Power Electronics

Day 7 – PWM inverters and rectifiers; ac-ac conversion; discontinuous modes

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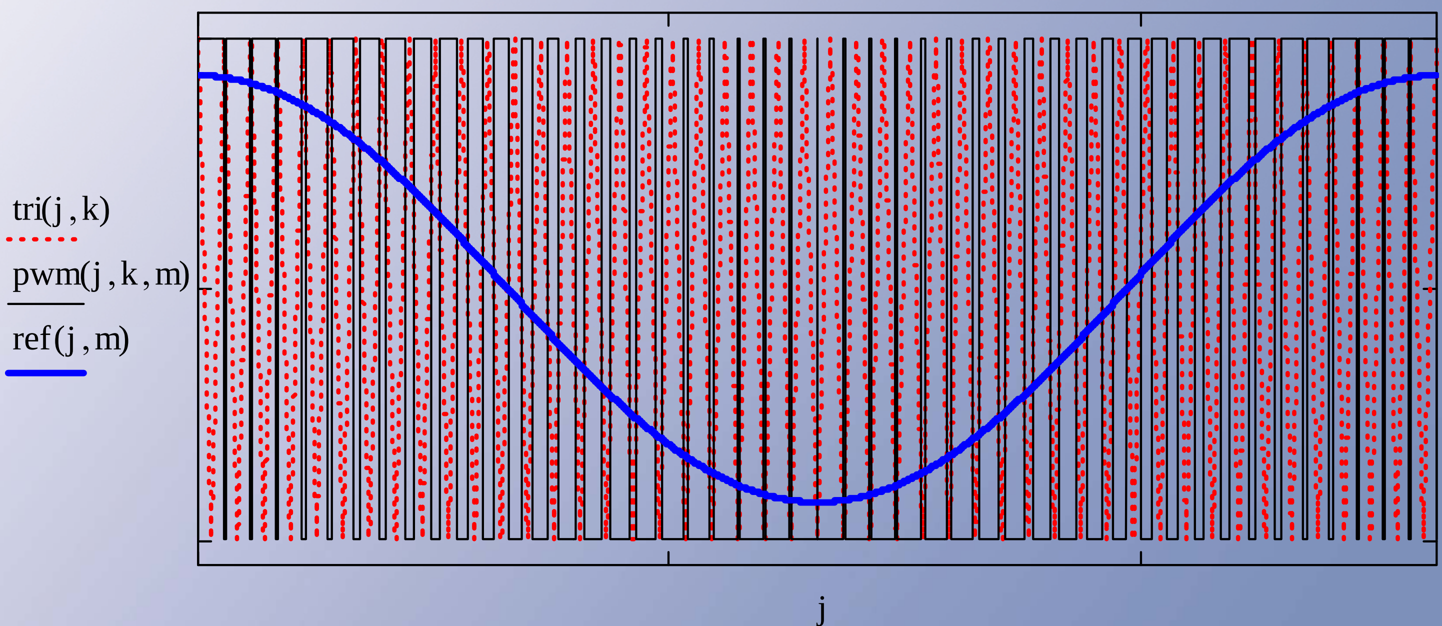
Duty Ratio -- Modulation

- Modulation

  In this case, we should be able to vary the duty ratio slowly.

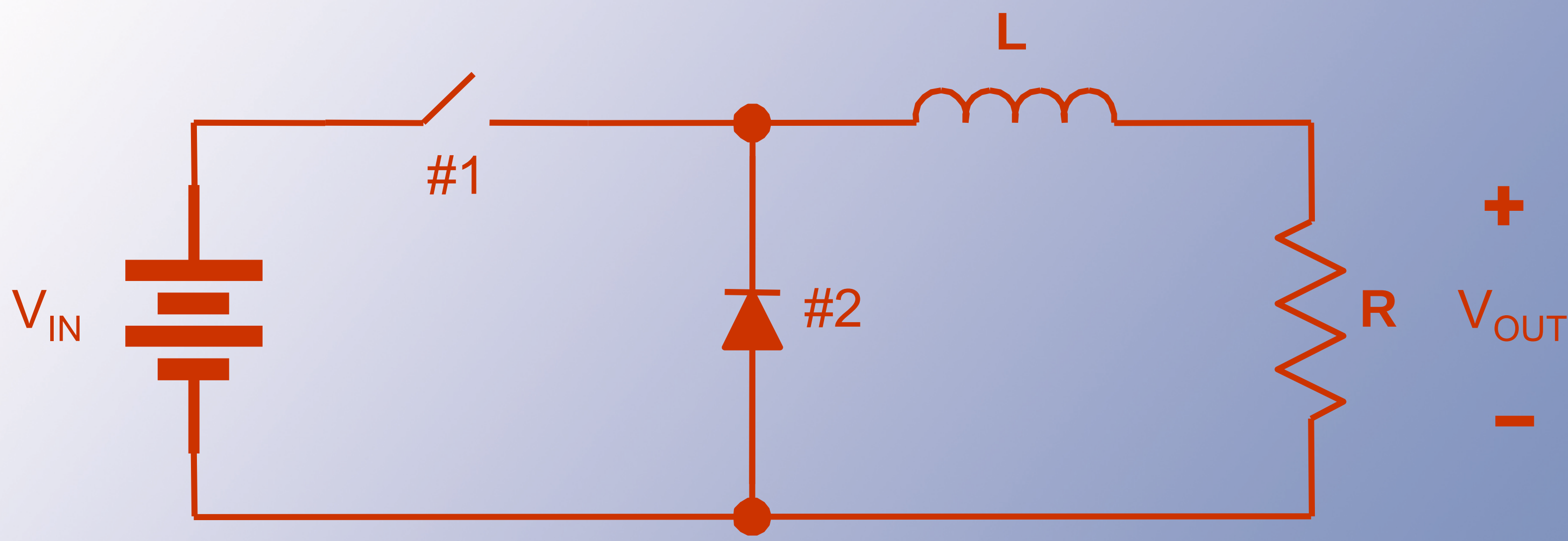
- This is PWM.



How Does it Work?

- Imagine a buck converter, switching at 200 kHz, with 1% ripple.
- If we slowly adjust D, the average output is $V_{out} = D\ v_{in}$.
- What if D is a 1 Hz waveform, like $D = 0.5 + 0.1 \cos(2\pi t)$?

How Does it Work?



f_{switch} at 200 kHz

Choose L for $\pm$1% ripple

Adjust D at 1 Hz

PWM

- Then we expect the output to be very close to DV_{in}.
- Now, vary D more generally:
  $d(t) = 0.5 + 0.5\ k\ m(t)$
- k is a constant between 0 and 1.
- m(t) is any time waveform between -1 and +1.

PWM

$D(t) = 0.5 + 0.5\ k\ m(t)$

k is "gain", $0 \le k \le 1$

m(t) is an arbitrary time function between –1 and +1

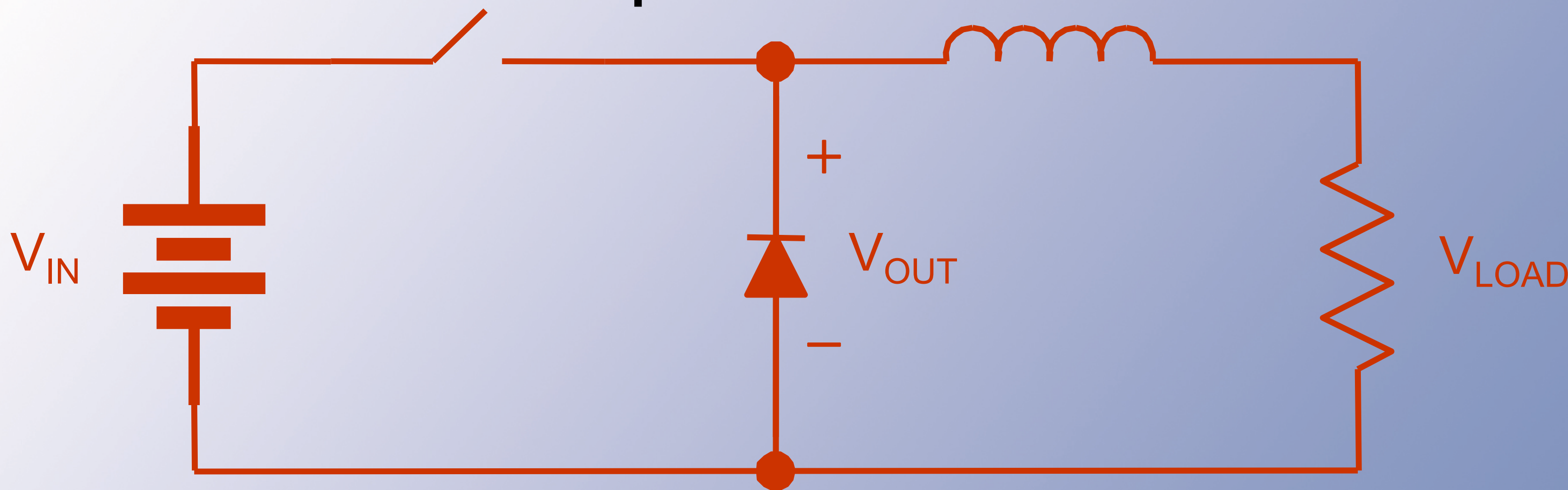
$V_{out}(t) = d(t)\ V_{in}$

This is a *moving average*

Output Series

- We can write the actual switch matrix output, $v_{out} = q(t)\ V_{in}$.
- This gives a useful Fourier series IF the frequencies in m(t) are well below the switching frequency.

Output Series



$$v_{OUT}(t) = q_1(t) v_{IN}$$

$$v_{OUT}(t) = [d_1 + \frac{2}{\pi} \sum \frac{\sin(n\pi d_1)}{n} \cos(n\omega_{SWITCH} t)] V_{IN}$$

$$v_{OUT}(t) = d_1 V_{IN} + \frac{2v_{in}}{\pi} \sum \frac{\sin(n\pi d_1)}{n} \cos(n\omega_{SWITCH} t)$$

Let $d_1 = 0.5 + 0.5\ k\ m(t)$

Output Series

$$v_{OUT}(t) = 0.5V_{IN} + 0.5k\,m(t)V_{IN}$$

$$+\frac{2V_{in}}{\pi}\sum \frac{\sin[n\pi \frac{1}{2} + \frac{1}{2}k\,m(t)]}{n}\cos(n\omega_{SWITCH}t)$$

- This is not in the form of a Fourier Series, since there is a term sin[m(t)].

Output Series

If $m(t) = \cos(\omega_{OUT} t)$,

Then terms are

$$\sin\left({}^{n\pi}/_{2}\, k \cos(\omega_{OUT} t)\right) \cos(n\omega_{SWITCH} t)$$

Bessel functions provide a way to reduce it

$$\sin(a \cos(\omega_{out} t)) \rightarrow \sum \pm 2 J_m(a) \cos(m\omega_{out} t)$$

(for odd values of m)

Output Series

- This means the first part becomes a set of terms in multiples of the output frequency.
- Now, we have terms like
  ( ) cos(m ω_{out} t) cos(n ω_{switch} t)
- Trig identities give terms
  ( ) cos[(n ω_{switch} ± m ω_{out})t]

Output Series

The Fourier terms include:

$$\text{dc} + {}^{kV_{in}}/_{2} \cos(\omega_{OUT} t)$$

$$+ {}^{2V_{in}}/_{\pi} \sum (\ ) \cos\left[\left(n\omega_{SWITCH} \pm m\omega_{OUT}\right)t\right]$$

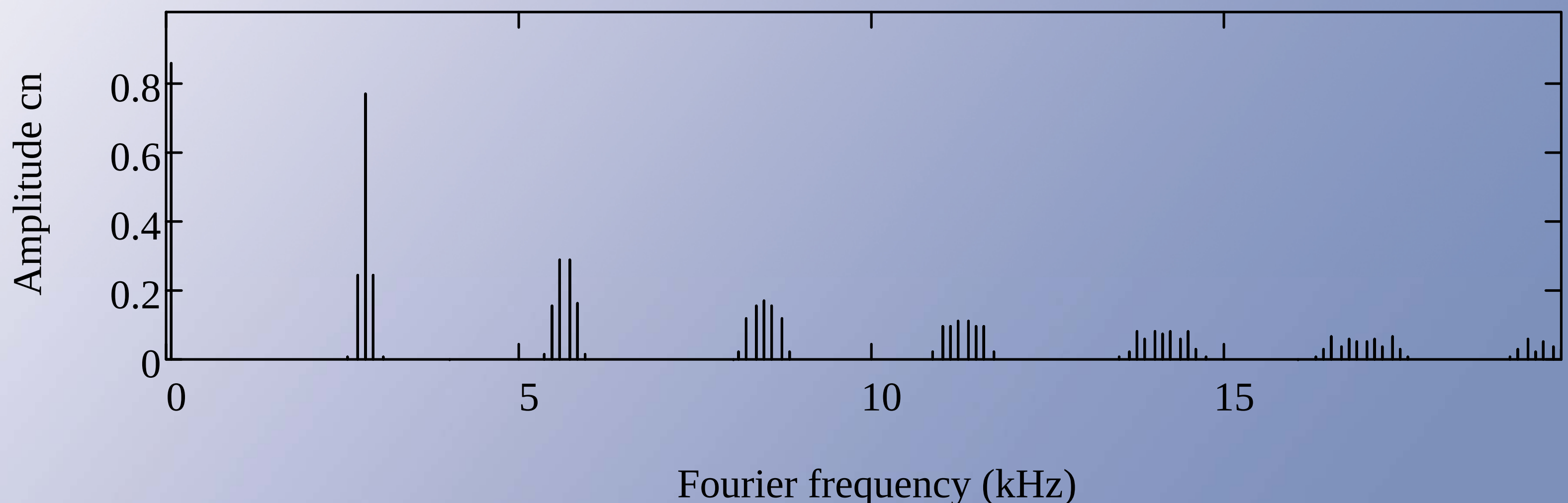
If $\omega_{switch} >> \omega_{out}$, we can filter out the series (low-pass), and are left with dc and $kV_{in}/2 \cos(\omega_{out} t)$

