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Policy Learning CS440/ECE448

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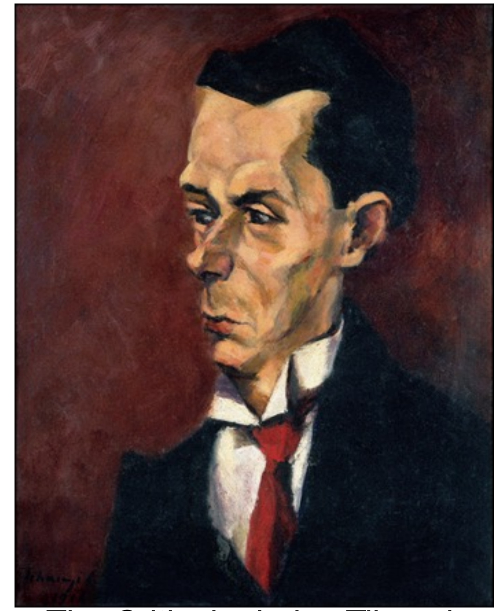

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Outline

- The prerequisite for policy learning: Stochastic policy
- Imitation learning
- Actor-critic
- REINFORCE

What should RL learn?

So far, we've been discussing $P(s'|s,a)$ and $q(s,a)$, because Bellman proved that the optimum policy can be computed by solving

$$u(s) = r(s) + \gamma \max_{a\in\mathcal{A}} \sum_{s'} P(s'|s,a)u(s')$$

…but the solution to a Markov decision process is not $P(s'|s,a)$ or $q(s,a)$. The solution is a policy, $a = \pi(s)$, that tells us what action to perform in state s.

Why not just learn $\pi(s)$ directly?

Stochastic policy

We can't just learn $a = \pi(s)$ directly, because it's not a differentiable function, so gradient descent and most other methods don't work.

To make it learnable, let's define a stochastic policy:

$$\pi_a(s) = P(a \text{ is the action the agent will perform} | \text{state } s)$$

…now, instead of always choosing the same action in state s, the agent will, instead, choose action a with probability $\pi_a(s)$.

Examples of stochastic policies

- Neural network, with hidden features $\boldsymbol{h}$:

$$\pi_a(s) = \frac{\exp(\boldsymbol{w}_a^T \boldsymbol{h})}{\sum_{k \in \mathcal{A}} \exp(\boldsymbol{w}_k^T \boldsymbol{h})}$$

- Epsilon-greedy exploration policy:

$$\pi_a(s) = \begin{cases} (1-\epsilon) + \dfrac{\epsilon}{|\mathcal{A}|} & a = \operatorname{argmax} q(s,a) \\ \dfrac{\epsilon}{|\mathcal{A}|} & \text{otherwise} \end{cases}$$

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How do we learn a stochastic policy?

Suppose we compute the probabilities $\pi_a(s)$ using a neural network with hidden features $\boldsymbol{h}$:

$$\pi_a(s) = \frac{\exp(\boldsymbol{w}_a^T \boldsymbol{h})}{\sum_{k \in \mathcal{A}} \exp(\boldsymbol{w}_k^T \boldsymbol{h})}$$

How should we learn the vectors $\boldsymbol{w}_k$?

Imitation learning

- Notice: This is like a classification problem!
  - Input: state s
  - Output: action a
- Solution**:** Learn to imitate a training database
  - Database consists of paired examples produced by a teacher doing the same task, e.g., a human:
$$\mathcal{D} = \{(s_1, a_1), \cdots, (s_n, a_n)\}$$
  - Gradient descent is used to minimize
$$\mathcal{L} = -\log P(\mathcal{D}) = -\sum_{t=1}^{n} \log P(a_t | s_t) = -\sum_{t=1}^{n} \log \pi_{a_t}(s_t)$$

Imitation learning

- $\pi_a(s)$ is learned in order to imitate a training database:

$$\mathcal{L} = -\log P(\mathcal{D}) = -\sum_{t=1}^{n} \log P(a_t|s_t) = -\sum_{t=1}^{n} \log \pi_{a_t}(s_t)$$

- Advantages: Usually converges quickly
- Disadvantages: Only learns what the teacher knows

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Review: Q-learning

$q(s,a)$ is the expected sum of all future rewards if we perform action a in state s :

$$q(s,a) = r(s) + \gamma \sum_{s'} P(s'|s,a)u(s')$$

$u(s)$ is the expected sum of all future rewards if we are in state s :

$$u(s) = \max_{a \in \mathcal{A}} q(s,a)$$

The Actor-Critic Algorithm

- Deep Q-learning gives us a network $q(s, a)$ which is very noisy, so we don't really want to trust it
- A policy network can directly estimate $\pi_a(s)$. But how do we train it, unless we imitate human behavior?

