ECE 420

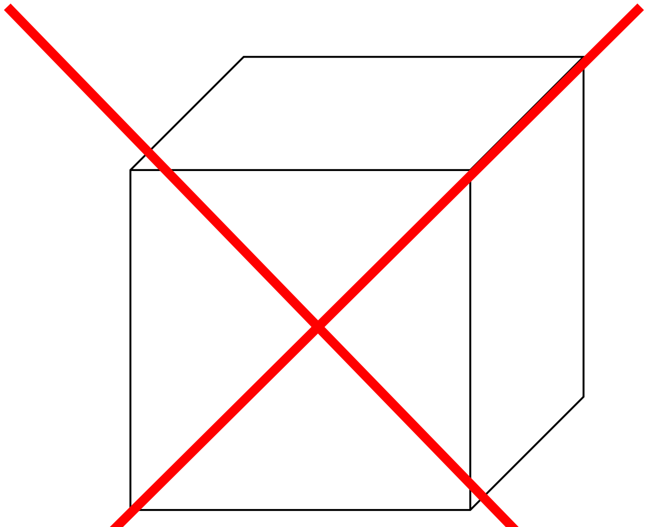
Lecture 6

Feb 25 2019

Now Entering

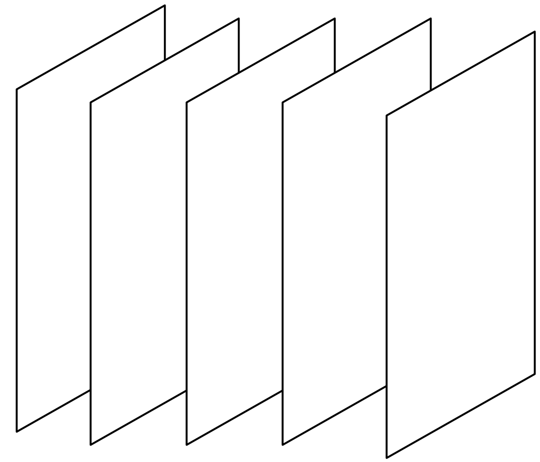
The Third Dimension!

3D Signal Processing



$f(x, y, z)$

No, not this one!



$f(x, y, t)$

This one!

Video Processing

- Not volumetric 3D signal processing, but processing of video streams
  - Set of 2D image frames
- Typical algorithms operate on a frame-by-frame basis with some state carried among frames
- Many video processing algorithms (some of these apply to still images as well):
  - Detection / recognition
  - Tracking
  - Compression
  - 3D reconstruction

Algorithm Performance

- Based on processing time per frame, we can express the performance of the algorithm in terms of frames per second
  - Very common metric in computer gaming / display systems
- Human visual system can perceive up to 1000 fps under certain circumstances
  - 13 – 20 fps: video motion becomes fairly fluid
  - 24 fps: broadcast TV / motion picture standard
  - 30 – 60 fps: gaming
  - 120+ fps: TV [with interpolation]

Algorithm Performance

- Insufficient FPS?
  - Live with it
  - Drop frames
  - Drop pixels
  - Drop frames and/or pixels and interpolate result
- Decreasing frame rate or resolution can potentially make things harder due to
  - lower temporal correlation
  - lower resolution
- Target FPS can put a significant limit on how much computation your algorithm can perform on each frame

2D DFT

$$X[k,\ell] = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} x[m,n]e^{-j2\pi(km/M+\ell n/N)}$$

Direct implementation: $O(N^4)$ [ouch!]

$$X[k,\ell] = \sum_{m=0}^{M-1} e^{-\frac{j2\pi km}{M}} \sum_{n=0}^{N-1} x[m,n]e^{-\frac{j2\pi\ell n}{N}}$$

Separable implementation: $O(N^3)$ [better!]