Output Series

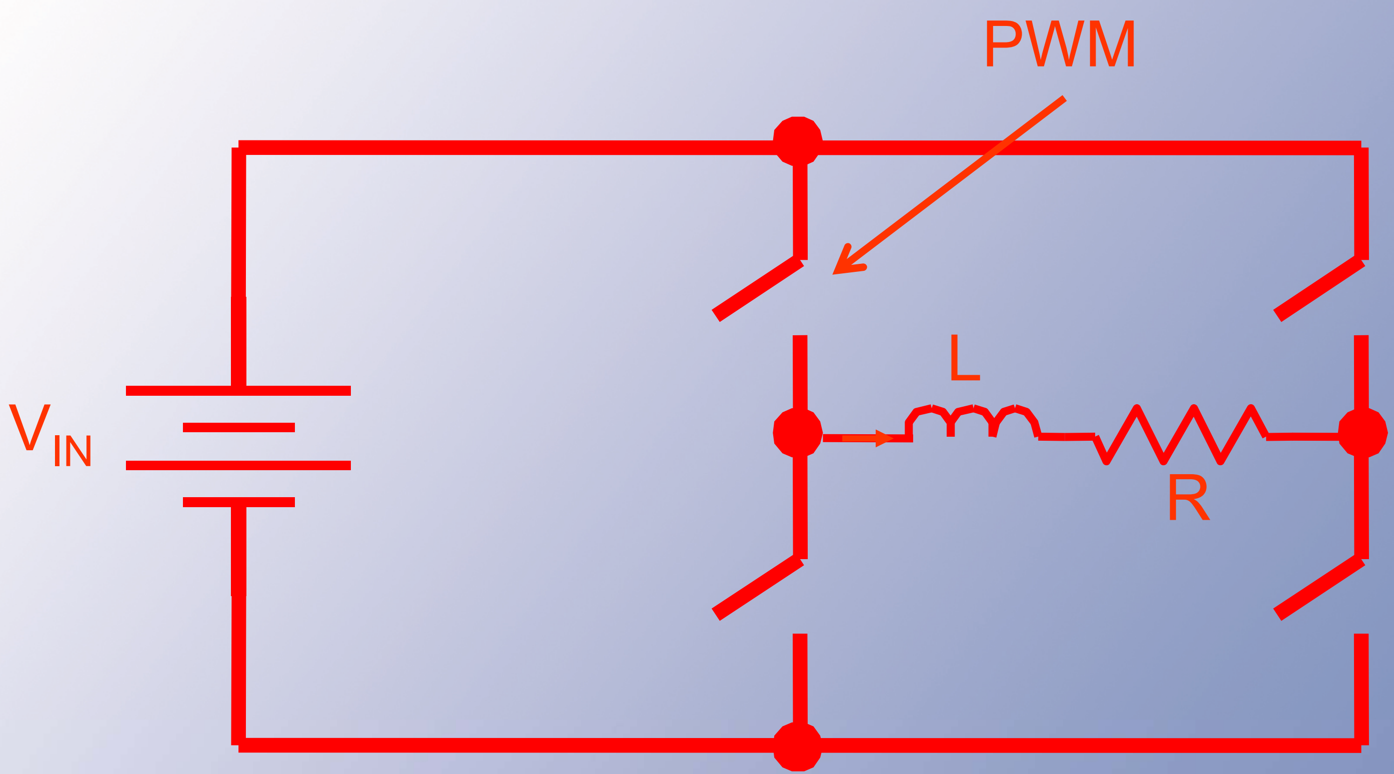
- Example:
- Switch at 200 kHz
  m(t) at 60 Hz
- We get dc, then 60 Hz, then
  n 200 kHz $\pm$ n 60 Hz
- Summary: 0 Hz, 60 Hz, 199940 Hz, 200060 Hz, 199880 Hz, 200120 Hz, etc.
- Wide separation → easy filtering

Inverter

- m(t) → "Modulating function" with ω_m.
- Switch much faster than $\omega_{m.}$
- Example, 60 Hz modulation, 2820 Hz switching.



Inverter



$I_{OUT}(t)$?

Inverter

- For two-level PWM, use $\pm V_{in}$.
- This requires $q_{11} = q_{22}$, so now $v_{out} = (2q_{11} - 1)\, V_{in}$.
- Let $d_{11}(t) = 0.5 + 0.5\, k \cos(\omega_{out} t)$.
- Now $v_{out} = kV_{in} \cos(\omega_{OUT} t)$ $+ \, 2V_{in}/\pi \sum (\ ) \cos\left[\left(n\omega_{SWITCH} \pm m\omega_{OUT}\right)t\right]$
- No dc. Low pass filter to get $kV_{in} \cos(\omega_{OUT} t)$

Components

- The unwanted components are near multiples of the switching frequency.
- Filtering involves a simple low-pass operation.
- Fast switching = high quality.

How to Create PWM?

- Presumably, we have a modulating function m(t).
- This gives a voltage level as a function of time.
- We must convert it to a pulse width – a time value based on level.

Doing This

- A triangle has a linear value as a function of time.
- PWM involves a comparison between a modulating function m(t) and a carrier function.
- A triangle carrier gives a linear change from level to width.

PWM Process

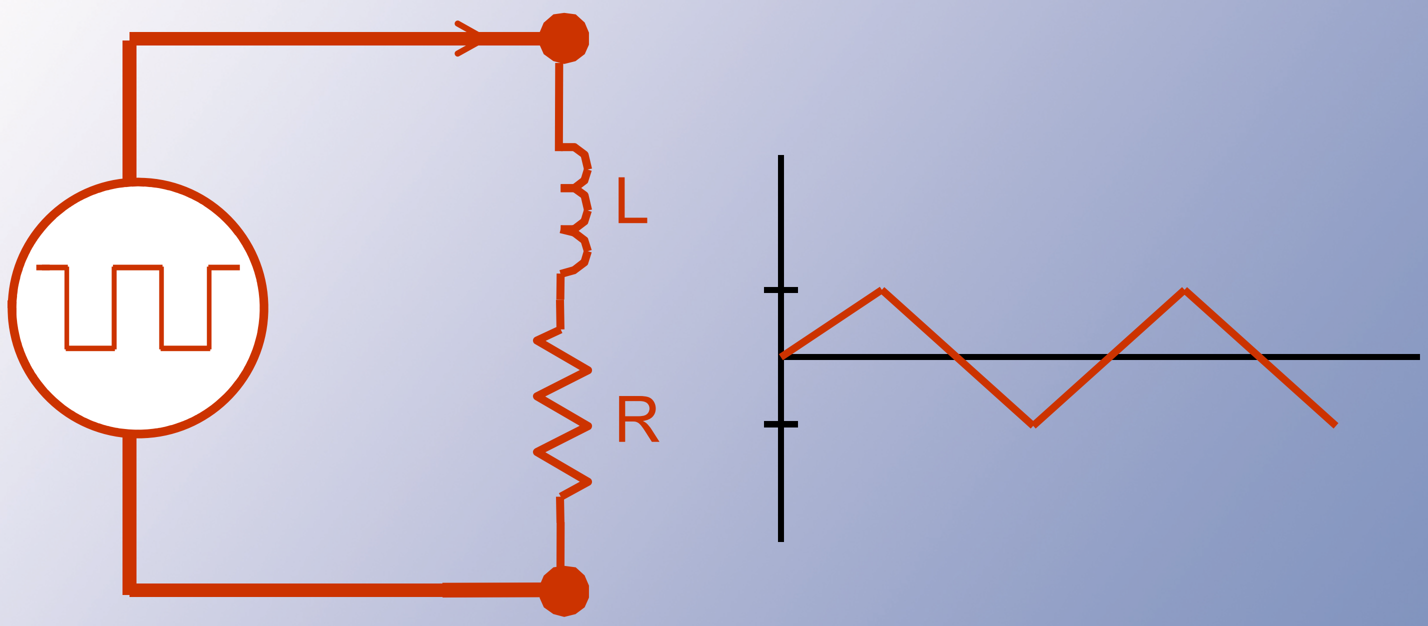
- It is actually very easy to create a triangle (at high frequency), then compare it to a desired function.
- If the carrier frequency is much higher than the modulating frequency, a successful PWM process results.
- The value *k* is called the *depth of modulation*.

PWM Process

OUT: 1) HIGH if m(t) > carrier
2) LOW if m(t) < carrier

→ q (t)

How to Create PWM?



$\Delta\, i = (\ )$

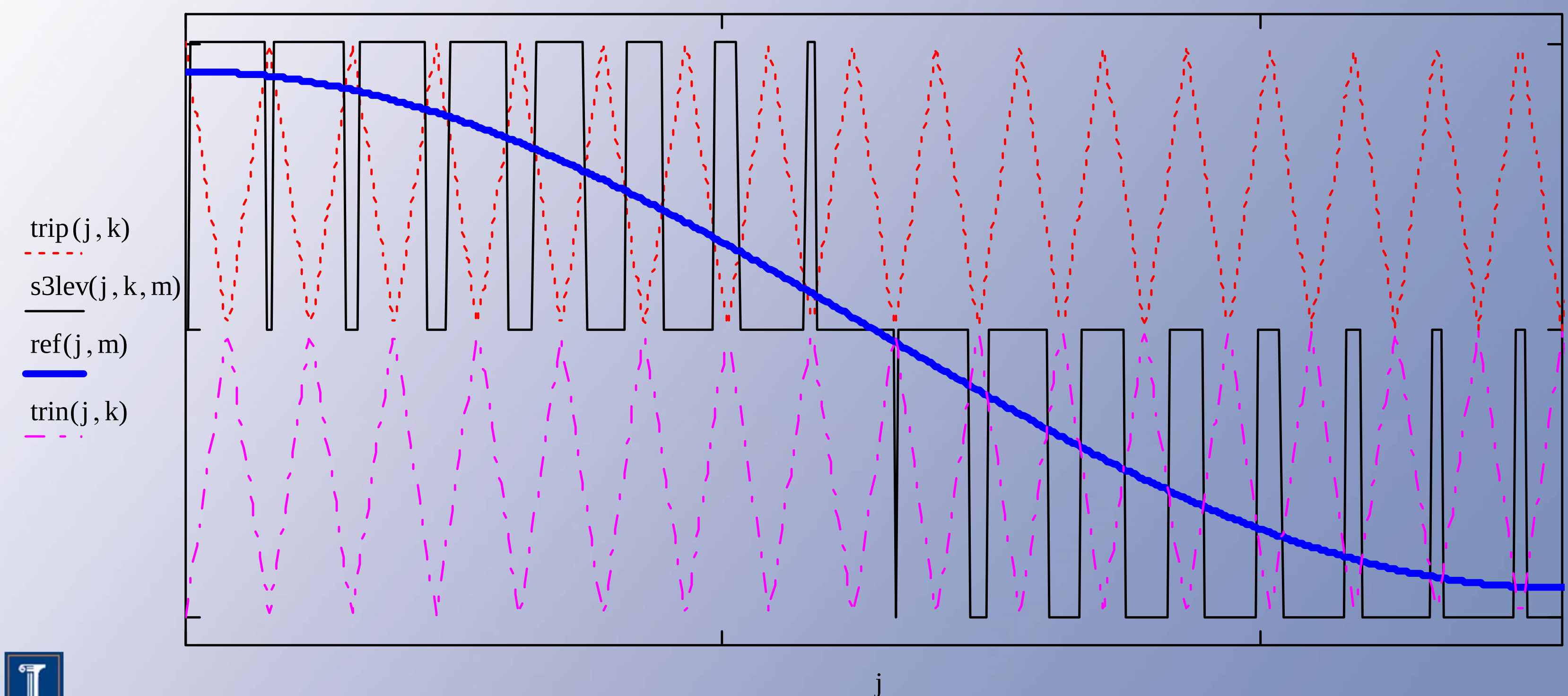
PWM, plus ripple on output current

Multi-Level PWM

- We can also switch among other levels: $\pm V_{in}$, 0, $\pm V_{in}/2$, etc.
- The case with zero is "three-level PWM."
- Some people use five-level and even seven-level PWM, sometimes more.

