

2/5/2025

$$x_i = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad y_i = 3 \quad w = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad b = 0$$

$$\tilde{x}_i = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad \tilde{w} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \eta = 0.01$$

$$f(x) = \tilde{w}^T \tilde{x} = 0 \quad = w^T x + b$$

$$E_i = f(x_i) - y_i = -3$$

$$\frac{\partial f}{\partial w} = E_i x_i = -3 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$w \leftarrow w - \eta \frac{\partial f}{\partial w} = w - \eta E_i x_i$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - (0.01)(-3) \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.03 \\ 0.06 \\ 0.03 \end{bmatrix}$$

After update

$$f(x_i) = w^T x_i + b = \tilde{w}^T \tilde{x}_i$$
$$= [0.03 \quad 0.06 \quad 0.03] \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= 0.03 + 0.12 + 0.03 = 0.18$$

$$E_i = f(x_i) - y_i = 0.18 - 3 = -2.82$$