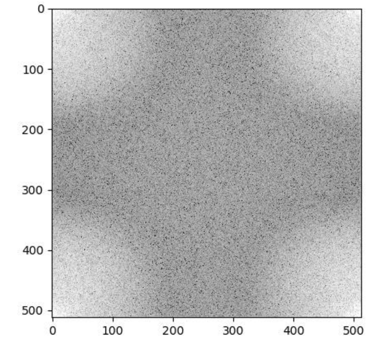
Replace direct sums with FFT

$$y[m,\ell] = F_n\{x[m,n]\}$$
$$X[k,\ell] = F_m\{y[m,\ell]\}$$

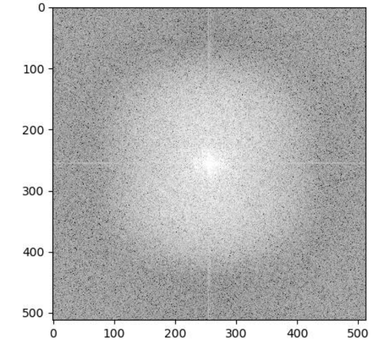
2D FFT: $O(N^2 \log N)$ [best!]

2D DFT

- 2D DFT samples span $[0, 2\pi)$ in each dimension
  - Samples are conjugate-symmetric about the origin
  - fftshift() moves the DC component to the image center for easier visualization
- Also images tend to have a *VERY* strong DC component, so some manipulation of magnitude values is necessary for visualization
  - log, sqrt, etc.
  - If your DFT looks empty, check the DC pixel!