3-Level PWM

- Switch between 0 and $+V_{in}$ when $m(t) > 0$
- Switch between 0 and $-V_{in}$ when $m(t) < 0$



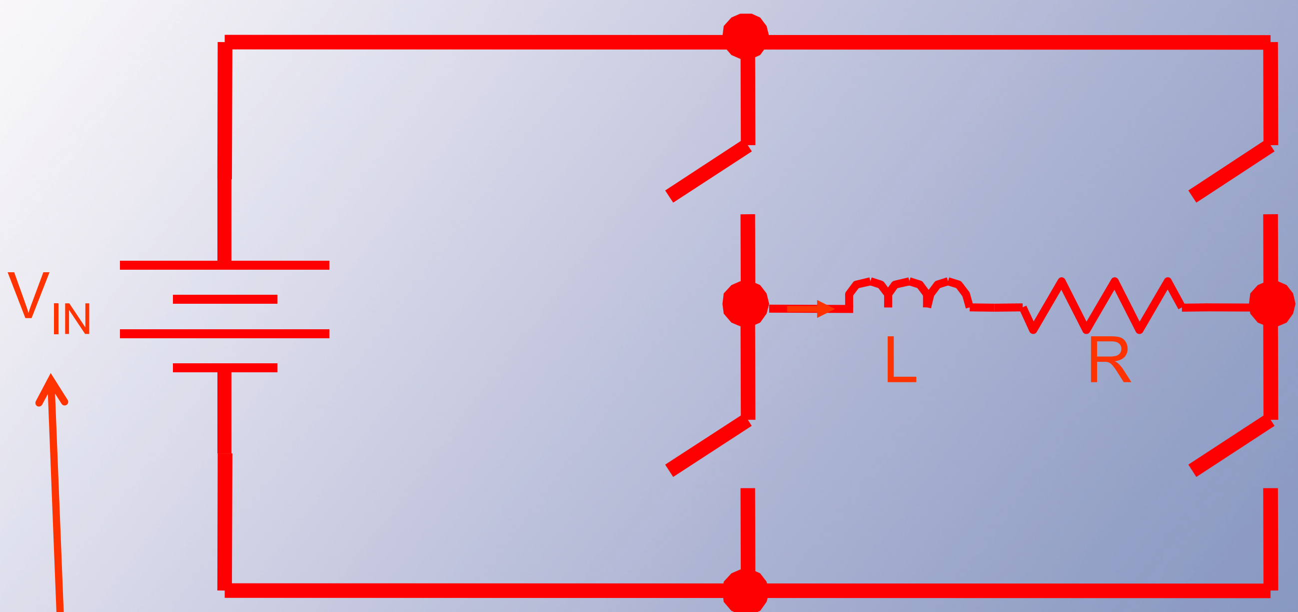
PWM Examples

- Consider a high-quality backup power application.
- We desire 120 V_{RMS} at 60 Hz into loads from 5 W to 500 W. The ripple around the nominal current sine wave should not exceed $\pm$10 mA.

PWM Example

- First, what bus voltage is needed?
- Since 120 V_{RMS} corresponds to 170 V_{peak}, we need at least 170 V at the input.
- This could come from a rectifier or from a backup battery set.

PWM Example

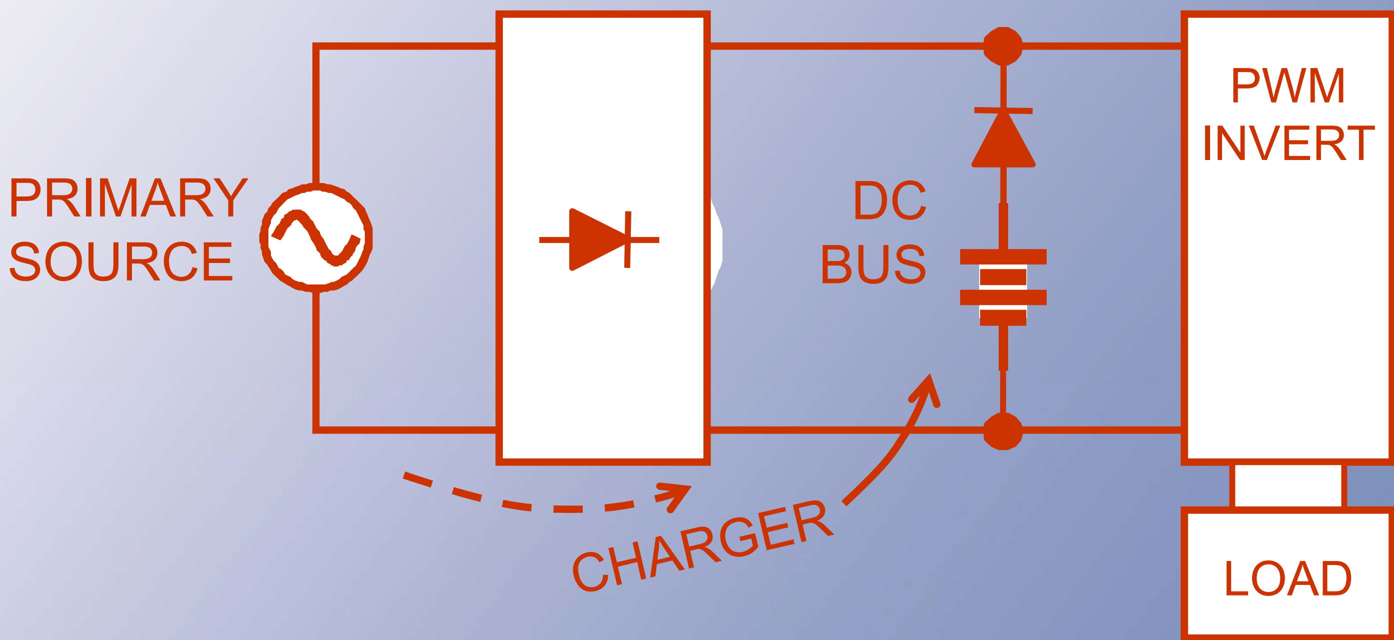


120 V_{RMS} → 170 V peak

V_{in} ~ 170 V

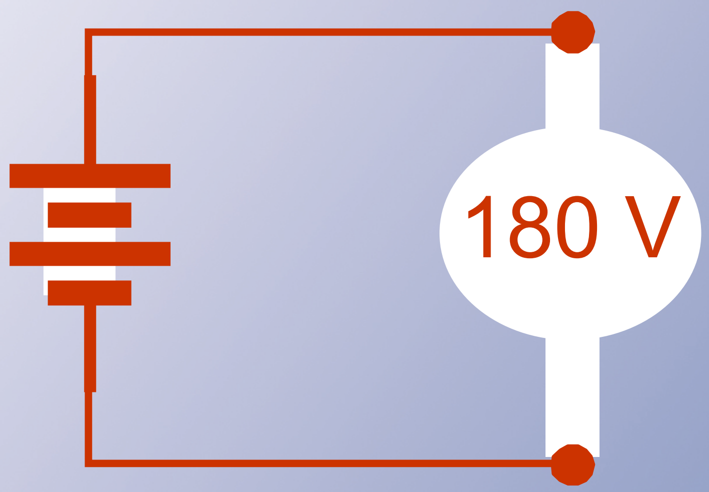
Backup power methods

1. Standby U. P. S.
   UPS → uninterruptible power supply
2. On-line U. P. S.



Backup power methods

3. Rectified input or battery input



PWM Example

- At this power level, it is reasonable to switch at 20 kHz or more. Let us choose 40 kHz (rather arbitrary).
- Depth of modulation is 100% for rectifier input, and about 94% for a 180 V battery.

PWM Example

f_{SWITCH} ?

5 W to 500 W

f_{SWITCH} ~ 20 kHz to 100 kHz

Choose ~ 40 kHz

$$m(t) = k \cos(120\pi t)$$

$$v_{OUT} \approx k\, v_{in} \cos(120\pi t)$$

PWM Example

f_{SWITCH} ?

$v_{in} \sim 170$ V

$k \sim 1$

100% depth of modulation

$v_{in} \sim 180$ V

$k \sim 0.94$

94% depth of modulation

Ripple

- To check ripple, consider the 0 modulation case.

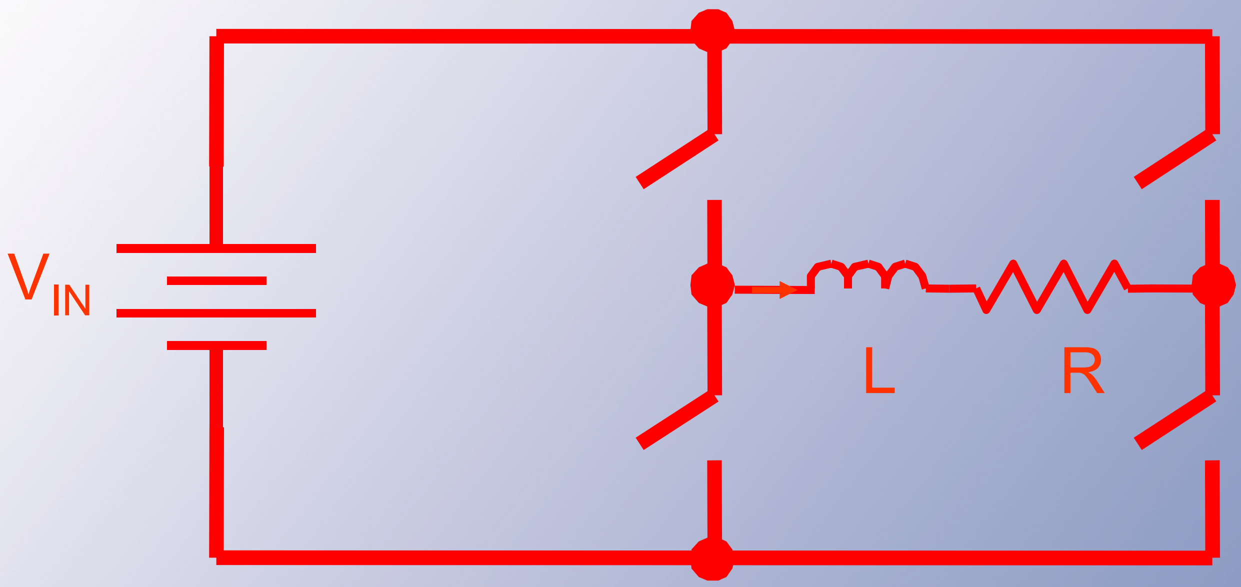
→ Then the signals are all ripple.

- A square wave (180 V peak) is imposed on an L-R circuit.
- The average output is intended to be zero.
- Thus $v_L = L\, di/dt$, $180\text{ V} = L\, \Delta i/\Delta t$.

Ripple Inductor

- The period is 25 us, so 180 V is exposed to the inductor for 12.5 us.
- We need $\Delta i < 0.02$ A.
- $L > 0.113$ H.
- This is quite large, and we could benefit from a capacitor.

Ripple Inductor



ωL ~ near 0Ω at 60 Hz

→ high at 40 kHz

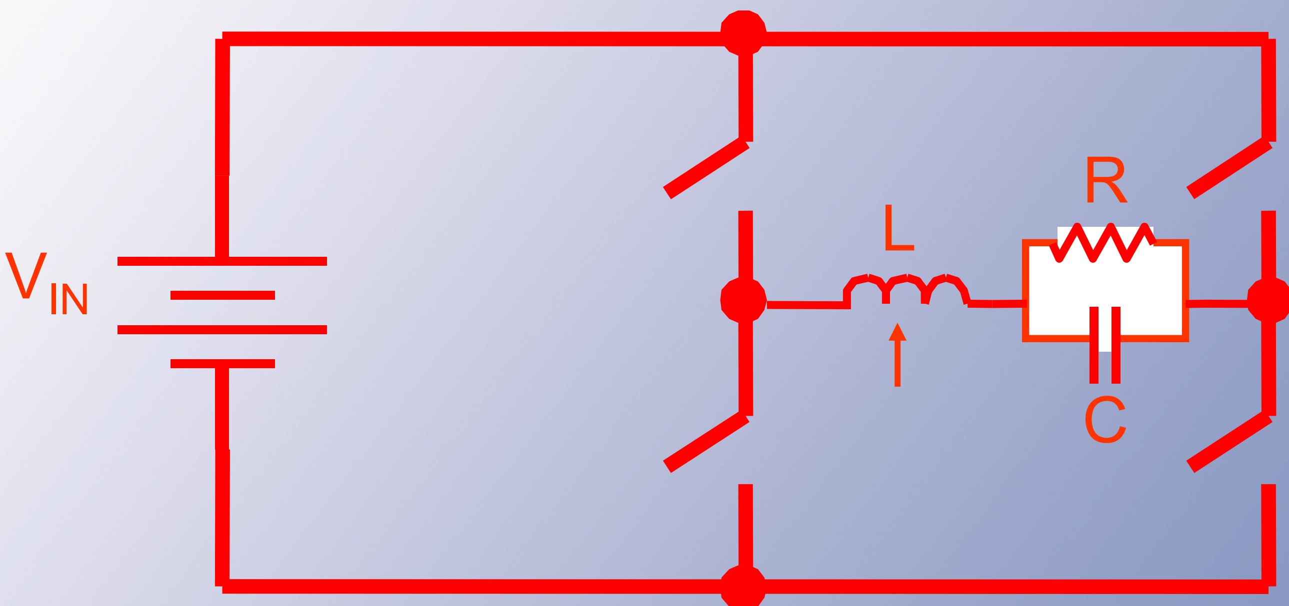
0.113 H at 60 Hz

(120 π) (0.113) ~ 42 Ω

More Detail

- Let L = 20 mH instead.
- This gives Δi = 0.113 A.
- A capacitor across the load will see this ripple current.
- From Chap. 3, the voltage ripple with be about T/(8C), so 40 uF could drop the ripple enough.

More Detail



$$C = 47\mu F$$

$$X_c = {}^{1}/_{(120\ \pi)(47\mu f)}$$

Design Sequence

- Select input so the maximum desired output can be reached.
- Select a switching frequency. Typical:
  - If P_{out} > 10 kW, the range today is 10-15 kHz.
  - If P_{out} > 1 kW, the range today is 10-40 kHz.
  - At lower power levels, 20-100 kHz.
- Set the modulation index to zero, then design for ripple level.
- Be sure the filter has little effect at f_{out}.