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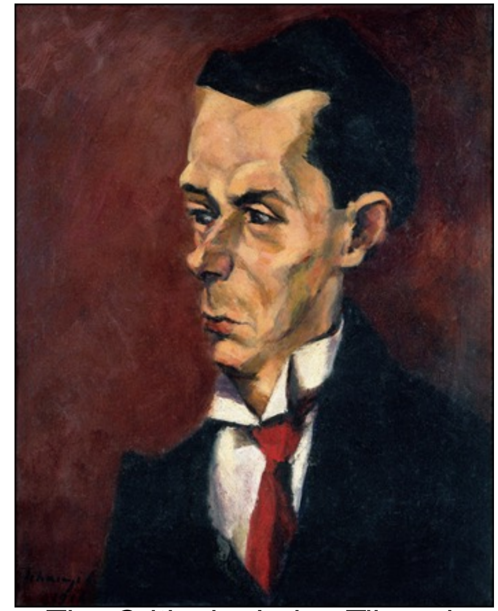
Actor-critic algorithm

Answer: train two neural nets!

- $q(s,a)$ is the **critic**, and is trained according to the deep Q-learning algorithm.
- $\pi_a(s)$ is the **actor**, and is trained to satisfy the critic



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The Actor-Critic Algorithm

Main idea:

- The **actor** decides what action to perform:

  $\pi_a(s) =$ Probability of doing action a in state s

- The **critic** is a deep Q-learning network that estimates the quality of that action:

  $q(s, a) =$ Expected sum of future rewards if (s, a)

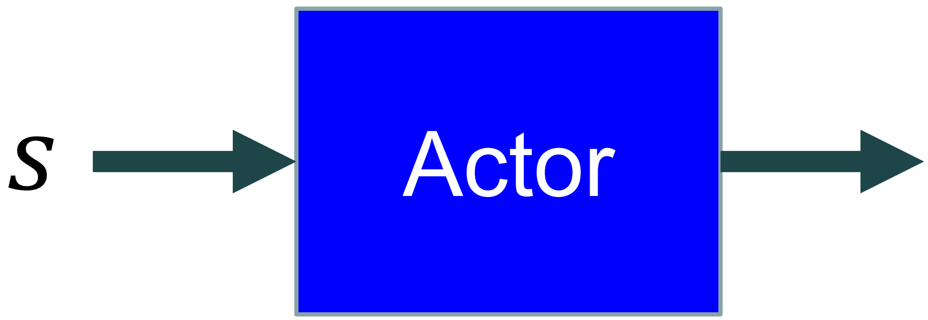
How to train the actor

The actor, $\pi_a(s)$, is trained to maximize the sum of all future rewards:

$$\mathcal{L} = -E\left[\begin{matrix}\text{sum of}\\ \text{all future}\\ \text{rewards}\end{matrix}\middle|\begin{matrix}\text{state}\\ s\end{matrix}\right] = -\sum_a P\left(\begin{matrix}\text{action}\\ a\end{matrix}\middle|\begin{matrix}\text{state}\\ s\end{matrix}\right) E\left[\begin{matrix}\text{sum of}\\ \text{all future}\\ \text{rewards}\end{matrix}\middle|\begin{matrix}\text{state } s,\\ \text{action } a\end{matrix}\right]$$