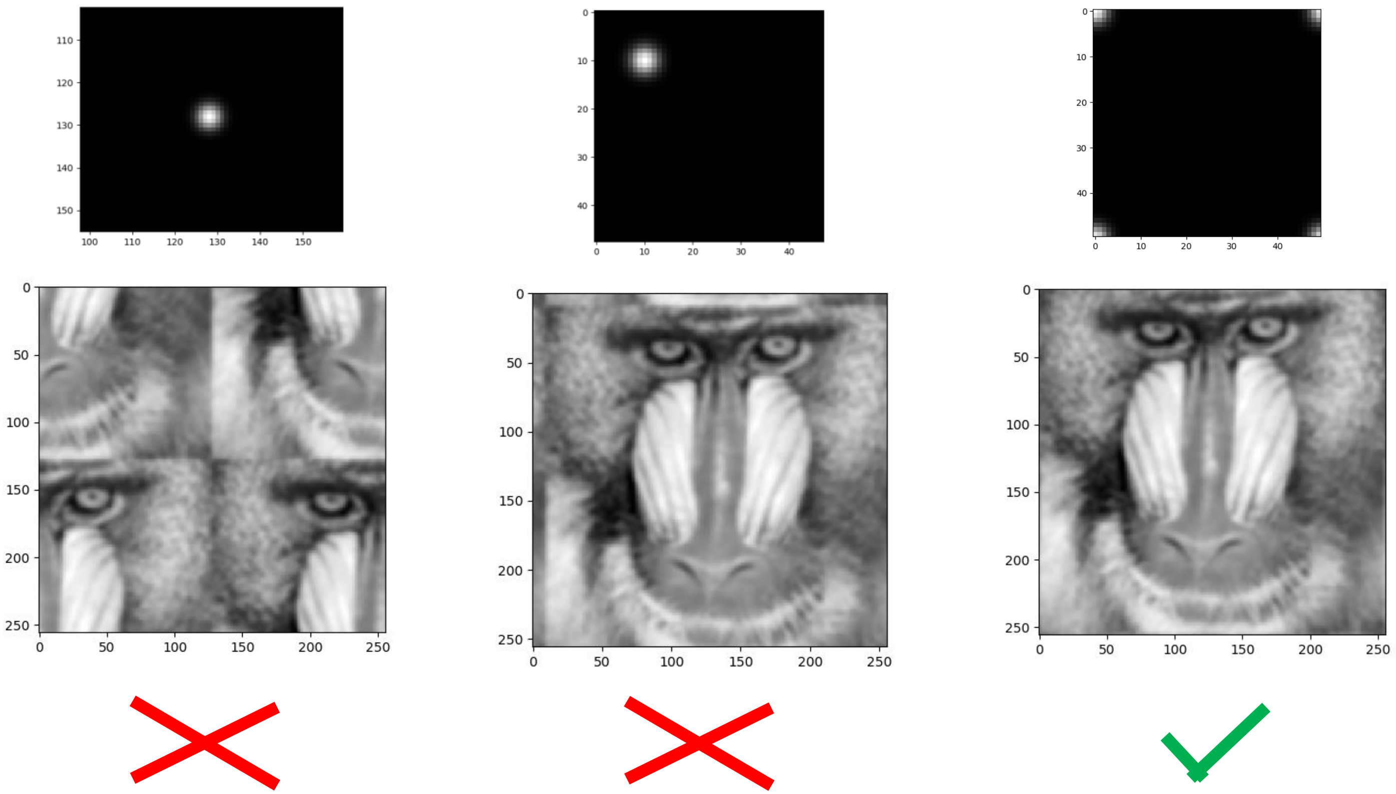
Magnitude 2D DFT



2D DFT after fftshift

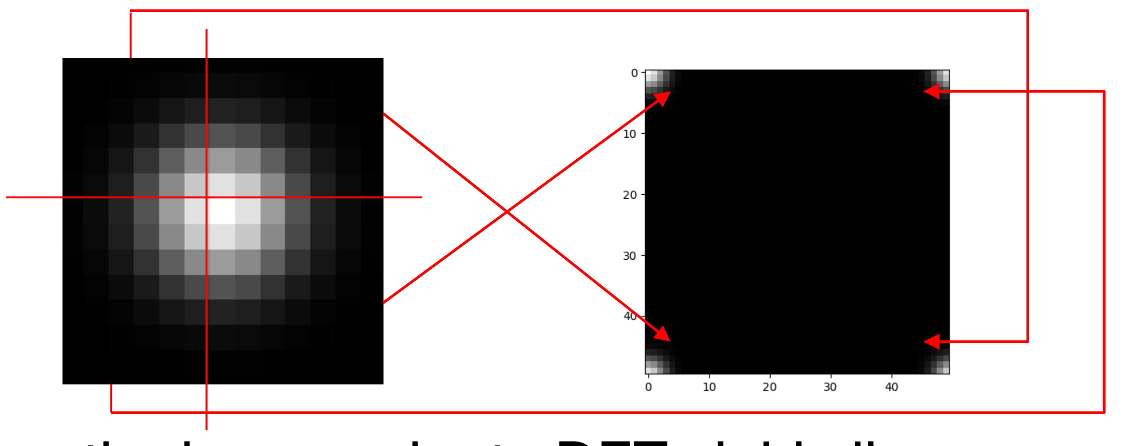
2D Convolution with DFT

- Multidimensional extension of the convolution theorem
  - $y[m,n] = x[m,n] ** h[m,n] = F_2^{-1}\{F_2\{x\}F_2\{h\}\}$
- When using the 2D DFT, we get 2D circular convolution



2D Convolution with DFT

- We want to apply a (mostly) zero phase filter $h[m, n]$
- The 'center' of h needs to be at the $[0,0]$ location
- Other patches of h wrap around
  - h is non-causal, which results in circular wrapping



- Zero padding the image prior to DFT yields linear convolution
  - Still need to rearrange h as above, or accommodate pixel shift
- Can also leverage ifftshift() to restructure h appropriately

Brief Review of Matrix Operations

- An m by n matrix has m rows and n columns
- Elements indexed as a_{ij} for element in row i and column j

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

- Input data (samples, state, etc.) represented as a column vector (m by 1 matrix)
- Higher dimensional input data (e.g. images) 'stacked' to form a 1D vector

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

- A matrix variable is usually written in bold, using lowercase ($\boldsymbol{x}$) for a column matrix and uppercase for a 'full' matrix operator ($\boldsymbol{A}$)

