

2025 Feb 10

$$\mathbf{v}_c = [1, 0, 0, \dots]^T$$

$$\mathbf{v}_t = [0, 1, 0, \dots]^T$$

$$\mathbf{v}_0 = [0, 0, 1, 0, \dots]^T$$

$$\mathbf{v}_h = [0, 0, 0, 1, 0, \dots]^T$$

$$\mathbf{v}_s = [0, 0, 0, 0, 1, 0, \dots]^T$$

$$\mathbf{v}_c^T \mathbf{v}_t = 0, \frac{1}{1+e} = \frac{1}{2}$$

$$\mathcal{L} = \sum_{t \in \{1, 1\}} -\ln \sigma(\mathbf{v}_c^T \mathbf{v}_{c+t}) + \frac{1}{N} \sum_{u \in V} -\ln(1 - \sigma(\mathbf{v}_c^T \mathbf{v}_u))$$

$$\frac{d \ln \sigma}{d \mathbf{v}_c} = (1 - \sigma) \mathbf{v}_{c+t}$$

$$\frac{d \ln(1 - \sigma)}{d \mathbf{v}_c} = -\sigma \mathbf{v}_i$$

$$\frac{d \mathcal{L}}{d \mathbf{v}_c} = -(1 - \sigma) \mathbf{v}_h - (1 - \sigma) \mathbf{v}_s + \frac{1}{N} \sum_{i=1}^N \sigma(\mathbf{v}_c^T \mathbf{v}_i) \mathbf{v}_i$$

$$= -(1 - \sigma) \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} - (1 - \sigma) \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \frac{1}{N} \sigma \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$\uparrow$ hot $\uparrow$ smells $\uparrow$ average

$$= -\frac{1}{2} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \frac{1}{2N} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$