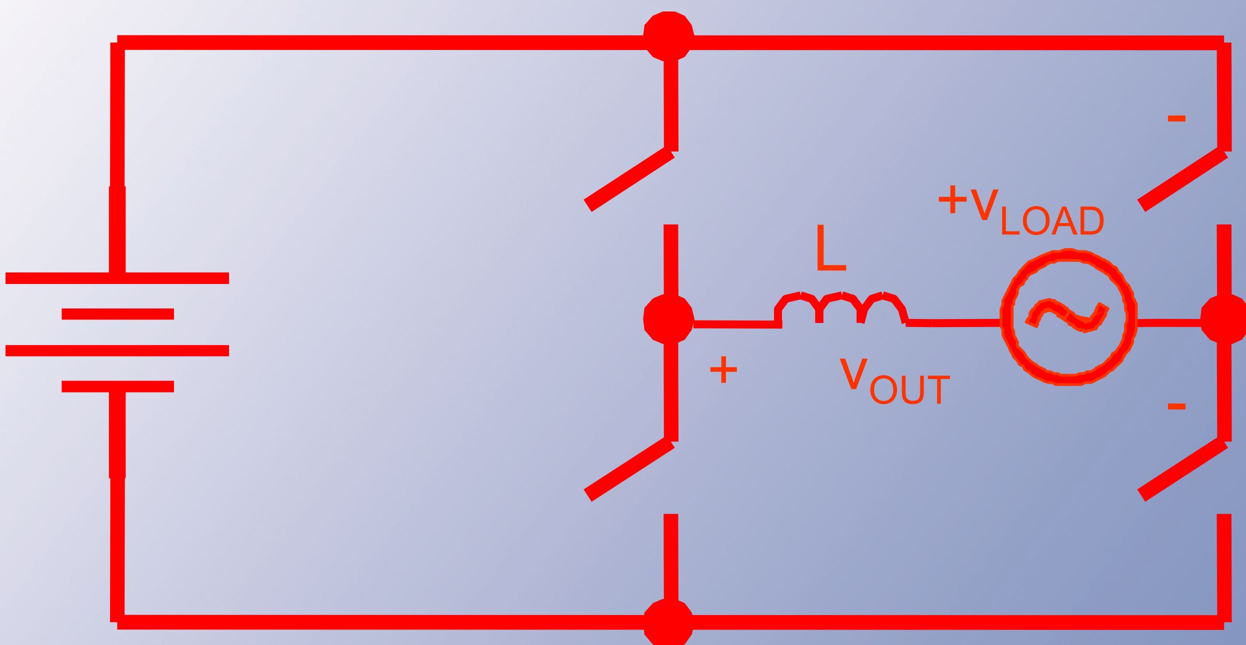
PWM Rectifiers

- We can always reverse the input and output source labels.
- This would become a rectifier application that involves dc voltage sources.
- The switches already handle ac current and dc voltage, so no change there.
- What if our “ac current source” is an ac voltage in series with L?

PWM Rectifiers

PWM

PWM Rectifiers



$v_{OUT} = kV_{in}\, m(t)$ after filtering

PWM Rectifiers

- This is the basis of *PWM rectifiers*.
- In these circuits, the input current is controlled by PWM to be nearly sinusoidal.
- In fact, we should be able to modulate to follow any current.

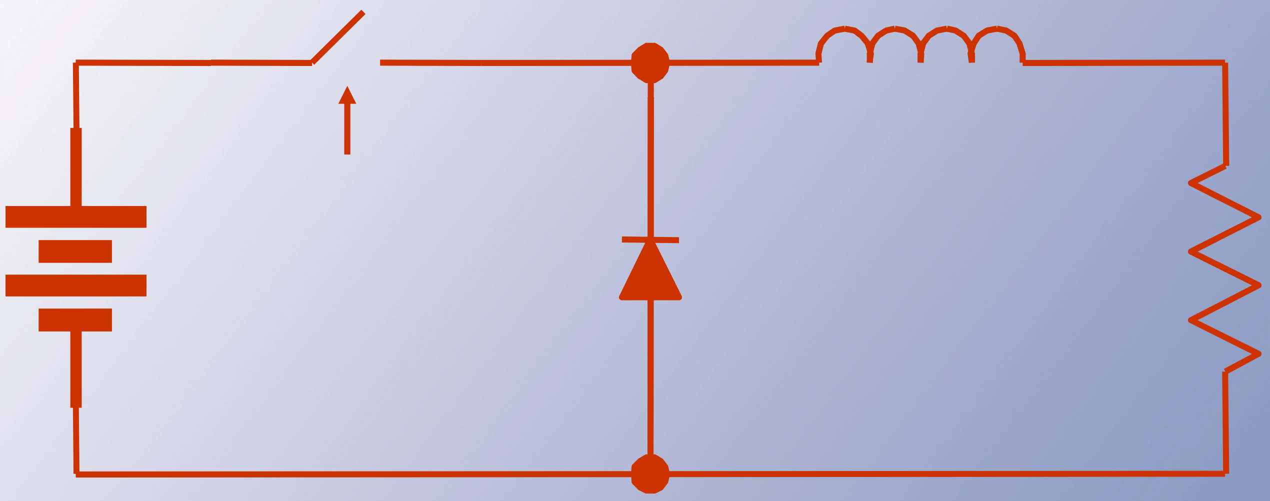
PWM Rectifier Circuit

- Take a simple version in one quadrant.
- A full-wave voltage is imposed, through an inductor as the input to a “reversed buck” converter.
- This is just a boost converter.

Boost Rectifier

- As long as the input waveform changes slowly, we can adjust the duty ratio to provide a given output.
- Recall that $V_{in} = D_2 V_{out}$.
- Now $v_{in} = |V_0 \cos(\omega_{in} t)|$.

Boost Rectifier



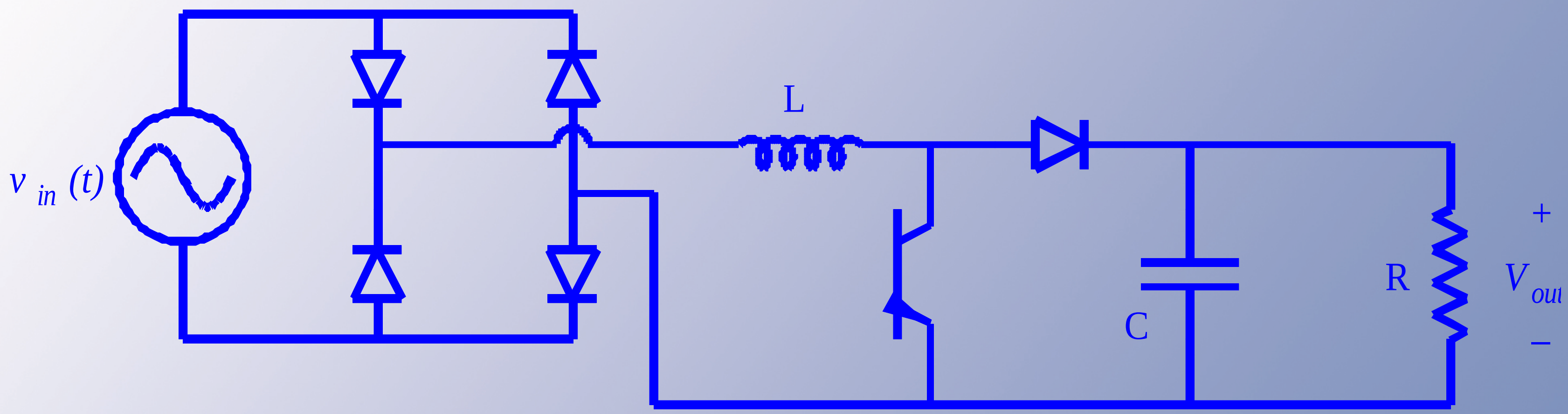
$D \sim {}^{1}/_{2} + km(t)$

$D \sim |\cos(\omega t)|$, ω low

0 to 100%

Switching is fast!

Boost Rectifier



If V_{OUT} > V_{in} peak

$V_{in_{boost}} \sim |V_0 \cos(\omega_{in} t)|$

V_{OUT} is fixed

Boost Rectifier

Boost on average,

$$V_{in} = D_2 V_{OUT}$$

Set $D_{2,}$, so that

$$d_2 = |{}^{V_0}/V_{OUT} \cos(\omega_{in} t)|$$

Boost Rectifier

- What if $d_2 = V_0/V_{out}\ |\cos(\omega_{in}\ t)|$?
- Then the input properly matches the intended input voltage.
- What about the current? As in the PWM case, the input current should follow the modulating function.

Boost Rectifier

$$\underbrace{|^{V_0}/_{V_{OUT}} \cos(\omega_{in} t)|}_{\text{SLOW}}\ V_{OUT} = v_{in}$$

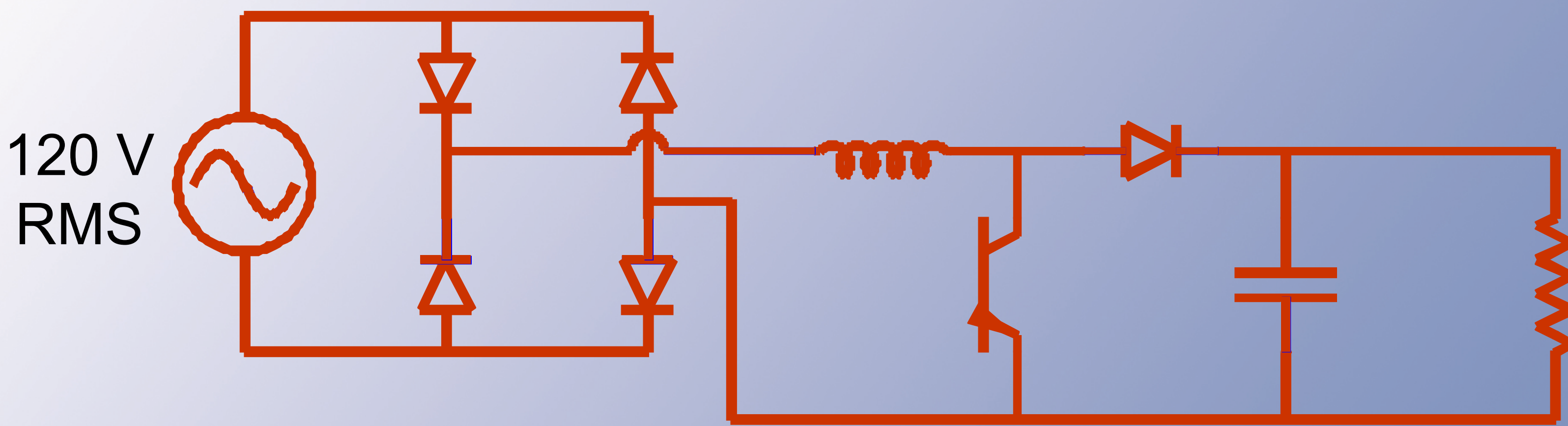
$$|v_0 \cos(\omega_{in} t)| \rightarrow v_{in}$$

Rectifier → no filter

Switching → FAST

Boost Rectifier

For instance:



→ DC-DC for output

Parts → small

Boost Rectifier

PWM Inverter

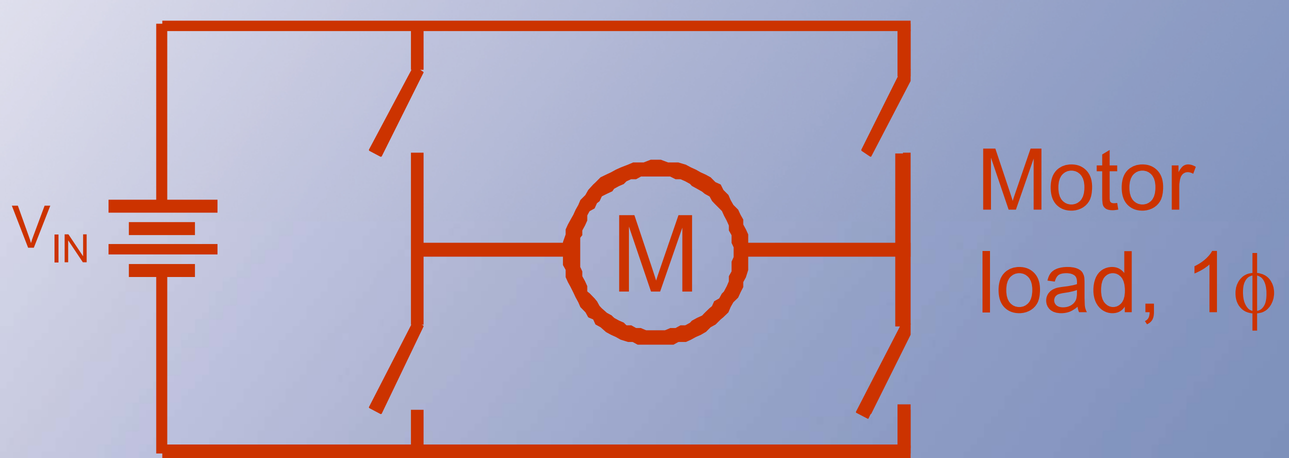
Output into an ideal current source

waveform m(t)

Inverter examples:

VSI with four alternative inputs:

1. Rectified 1ϕ source 230V
2. Rectified 3ϕ source 208V
3. Batteries
4. Solar panel source



Inverter examples:

Motor:

230 V, 1ϕ, 60 Hz, 5 HP

$V_{VSI} \sim {}^{4V_{in}}/_{\pi} \cos\left({}^{\delta}/_{2}\right) \cos\left(\omega_{SWITCH}\, t\right)$

230 V → $230\sqrt{2}$ ~ 325 V

208 V → $208\sqrt{2}$ ~ 295 V

Inverter examples:

$V_{bat} \sim 300$ V (25 x 12 V)

$V_{solar} \sim 300$ V (600 cells x 0.5 V/cell)

$$^{4V_{in}}/_{\pi} \cos\left(^{\delta}/_{2}\right) = 230\sqrt{2}$$

Want an output of 230 V RMS
(325 V peak)

VSI Example

- Source Delta

Source	Delta
230 V ac	76°
208 V ac	60°
300 V dc	63°

- In general, any bus potential down to 255 V can be supported.
- For 208 V 3ϕ with filter, bus is 243 V, and δ = 0 gives 219 V RMS (works).
- That extra 27% is quite useful.