Brief Review of Matrix Operations

- Addition/subtraction is element wise application of operation

$$A + B = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \\ a_{31} + b_{31} & a_{32} + b_{32} \end{bmatrix}$$

- Multiplication is inner products between rows and columns of respective matrices

$$C = AB, \qquad c_{ij} = \sum_k a_{ik} b_{kj}$$

- Instead of 'division' we talk about matrix inverse A^{-1}

$$A^{-1}A = I$$

Brief Review of Matrix Operations

- Identity matrix I is 1 on the diagonal and 0 everywhere else

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- A matrix is *diagonal* if its non-zero elements are on the diagonal only

$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$$

- The inverse of a diagonal matrix is easily calculated

$$A^{-1} = \begin{bmatrix} 1/a_{11} & 0 & 0 \\ 0 & 1/a_{22} & 0 \\ 0 & 0 & 1/a_{33} \end{bmatrix}$$

Brief Review of Matrix Operations

- Matrix transpose flips elements about the diagonal

$$A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \end{bmatrix}$$

- Hermitian A^H is a matrix transpose with conjugation of each element

- Norm is defined as $\|A\| = \sqrt{\sum_{i,j} a_{ij}^2}$

- An operator is defined as linear if

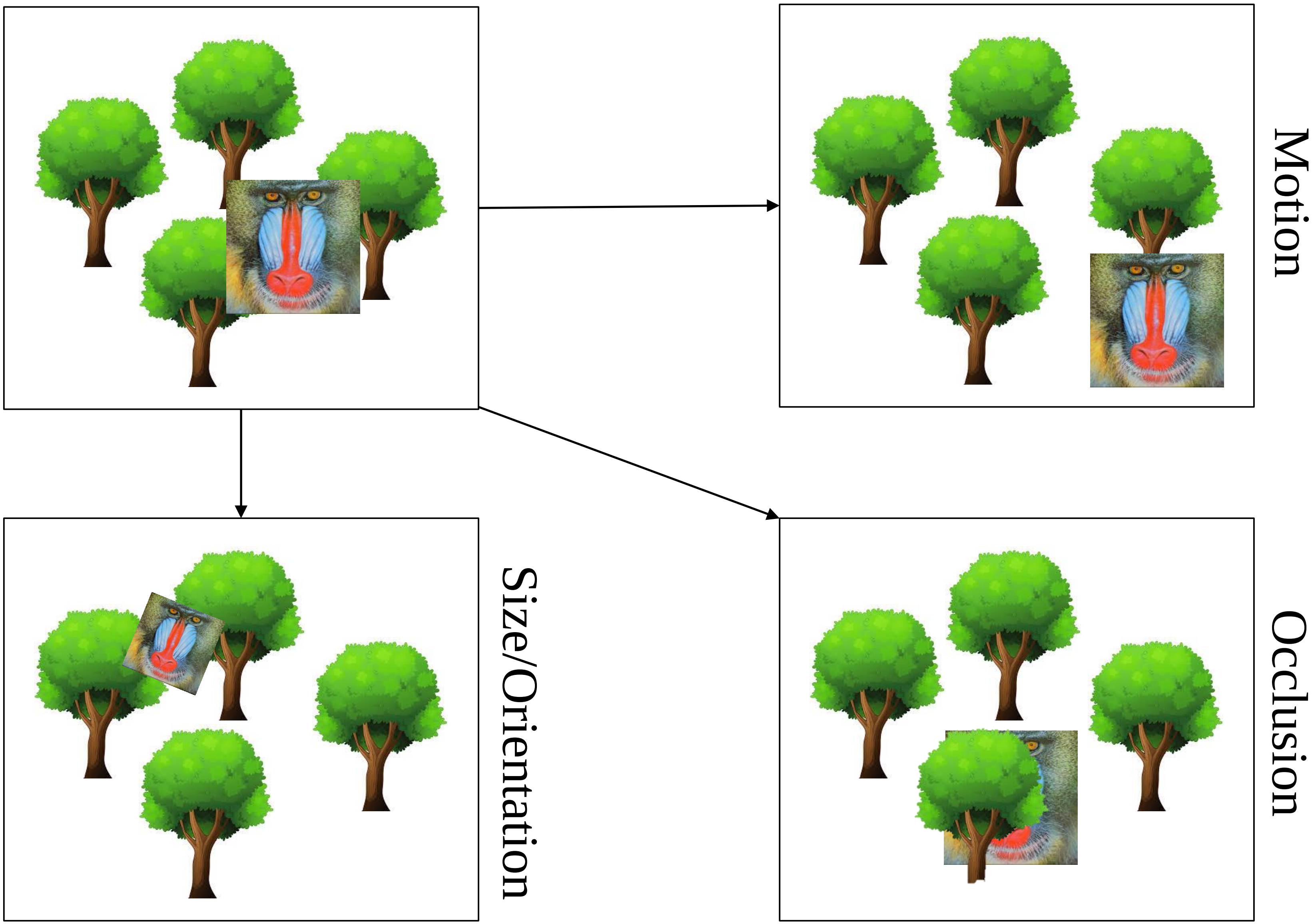
$$Ax + Ay = A(x + y), \qquad A\alpha x = \alpha Ax$$

