi_0(1 - \pi_0)\|\vec{\mu}_0 - \vec{\mu}_1\|^2$$

The remaining eigenvalues are

$$\lambda_d = 0, \quad d > 1$$

and have eigenvectors which are orthogonal to $\vec{v}_1$.