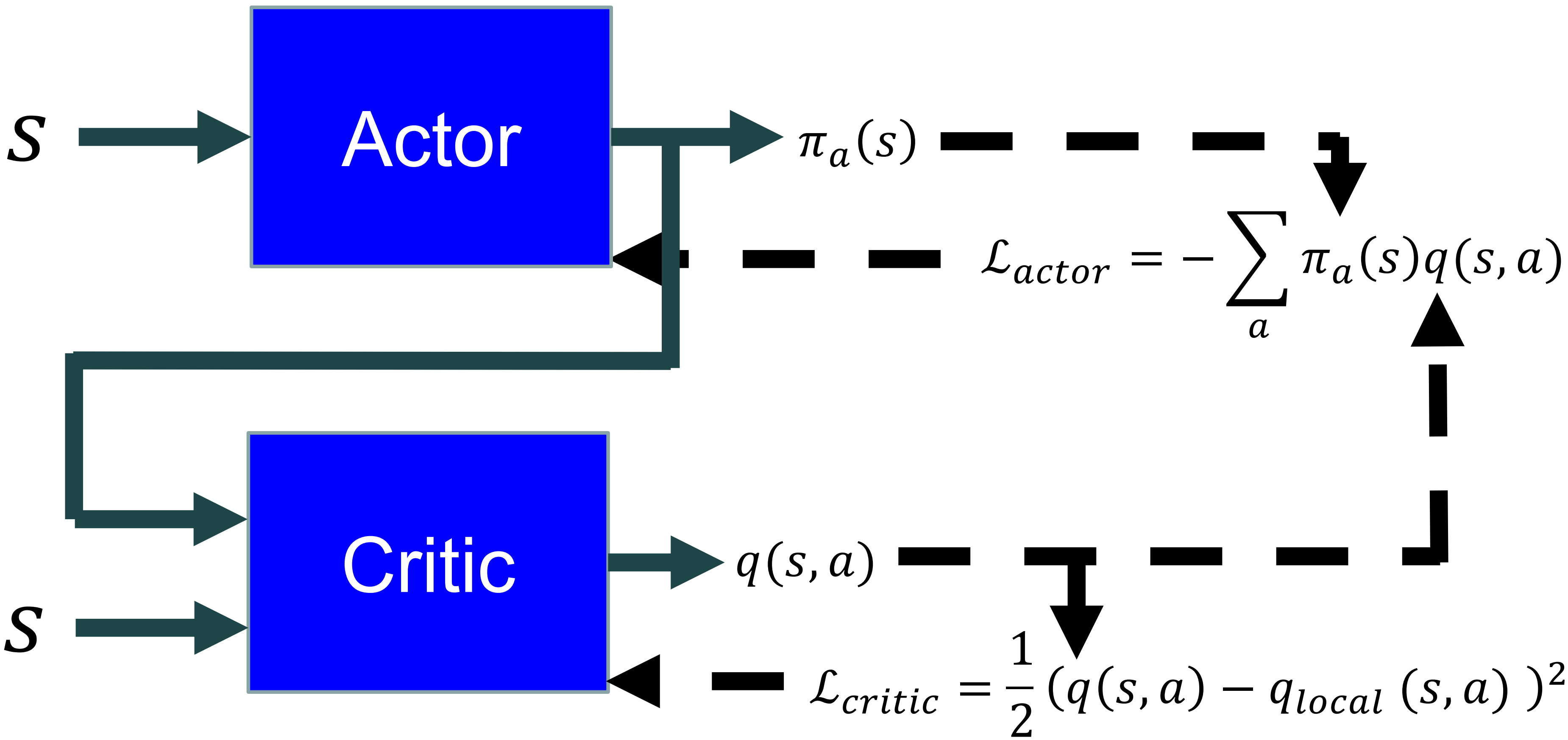
$$= -\sum_a \pi_a(s) q(s,a)$$

The Actor-Critic Algorithm: Forward-Prop



$$\pi_a(s) = P(a|s)$$

The Actor-Critic Algorithm: Back-Prop



Quiz

Try the quiz!

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Why don’t we do the obvious thing?

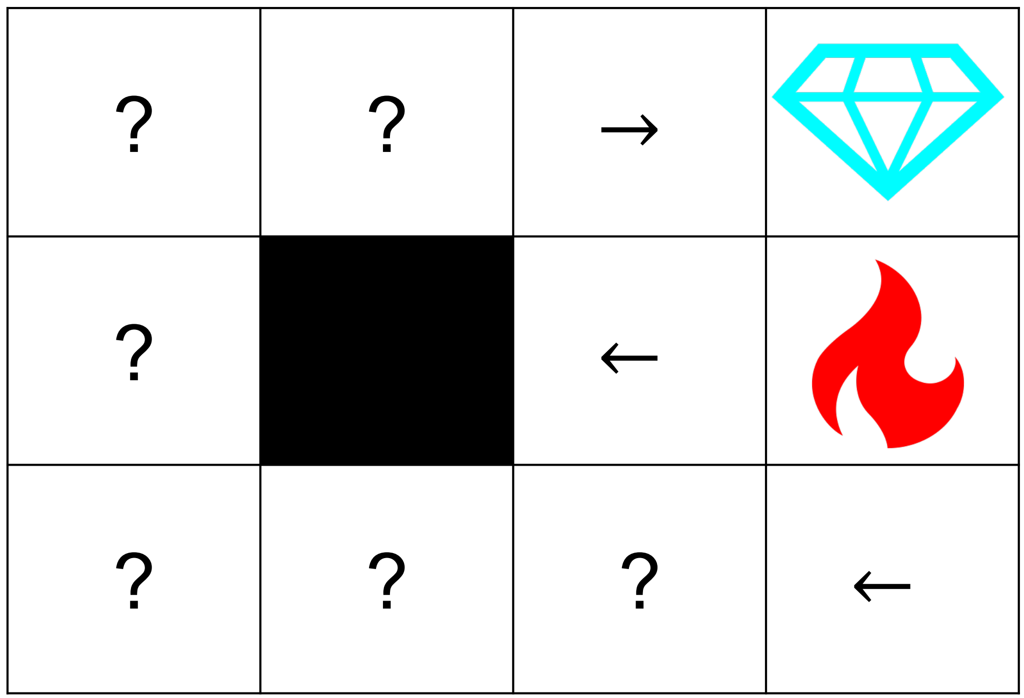
Why don’t we just use the following algorithm?