- All linear operators can be written as a matrix!

Detection vs. Tracking

- Detection
  - Usually posed as a single-frame / image problem
  - Is there a particular object present?
    - Where is it?
    - What is it?
- Tracking
  - Given a starting location/description (seed)
  - Follow object as it traverses scene
  - May also want to estimate/report changes in “pose”
    - How is it oriented / configured?
  - Tracking will typically involve some detection

Challenges in Tracking

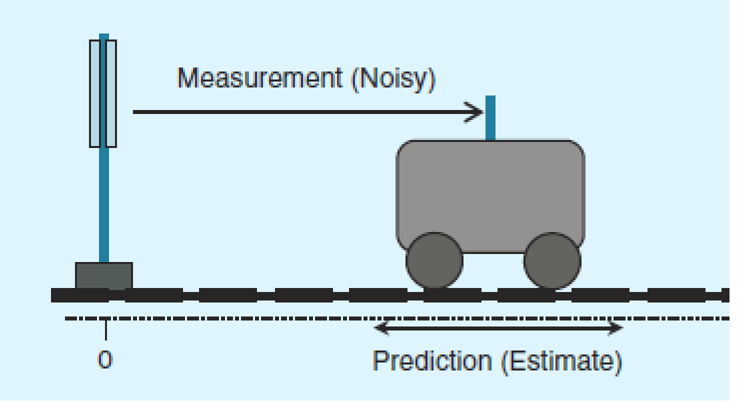


Kalman Filter

- General problem statement:
  - Given a model of the system state evolution, estimate progression of system state over time, given system measurements
- State update equation
  - $x_t = F_t x_{t-1} + B_t u_t + w_t$
  - x_t - system state vector
  - F_t - state transition matrix
  - u_t - system control vector
  - B_t - control input matrix
  - w_t - process noise (with covariance Q_t)

Kalman Filter

- State measurement
  - $z_t = H_t x_t + v_t$
  - z_t - measured data
  - H_t - measurement matrix
  - v_t - measurement noise (covariance R_t)