Ac Regulators

- A true ac-ac converter gathers energy at one frequency and delivers it at another.
- The actual most common “ac-ac converters” are only partial in the senses we usually use.

Ac Regulators

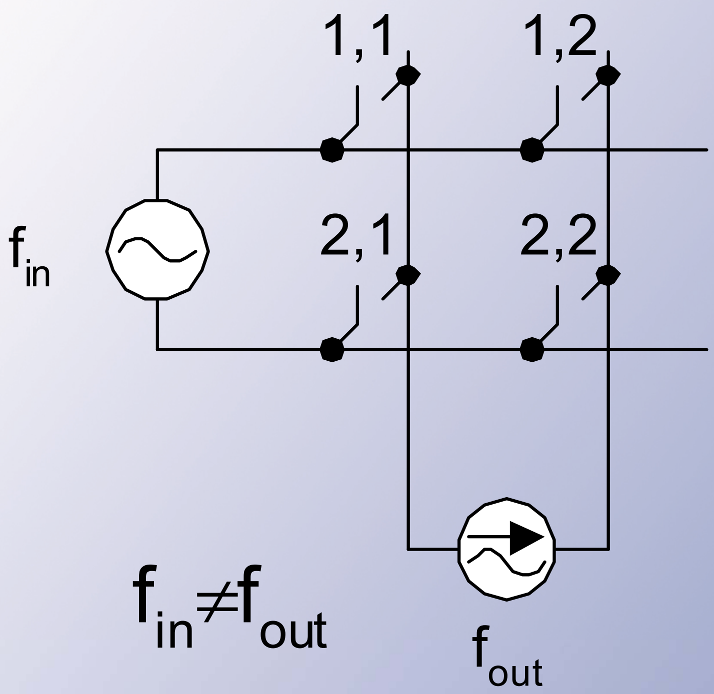
- We might want to control energy flow without frequency change.
- An *ac regulator* is a converter that manipulates energy flow between a source and load in a single-frequency ac system.

Applications

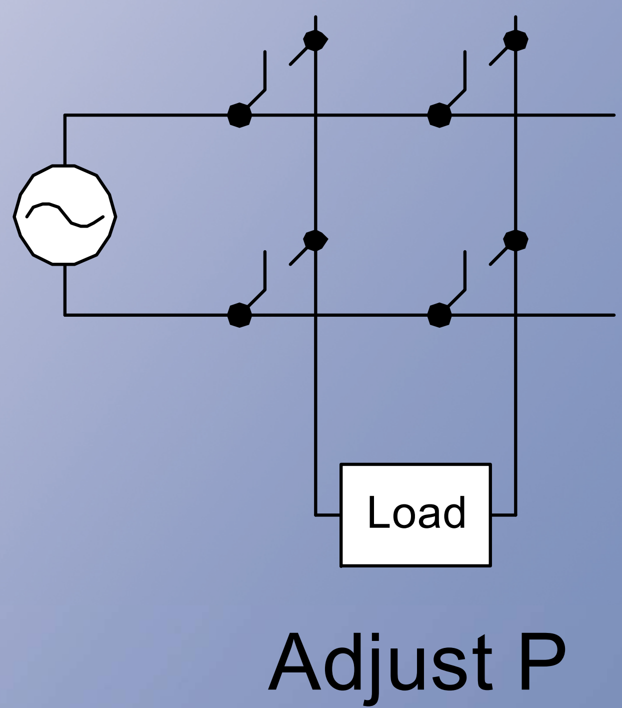
- Applications include incandescent light dimmers, heater controls, microwave ovens, hand tools, and some motor starters.
- Most ac regulators rely on a resistive load, or maybe a very slightly inductive load.

Ac Regulators

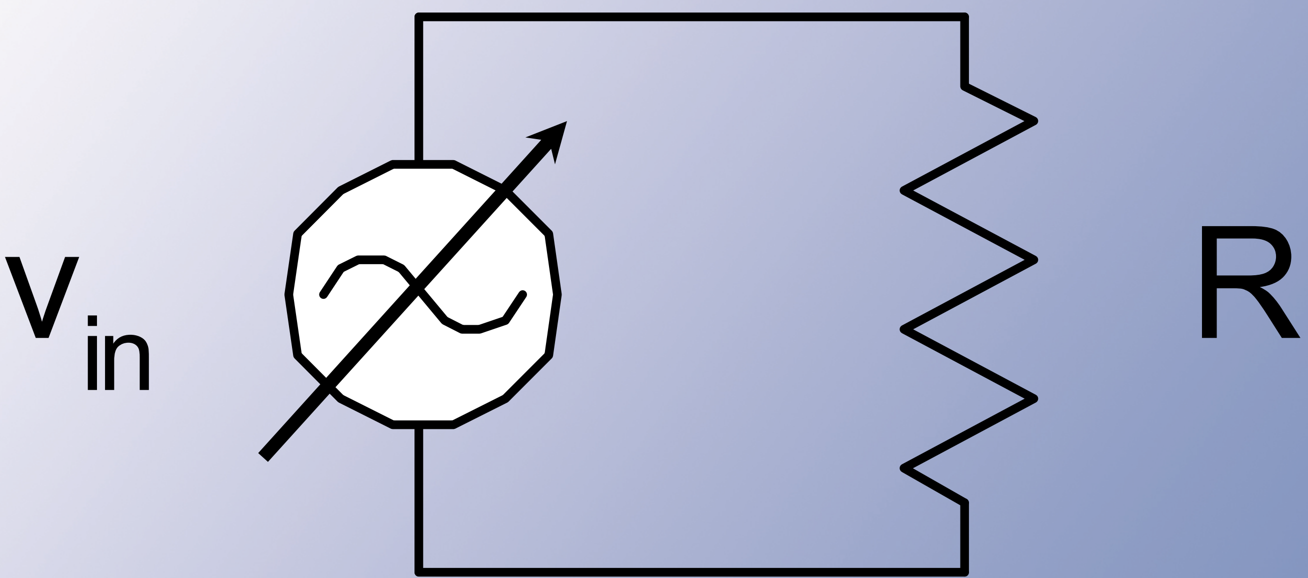
General ac-ac converter



More basic ac regulator function



Ac Regulators



Resistive version

Ac Regulators

- Power: <P> = < v(t) i(t) >
- For a resistive load: <P> = < v(t) v(t)/R > and <P> = (1/R) < $v(t)^2$ >.
- Recall that the RMS value is

$$V_{ms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

- So <P> = (1/R) ($v_{RMS}{}^2$)

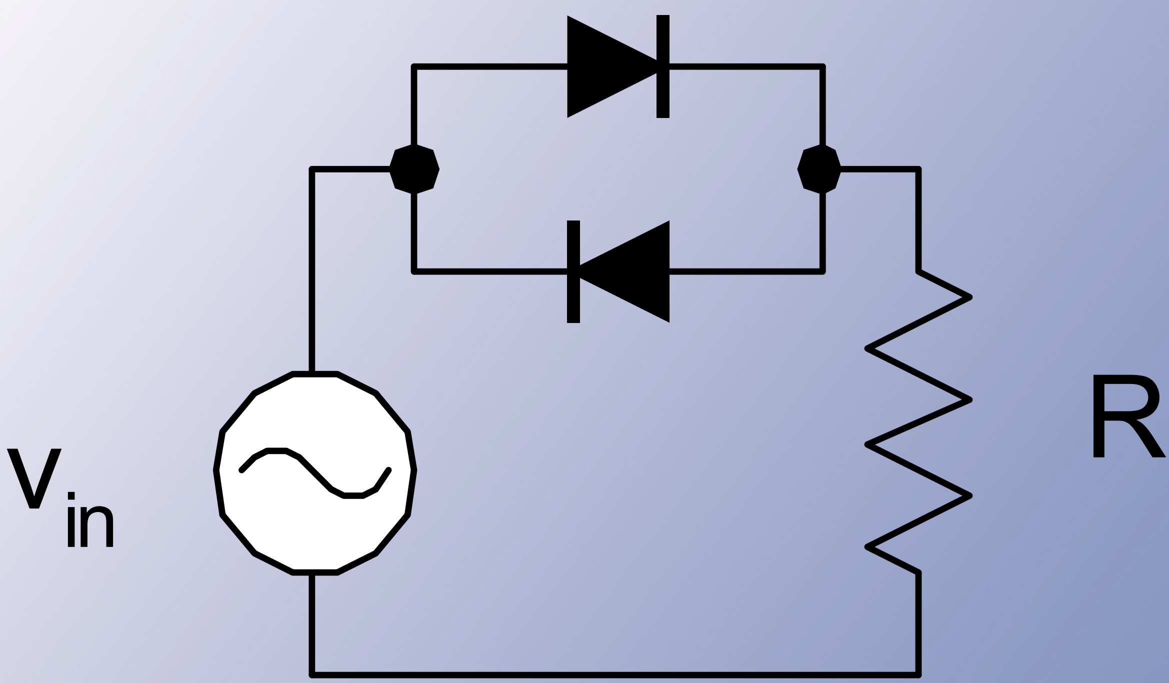
Circuits and Cases

- If a load is resistive, we can vary the power by altering the connection time.
- Resistive loads make this easy and predictable with SCRs.
- Slightly inductive loads can be handled, but less predictably.

Resistive Loads

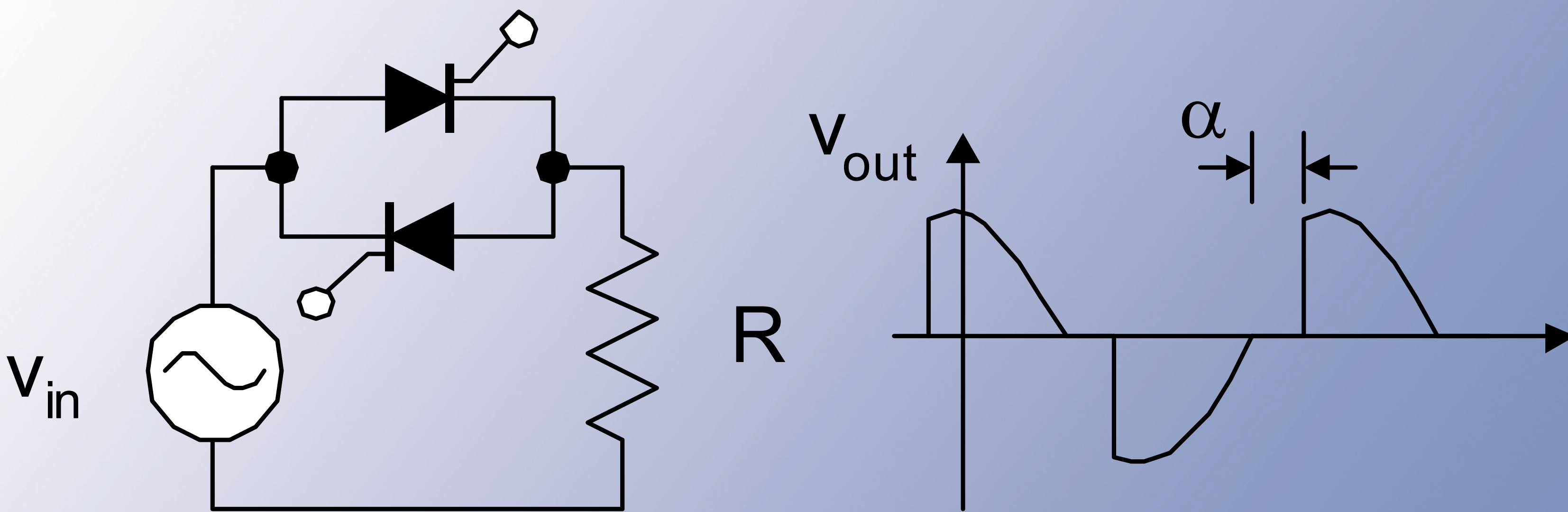
- *In this case, there is no single wanted component*.
- All harmonics deliver energy into a resistor.

Concept



With ideal diodes, there is no change in power flow (no turn-on delay).

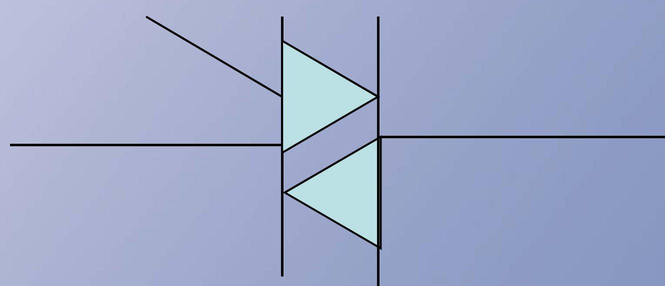
Basic Regulator



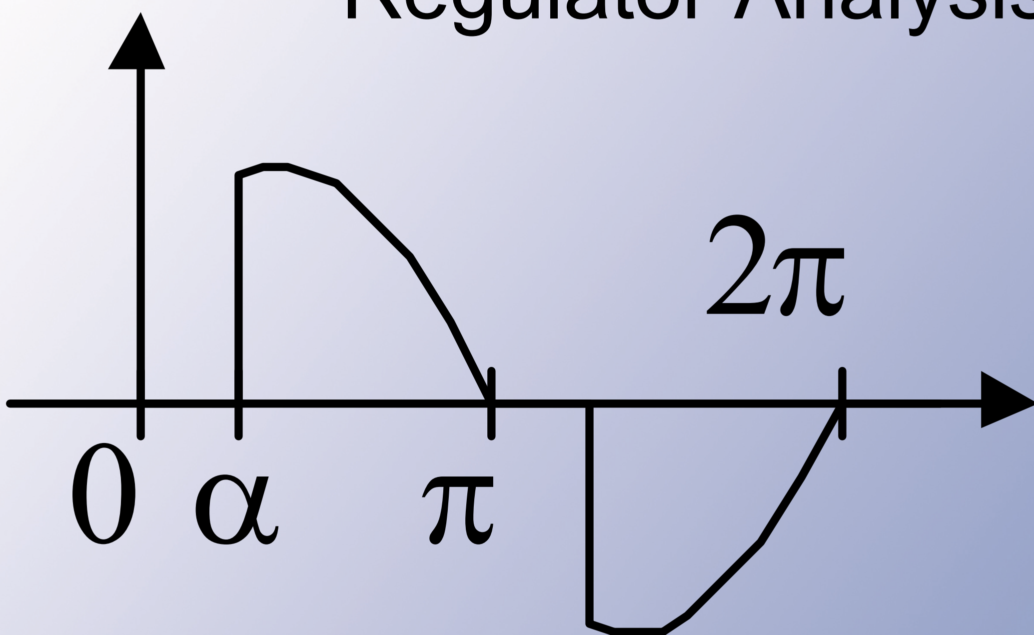
- Use SCRs instead. Now a delay angle can be added. Power decreases with delay.