- Choose action a with probability $\pi_a(s)$
- If we get a positive reward, then increase $\pi_a(s)$

Let’s call this “learning a policy to maximize instant gratification”

The problem with instant gratification…

…is that it doesn't learn about rewards that take time. For example, in gridworld, it learns the following policy:



Episodes: A cure for instant gratification

- Start the agent in state s_1 at time 1, and for $t = 1$ to T:
  - Compute the probabilities $\pi_a(s_t)$
  - Choose action a_t with probability $\pi_{a_t}(s_t)$
  - Observe the resulting state s_{t+1}
- Store the episode

$$\{(s_1, a_1), \cdots, (s_T, a_T), r\}$$

…where r is the total of all rewards accumulated.

REINFORCE: A cure for instant gratification

Suppose that the neural net is parameterized by a matrix $\boldsymbol{W}$ whose elements are w_{ij}. We want to adjust $\boldsymbol{W}$ to maximize r, i.e., we want

$$\boldsymbol{W} \leftarrow \boldsymbol{W} + \eta \frac{\partial r}{\partial \boldsymbol{W}}$$

Unfortunately, we don't know $\frac{\partial r}{\partial \boldsymbol{W}}$. But Williams showed that, for any arbitrary constant b, $\frac{\partial r}{\partial \boldsymbol{W}}$ is in roughly the same direction as:

$$\Delta w_{ij} = (r - b) \sum_{t=1}^{T} \frac{\partial \log \pi_{a_t}(s_t)}{\partial w_{ij}},$$

REINFORCE: A cure for instant gratification

$$\boldsymbol{W} \leftarrow \boldsymbol{W} + \Delta \boldsymbol{W}, \qquad \Delta w_{ij} = \eta(r - b) \sum_{t=1}^{T} \frac{\partial \log \pi_{a_t}(s_t)}{\partial w_{ij}}$$

REward Increment=Nonnegative Factor×Offset Reinforcement×Characteristic Eligibility
REINFORCE

That's a complicated formula. But unlike $\frac{\partial r}{\partial \boldsymbol{W}}$, it consists entirely of things we know. Given the record of one episode, $\{(s_1, a_1), \cdots, (s_T, a_T), r\}$, all the terms in Δw_{ij} can be computed.

What's the constant for?

$$\boldsymbol{W} \leftarrow \boldsymbol{W} + \Delta\boldsymbol{W}, \qquad \Delta w_{ij} = \eta(r-b)\sum_{t=1}^{T}\frac{\partial \log \pi_{a_t}(s_t)}{\partial w_{ij}}$$

Usually, REINFORCE is implemented in a minibatch of four or five episodes. The constant b is set equal to the minibatch-average of r, so that:

- If an episode's r is better than average, w_{ij} is adjusted to increase $\pi_{a_t}(s_t)$ for its actions
- If an episode's r is worse than average, w_{ij} is adjusted to decrease $\pi_{a_t}(s_t)$ for its actions

Modern Policy Learning Algorithms

$$\boldsymbol{W} \leftarrow \boldsymbol{W} + \Delta \boldsymbol{W}, \qquad \Delta w_{ij} = \eta(r - b) \sum_{t=1}^{T} \frac{\partial \log \pi_{a_t}(s_t)}{\partial w_{ij}}$$

- The general idea of REINFORCE is that we rollout a complete episode, find its total reward, then adjust $\boldsymbol{W}$ according to (reward × derivative of the log probability of the episode's actions).
- This general framework is used by all modern RL, including:
  – PPO: Proximal Policy Optimization
  – DPO: Direct Policy Optimization

Summary

- Policy learning

$$\pi_a(s) = P(a \text{ is the action the agent will perform} | \text{state } s)$$

- Imitation learning: Given a database of teacher actions,

$$\mathcal{L} = -\log P(\mathcal{D}) = -\sum_{t=1}^{n} \log \pi_{a_t}(s_t)$$

- Actor-Critic:

$$\mathcal{L}_{critic} = \frac{1}{2}\left(q(s,a) - q_{local}\,(s,a)\,\right)^2, \qquad \mathcal{L}_{actor} = -\sum_{a} \pi_a(s) q(s,a)$$

- REINFORCE: Given a stored episode $\{(s_1, a_1), \cdots, (s_T, a_T), r\}$,

$$\boldsymbol{W} \leftarrow \boldsymbol{W} + \Delta\boldsymbol{W}, \qquad \Delta w_{ij} = \eta(r-b) \sum_{t=1}^{T} \frac{\partial \log \pi_{a_t}(s_t)}{\partial w_{ij}}$$