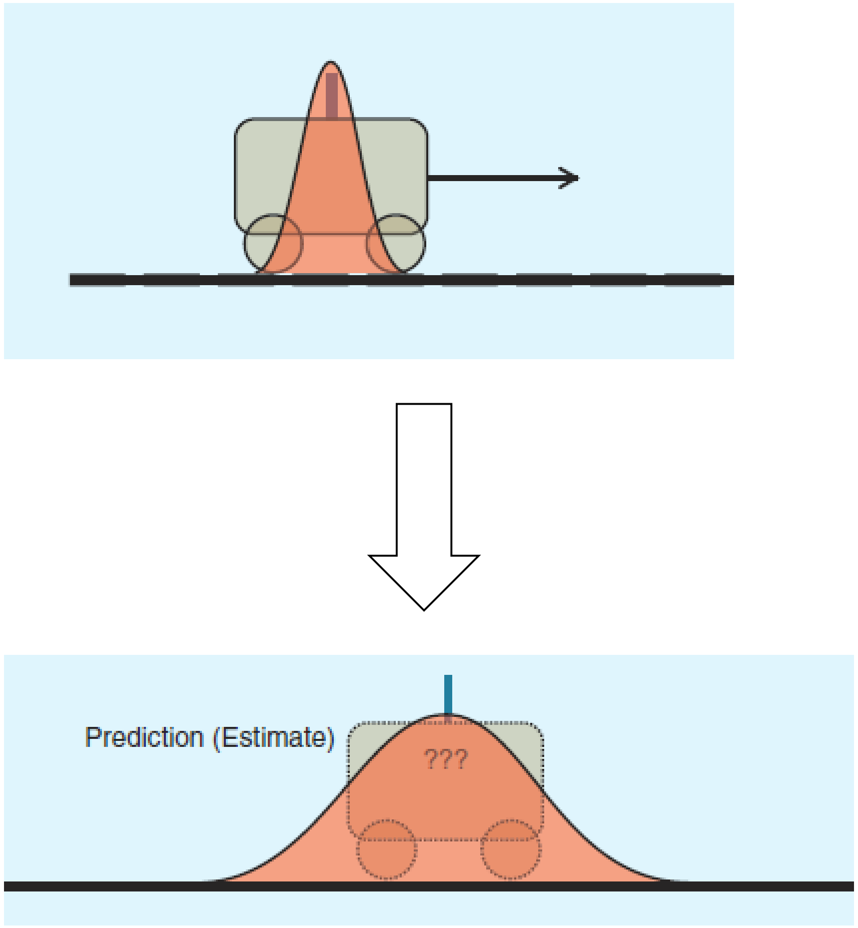


- Kalman filter algorithm has two parts
  - Prediction step
  - Measurement update step
- For notational simplicity, let $H_t = I$

Kalman Filter - Prediction

- Given past state estimate, calculate new state estimate
  - $\hat{x}_{t|t-1} = F_t \hat{x}_{t-1|t-1} + B_t u_t$
- Notation $\hat{x}_{a|b}$
  - Estimate of x at time $t = a$ given measurements up to time $t = b$
- This update propagates the estimated state forward
- Key to the Kalman filter is keeping track of the *certainty* of our estimates
  - $P_{t|t-1} = Var\left[x_t - \hat{x}_{t|t-1}\right] = F_t P_{t-1|t-1} F_t^T + Q_t$
- Note that at this point we have updated the state without any feedback from the system