Regulator Analysis

- We define a delay angle α, based on diode waveforms as α=0.
- With a resistive load, turn-off occurs at a later zero crossing.
- Alternative device: the *triac* acts as reverse-parallel SCRs with a single gate. Good for ac regulators.



Regulator Analysis



- <P> = v_{RMS}^2/R. The RMS voltage is

$$v_{RMS} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi} V_0^2 \sin^2\theta \, d\theta}$$

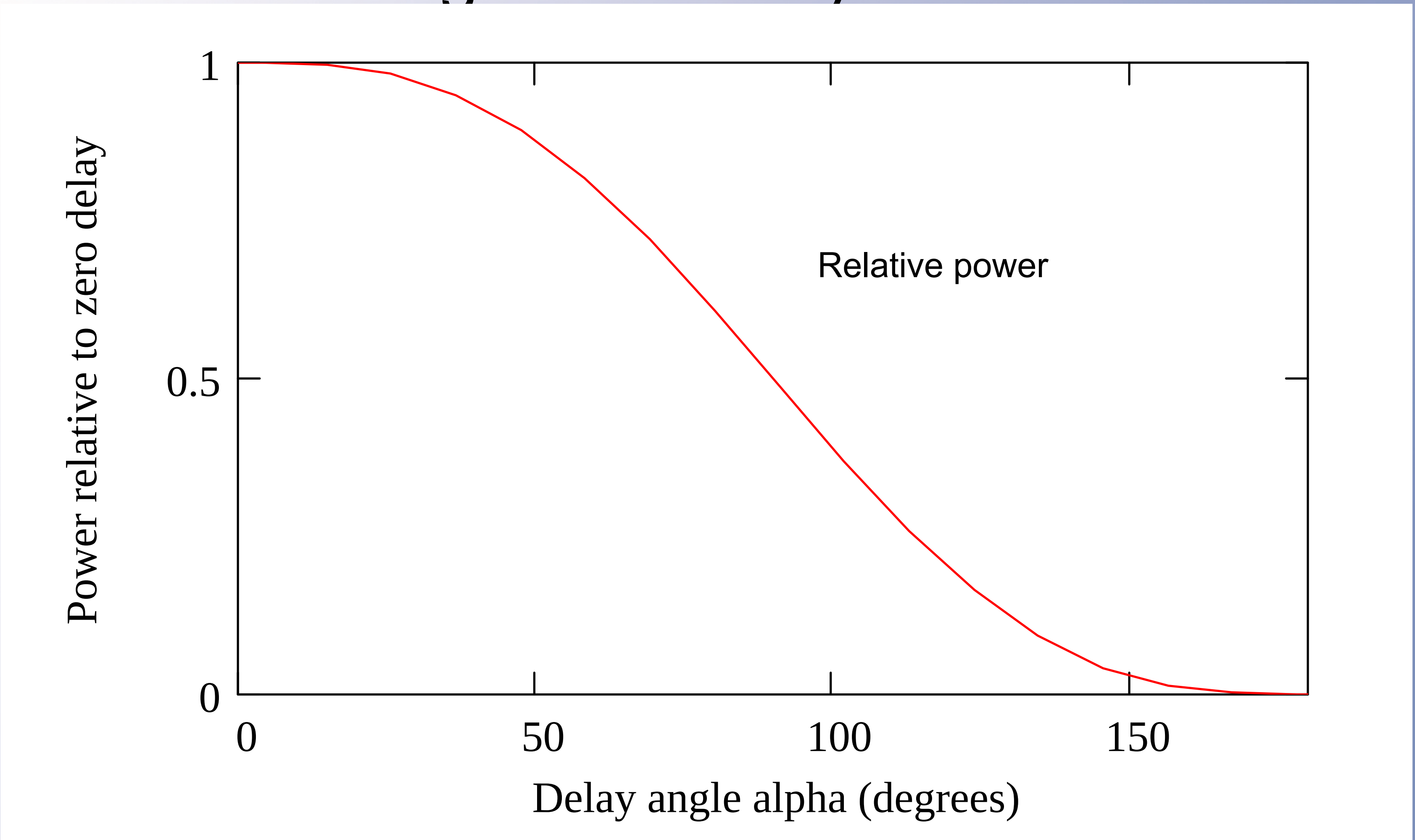
Regulator Analysis

- This integral yields

$$v_{RMS} = V_0 \sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}}$$

- The average power is the square of this divided by R.
- The valid range is $0° \leq \alpha < 180°$

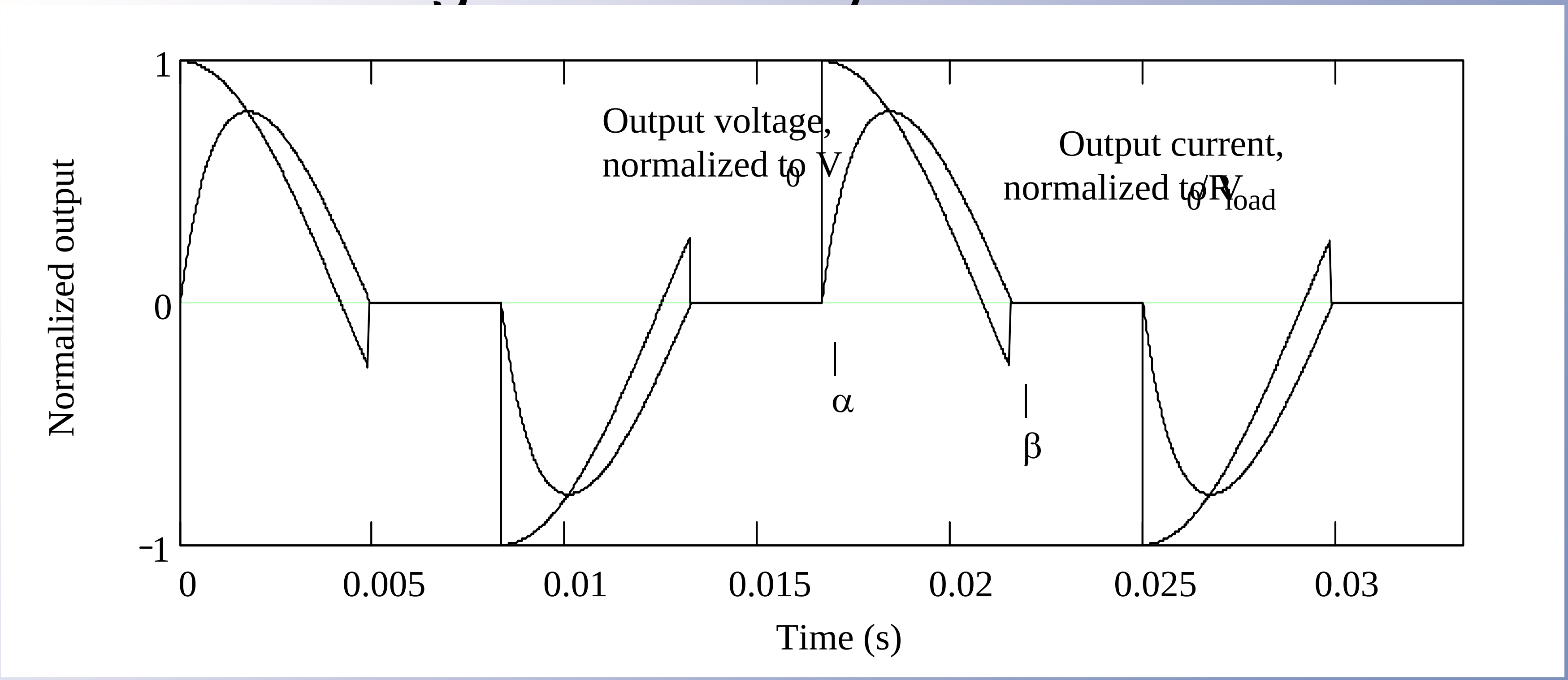
Regulator Analysis



Inductive Loading

- When the load is inductive, turn-off is delayed.
- Turn-off occurs when current reaches zero, which will be delayed from the voltage zero.
- The power depends on L.
- Turn-off angle shown as β.

Regulator Analysis with L

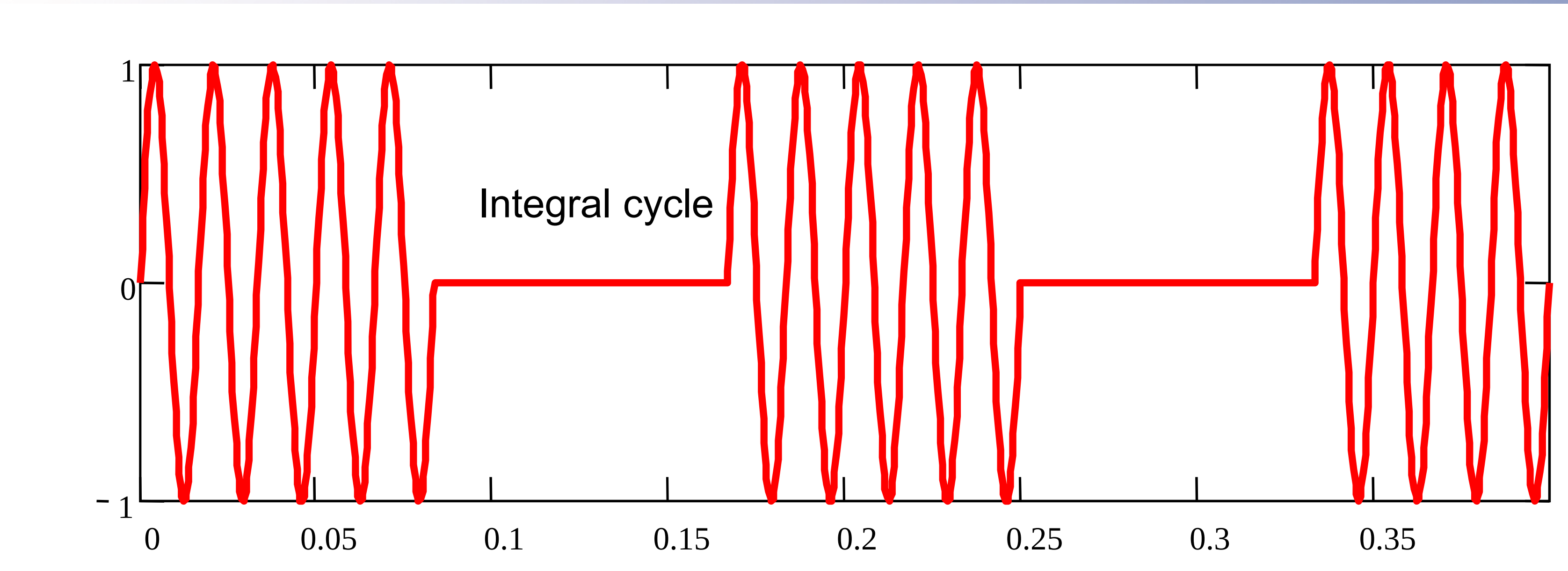


- The power will be lower than for the same resistor alone.

Integral Cycle Control

- It is also possible to use SCRs or triacs to "meter" out the ac waveform on a cycle-by-cycle basis.
- For example, turn the waveform on for 5 cycles, then off for 5 cycles.

Integral Cycle Control



This will deliver adjustable power in a direct way.

Integral Cycle Control

- This is a simple way to control energy flow to some types of loads.
- It cannot be used for lighting or for motors, but is sufficient for heating.
- If we switch on multi-second time scales, this works for many loads.

Integral Cycle Control

- The trouble with this is *subharmonics* -- frequency terms *below* both the input and output values.
- For example, a 1-cycle on, 9-cycle off arrangement generates 6 Hz in a 60 Hz application.

Final Comments

- Notice that ac regulators function by allowing all switches to turn off.
- There are times when no KCL path is required.
- This action is called *discontinuous mode*, since current paths are not always required.
- Ac regulators are a common example.

Discontinuous Mode

- In other converters, we used large L and large C to form near-ideal sources and loads.
- In these cases, KVL and KCL make the switch action definite and pre-determined.
- In DCM, the switch action depends on load.

How Large?