Kalman Filter - Prediction



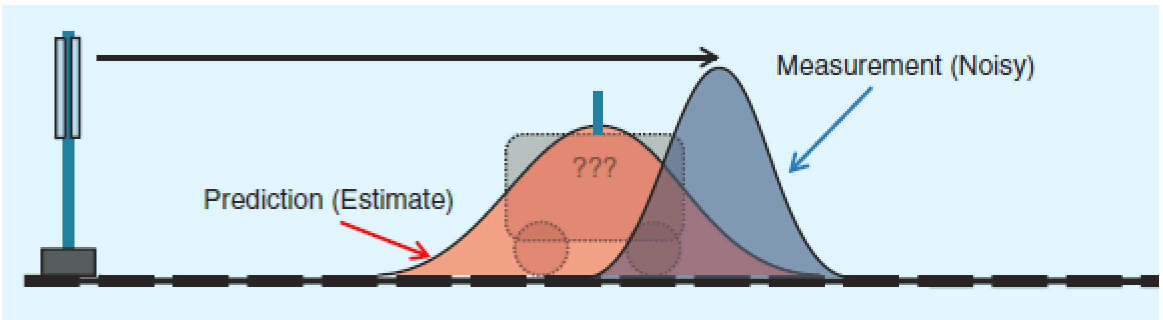
$\hat{x}_{t-1|t-1}$

$\hat{x}_{t|t-1}$

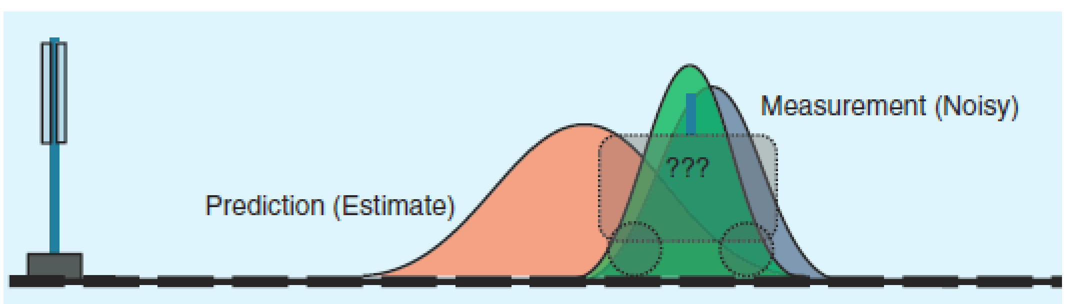
Kalman Filter - Measurement update

- Given noisy measurements, update the state estimation
  - $\hat{x}_{t|t} = \hat{x}_{t|t-1} + K_t(z_t - \hat{x}_{t|t-1})$
  - $K_t = P_{t|t-1}\left(P_{t|t-1} + R_t\right)^{-1}$
- Note that at no point in time do we assume a perfect state value
  - Every vector has an associated uncertainty with it
- Updated certainty of estimate
  - $P_{t|t} = Var\left[x_t - \hat{x}_{t|t}\right] = P_{t|t-1} - K_t P_{t|t-1}$
- How did these updates come about?

Kalman Filter - Measurement Update



$\hat{x}_{t|t-1}, z_t$



$\hat{x}_{t|t}$

Fusing Measurements

- Consider two noisy measurements x_1, x_2 with different variances σ_1^2, σ_2^2
  - How should these be ‘optimally’ combined?
- Consider a linear combination of the two measurements that minimizes the variance of the combined estimate
  - $\hat{x}_{opt} = \min_{\alpha} Var[(1-\alpha)x_1 + \alpha x_2]$
- This is achieved by ‘Kalman Gain’ K
  - $\alpha = K = \sigma_1^2/(\sigma_1^2 + \sigma_2^2)$
- Yielding
  - $\hat{x}_{opt} = x_1 + K(x_2 - x_1)$
  - $Var[\hat{x}_{opt}] = (1-K)\sigma_1^2$

Fusing Measurements - Kalman Filter

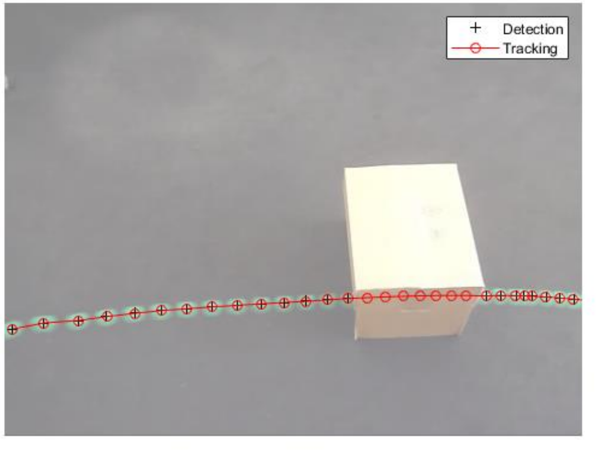
- In the Kalman Filter derivation, we want to estimate $\hat{x}_{t|t}$ given
  - $\hat{x}_{t|t-1}$, which has variance $P_{t|t-1}$
  - z_t, which has variance R_t
- Applying the ‘optimal’ fusion of these two measurements from the scalar case