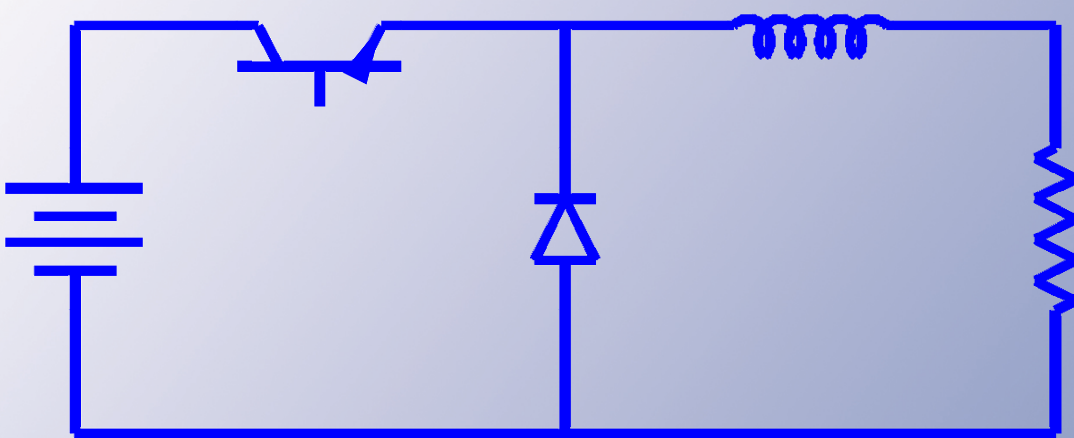
- When is L “large enough”?
- So far, we have some time constant arguments.
- The time constant should be much larger than the period, $L/R = \tau >> T$.
- Similar arguments for C.

Exceptions

- In ac regulators, we prefer small L, and make sure all switches turn off part of the time.
- In dc-dc converters, light loads imply that sometimes it might be hard to maintain current flow for a given inductor.
- Limit example: buck converter with open-circuit output?

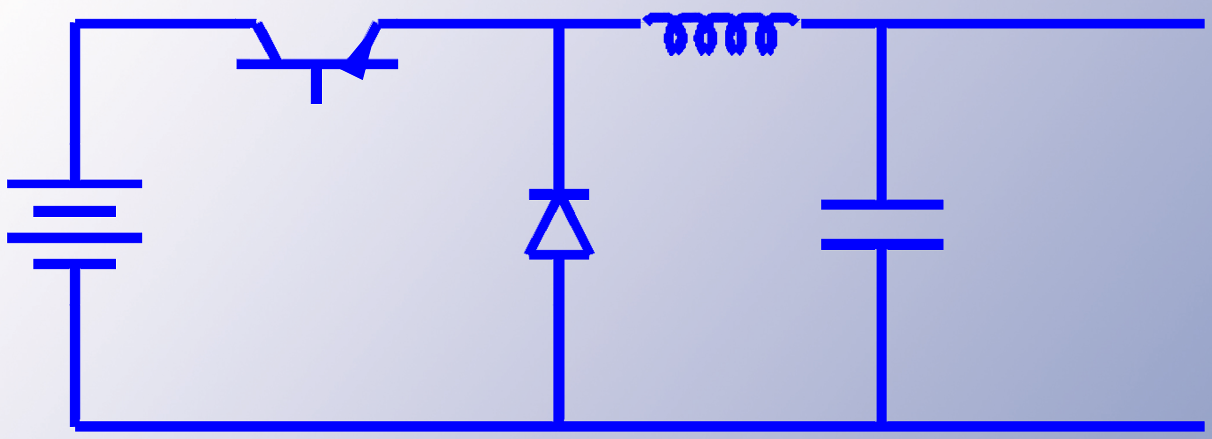
Buck Converter

- $V_{out} = D_1 V_{in}$



- What if the load is disconnected?

Open Circuit Output



- When the switch turns on, the capacitor will charge to $+V_{in}$, like a classical rectifier.
- It never discharges. $V_{out} = V_{in}$ at all duty ratio values.
- So what happened to D_1V_{in}?

Discontinuous Mode

- What if an inductor energy or capacitor energy reaches zero at some point during a cycle?
- If inductor current drops to zero, no current path is needed (KCL).
- The path is discontinuous, and the converter is in discontinuous mode.

Implications

- In discontinuous mode, the KVL and KCL constraints change.
- We can have intervals with all switches off or all switches on without violations.

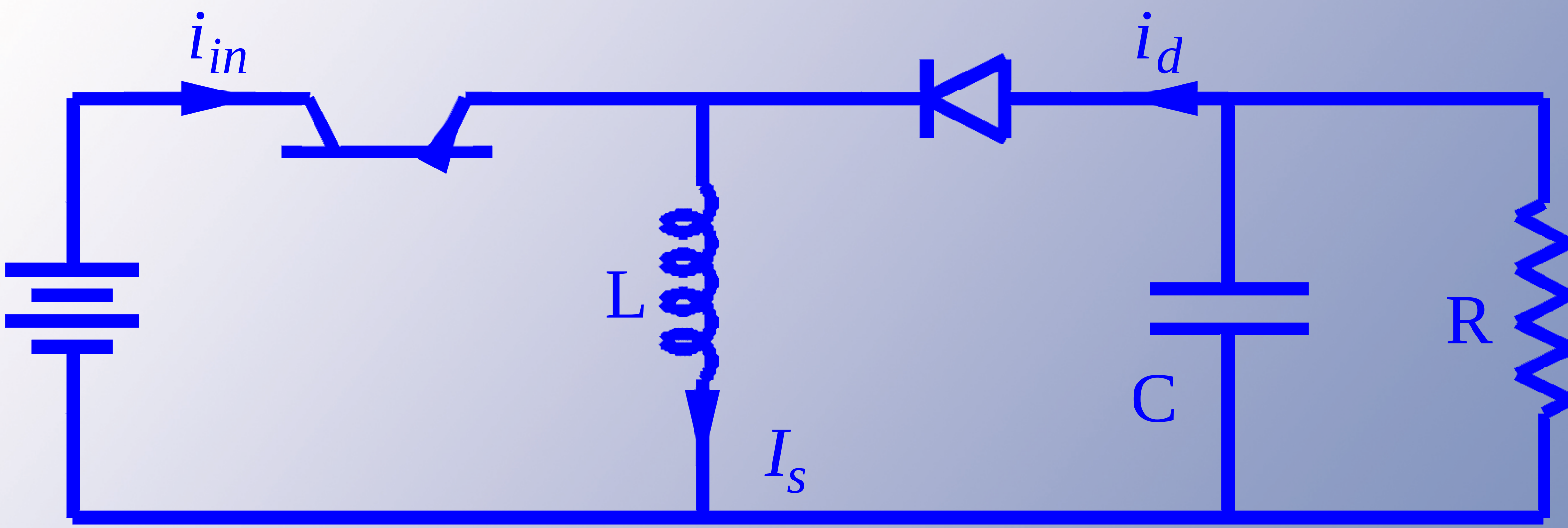
Some dc-dc Converters

- In a buck, boost, or buck-boost converter, discontinuous mode generally means that $q_1 + q_2 \leq 1$.
- The average result is $D_1 + D_2 < 1$.
- We have lost one of the equations used previously for analysis – have an extra unknown.

Buck-Boost Example

- Even so, in any converter, $<v_L> = 0$.
- This holds true no matter what the inductor value might be.
- In some converters, DCM applies to a capacitor, and there could be times when two switches can be on without violating KVL.
- It is still true that $<i_C> = 0$.

Buck-Boost Example



- Low output ripple requires C large.
- What about the choice of L?

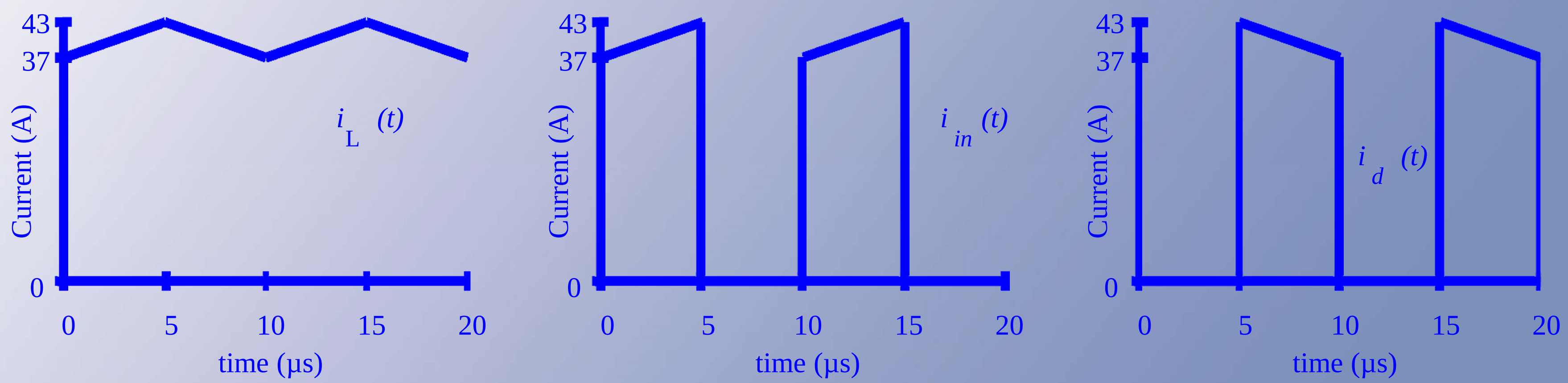
Buck-Boost Example

- Large L --> I_L ~ constant.
- $v_L = q_1 V_{in} + q_2 V_{out}$
- $\langle v_L \rangle = 0 = D_1 V_{in} + D_2 V_{out}$
- $i_{in} = q_1 I_L$, $\langle i_{in} \rangle = D_1 I_L$
- Input power $P_{in} = D_1 I_L V_{in}$
- $i_d = q_2 I_L$, $\langle i_d \rangle = D_2 I_L = I_{load}$
- Output power $P_{out} = D_2 I_L V_{out}$
- $I_L > 0$, $q_1 + q_2 = 1$, $D_1 + D_2 = 1$.

Same relationships as before in Chap. 4.

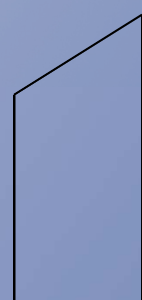
Buck-Boost Example

- Now use a smaller L, but still large enough to maintain positive energy.
- We still have $D_1 + D_2 = 1$, and analysis shows that the averages have not changed.



Buck-Boost Example

- We still have $v_L = q_1 V_{in} + q_2 V_{out}$, and $\langle v_L \rangle = D_1 V_{in} + D_2 V_{out} = 0$.
- BUT, consider that $\langle i_{in} \rangle = \langle q_1 i_L \rangle$ might not be equal to $\langle q_1 \rangle \langle i_L \rangle$, since i_L now varies significantly.
- Try one of these waveforms: it turns out that $\langle i_{in} \rangle$ is still $D_1 \langle i_L \rangle$.
- We can check others.

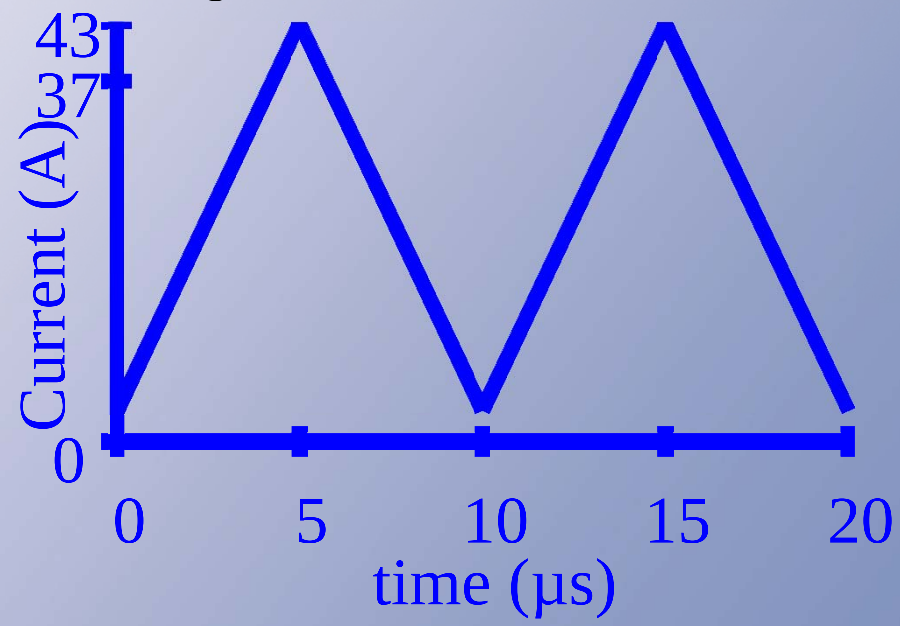


Buck-Boost Example

- All the original relationships hold with $\langle i_L \rangle$ taking the place of I_L.
- The switches still must alternate to provide a current path.

Buck-Boost Example

- Drop the inductor still more, until the current is just barely above 0.
- We still require $D_1 + D_2 = 1$ to meet KCL.
- The average relationships still hold!



Buck-Boost Example

- The relationships continue to be valid until i_L just touches zero.
- Notice that provided L is large enough to enforce $i_L > 0$, the average relationships still hold.
- As long as $i_L > 0$, the average relationships are the same as for L --> ∞!

Buck-Boost Example

- This is the ultimate answer to “how large?”.
- If the inductor is big enough to maintain current flow (so that its energy never drops to 0), the average relationships match those for $L \rightarrow \infty$.
- The *smallest* inductor that enforces $i_L > 0$ is called *critical inductance*.