Variable	Scalar Fusion	Kalman/Vector Fusion
K	$\sigma_1^2/(\sigma_1^2 + \sigma_2^2)$	$P_{t\|t-1}\left(P_{t\|t-1} + R_t\right)^{-1}$
$\hat{x}_{t\|t}$	$x_1 + K(x_2 - x_1)$	$\hat{x}_{t\|t-1} + K_t(z_t - \hat{x}_{t\|t-1})$
$P_{t\|t}$	$(1 - K)\sigma_1^2$	$(I - K_t)P_{t\|t-1}$

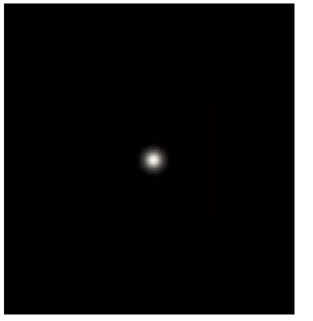
- The attractive feature of Kalman filtering is its simple, recursive form

Example of Kalman Video Tracking

- Consider tracking a ball
- Provided an initial location
- Estimate new ball location
- Check for ball near new location, update based on discrepancy
- If no ball detected, continue propagating state without measurement reinforcement



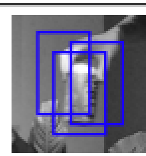
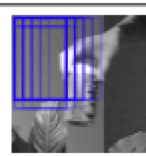
Correlation Filter Tracking



- Not correlation of image patches with each other but rather correlation with a classifier filter
- In a *training phase* a target image/patch is provided which is used to construct the classifier filter
  - The filter is designed so that its response to the training image is similar to a predefined regression target image (e.g a Gaussian)
- In the *tracking phase* applies the classifier filter to patches in the image
  - Large responses = high correlation = the object we are looking for!

Correlation Filter Tracking

- Selecting which sections of the image to test can be tricky
  - Correlation evaluation can be costly per patch
  - Insufficient patch coverage leads to loss of tracking performance
- Test all the patches using the DFT / convolution
  - Apply a window to attenuate circular wrapping effects
- Look for maximum response and update classifier filter
- FFT implementation allows for very efficient tracking algorithm

	Storage	Bottleneck	Speed
Random Sampling (p random subwindows) 	Features from p subwindows	Learning algorithm (Struct. SVM [4], Boost [3, 6]...)	10 - 25 FPS
Dense Sampling (all subwindows, proposed method) 	Features from one image	Fast Fourier Transform	320 FPS

OpenCV

- Open Source Computer Vision Library
- Implements main different computer vision algorithms with focus on real-time applications
    - Can leverage multiple cores, hardware accelerators
- Among other areas has support for facial and gesture recognition, object identification, segmentation, motion tracking, machine learning, image filtering and transforms, drawing
- C++, Python and Java Interfaces
- Active community with continual contributions
- Goal is not to reinvent the wheel

Lab 7

- Video Processing
- Utilize KCF to track an object of interest
  - Identified at start of algorithm’s execution by user
- Leverages OpenCV to do the heavy lifting

Assigned Project Lab Proposals

- Due March 4, 2PM
- Expectations for proposal:
  - Overview of the algorithm to be implemented, including citation of sources.
  - Plan for testing and validation of the algorithm's implementation.
  - Rough idea(s) for Final Project applications of the algorithm.
- Feedback to be provided prior to starting on Assigned Project Lab
  - The earlier the proposal is submitted, the sooner it can be returned and the more time you have to adjust based on feedback

Outline of Rest of Semester

- 3/17: Final Project Proposals Due
- Week of 3/25: Final Project Proposal Presentation + Assigned Lab Demo
- Week of 4/22: Final Project Demo
- 4/29: In-class Final Lecture Cumulative Quiz
- 5/3: Final Project Report and optional Video Due

This week

- Lab 6: Image Processor Quiz/Demo
- Lab 7: Video Tracker
- Assigned Project Lab Proposals Due March 4
  - Sign up groups as soon as you have them worked out
  - Submission of proposals on Compass (available soon)