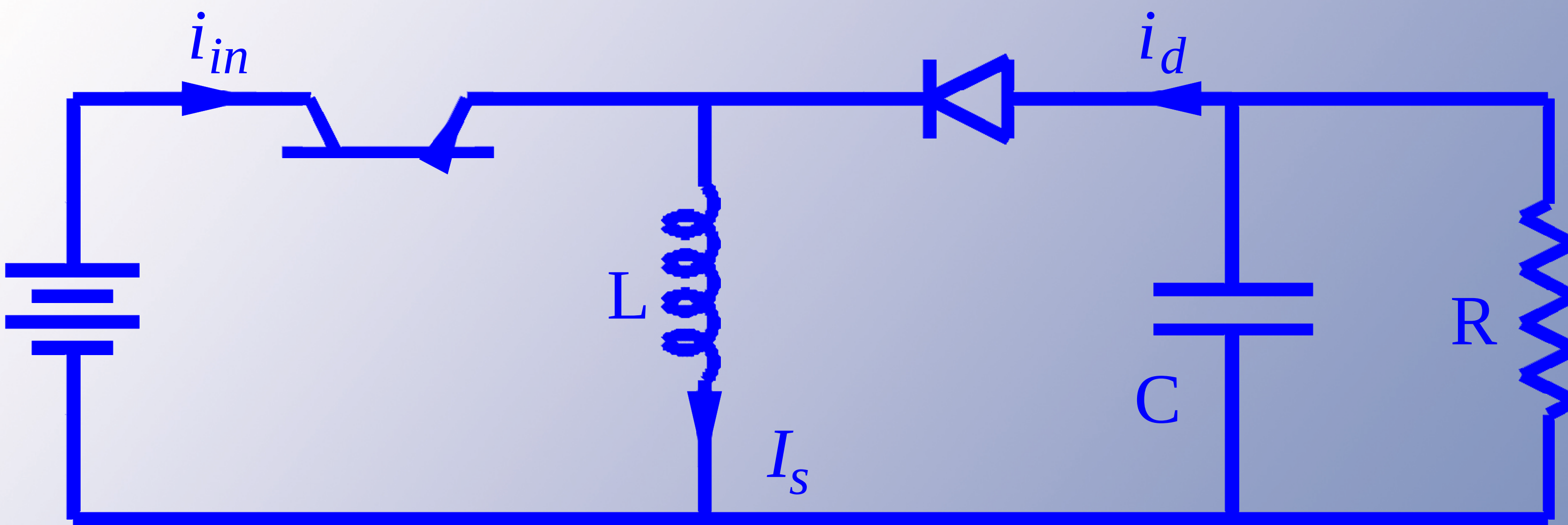
Critical Inductance

- So, if $L > L_{crit}$, then the inductor is "big enough" to support the ideal relationships.
- Similar ideal: a capacitor that is large enough to maintain its energy above zero will support the ideal relationships.
- The *smallest* capacitor for which $v_C > 0$ would be the *critical capacitance*.

Buck-Boost Example

- What if L is smaller, $L < L_{crit}$?
- Now, with switch #1 on, the inductor current ramps up linearly.
- When the diode turns on, the current falls to zero.
- Then both switches turn off.

Buck-Boost Case

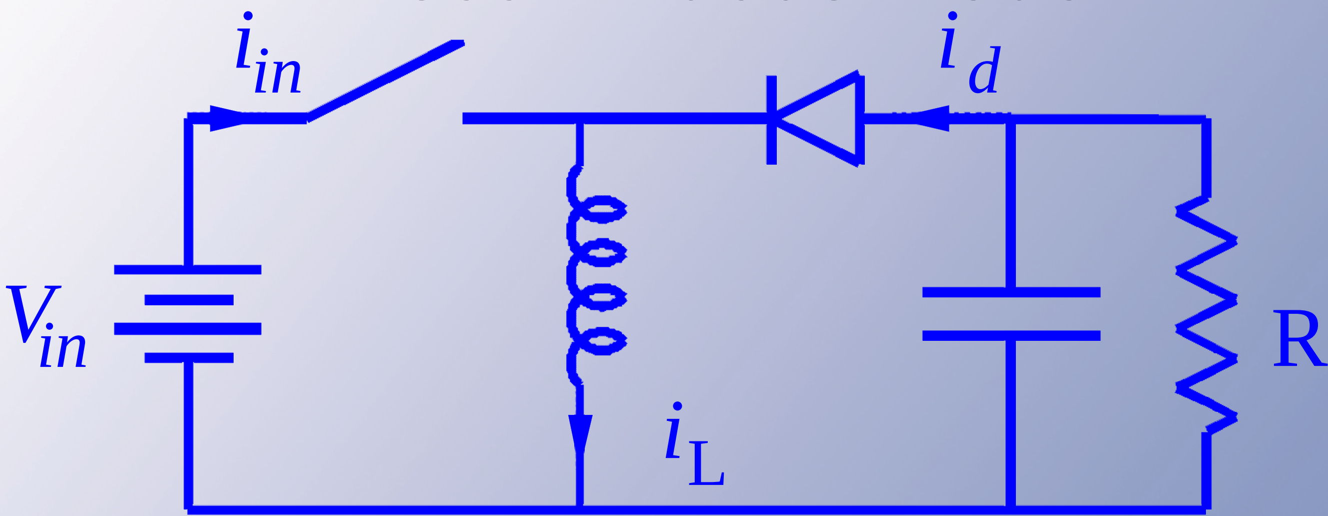


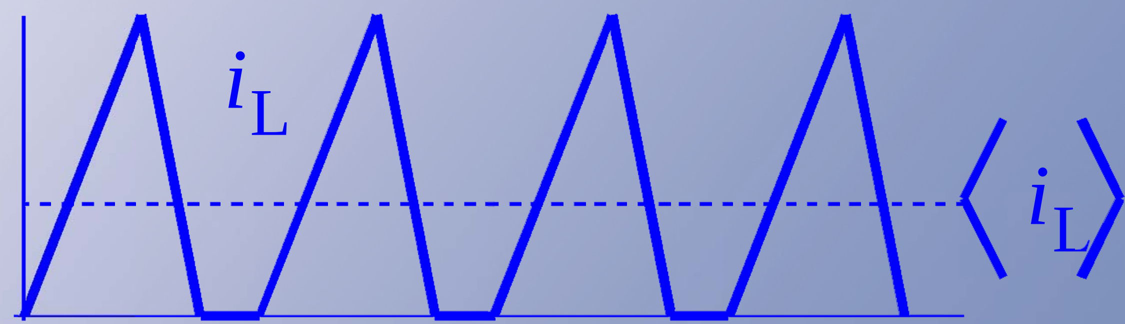
- Low output ripple --> C large.
- The action when L < L_{crit}.

Exploring Relationships

- The buck-boost case.
- The peak inductor current is $V_{in}D_1T/L$. The average input current is $D_1\ i_{L(peak)}/2$.
- D_2 is now unknown.
- But because $P_{in} = P_{out}$, we can find V_{out} in terms of V_{in}, D_1, R, T, L.

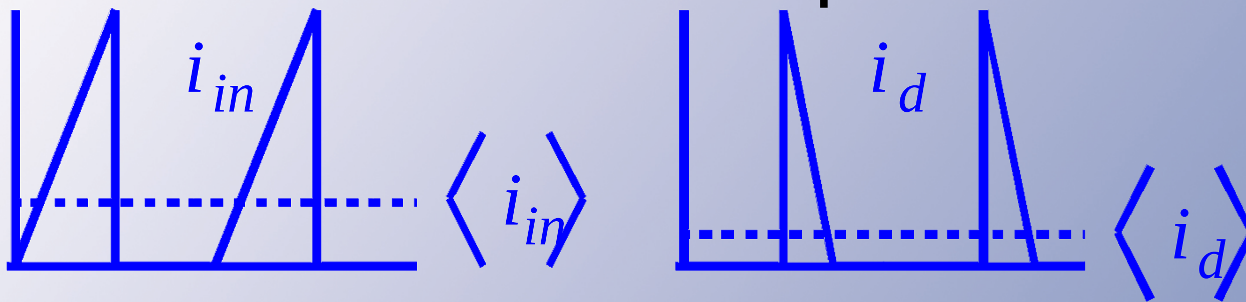
Discontinuous Mode





Now $\langle i_{in} \rangle \neq D_1 \langle i_L \rangle$.

Relationships



$$D_1 V_{in} = -D_2 V_{out}$$

- $\langle i_{in} \rangle = \langle q_1 i_L \rangle$,
- The average can be integrated to give $\langle i_{in} \rangle = 1/2\ D_1\ i_{L(peak)}$.
- On the output side, $\langle i_d \rangle = 1/2\ D_2\ i_{Lpeak}$
- But now, what about D_2? Don't know it.

Begin Day 8 -- Relationships

- What is the peak current?
  - Since $v_L = L\,di/dt$, with #1 on we find
  - $i_{peak} = V_{in} D_1 T/L$
- The expression $P_{in} = P_{out}$ gives us the second equation needed to complete the analysis.
- $P_{in} = \langle V_{in}\, i_{in} \rangle = V_{in} \langle i_{in} \rangle$
  $= V_{in} (D_1/2)(V_{in} D_1 T/L)$

Output Values

- Now, $P_{in} = V_{in}^2 D_1^2 T/(2L) = P_{out}$
- $P_{out} = V_{out}^2/R$
- We have $$\frac{V_{out}^2}{R} = V_{in}^2 \frac{D_1^2 T}{2L}$$
- The solution is

$$V_{out} = -D_1 V_{in} \sqrt{\frac{RT}{2L}}$$

when $D_1 + D_2 < 1$.

Output Values

- The output magnitude is higher than would be expected with large L.
- In the end, the relationships listed in the text are the results.

Summary of Relationships

- We can analyze any dc-dc converter based on energy conservation.
- Conservation holds even when average relationships become more complicated.

Why Does It Matter?

- Why not choose sufficient L or C to avoid discontinuous mode?
  - At light loads, this is not always possible.
  - There are certain advantages to discontinuous mode.

Advantages - DCM

- For current, each cycle is the same whether first starting or in steady state. Response is very fast.
- Low L.
- Extra output voltage (is this is useful?).

Disadvantages - DCM

- Output depends on load.
- Tendency toward magnetic saturation.
- Sometimes hard to keep the output voltage constrained.

Light Load

- In general, there is always a DCM when the load power is low enough.
- This would suggest that we cannot assume continuous mode for design.
- One problem is that ripple is extreme in DCM.

Ballast Load

- We can choose a minimum load to ensure continuous mode.
- One alternative is a *ballast load*: an extra resistance inside the converter that keeps it out of discontinuous mode.
- We keep it low (e.g. 1% load).

Definition

- The minimum value of L that is sufficient to maintain positive current is called the *critical inductance* L_{crit}.
- If $L \geq L_{crit}$, current flow is maintained, and average relations hold as before.

Critical Inductance

- Notice that if $L = L_{crit}$, the inductor current ripple is $\pm$100% of the average current.
- Now we see the implications: If $L > L_{crit}$, the converter action is pre-determined, and follows basic average relationships.

Critical Inductance

- Critical inductance is an excellent *design* tool.
  - It is easy to compute.
  - Ripple is ±100% when $L = L_{crit}$.
- Ripple is inversely proportional.
  - If $L = 10\ L_{crit}$, then ripple is ±10%
  - If $L = 50\ L_{crit}$, then ripple is ±2% …

For Converters

- Most converters have a range of operation rather than a single point.
- We need to define *critical inductance for a converter*, L_{crit}, such that $i_L > 0$ for *all allowed operating conditions*.

For Converters

- To ensure continuous mode, $L \geq L_{crit}$ (for the converter).
- In contrast, to ensure DCM, L must be less than the lowest critical value.

Capacitance

- In most circuits, ripple means the capacitors are far above the critical values.
- Exceptions are converters that have transfer voltage sources.
- The boost-buck is one example.

Capacitance

- If the capacitor in a boost-buck allows its energy to drop to zero, there are times when both switches can be on without KVL problems.
- Diodes do this automatically.
- Now $D_1 + D_2 > 1$.

Analysis

- The analysis is similar, since energy is conserved.
- For this converter, the output falls if the capacitor is too small for continuous mode.
- We can also have discontinuous modes involving the inductors.

Concepts

- The concepts of critical inductance and capacitance apply to all types of converters.
- Many applications deliberately use discontinuous mode. It is essential for ac regulators.

Load Example

- A buck converter for 48 V to 12 V conversion operates at 50 kHz. It has L = 100 uH, C = 100 uF. What is the minimum load to avoid DCM?
- The answer is that load for which L matches L_{crit}.

Load Example

- Consider just a load current.
- Since the load current matches $\langle i_L \rangle$, when $L = L_{crit}$, the peak inductor current is twice I_{load}.
- The duty ratio is 25%.
- When #1 is on, v_L = 36 V.

Load Example

- 36 V = L di/dt = L_{crit} $(2I_{load})/(D_1T)$.
- With L_{crit} = 100 uH and D_1T = 5 us, we have I_{load} > 0.9 A.
- The load power would need to be at least 10.8 W.
- We could add a ballast load to meet this minimum, although this would only be appropriate if P_{out} >> 10.8 W.

Ballast Load Case

- To make $P_{out(min)}$ smaller, we have several choices:
  - Larger inductor
  - Faster switching
  - Ballast load
  - Some combination

Ballast Load Example

- Example: Raise the switching frequency to 100 kHz. The inductance now matches the critical value at 5.4 W load.
- Raise the inductor to 250 uH instead. Now the minimum load is 2.16 W.
- Add a 56 Ω ballast resistor. This will draw 2.57 W and drops the minimum output power to zero.

Boost Example

- A boost converter has input in the range of 8 V to 25 V, and an output of 50 V. The allowed load ranges from 0 W to 200 W. The switching frequency is 50 kHz. The output capacitor is large.
- Find L_{crit} for this converter.
- Also find the inductance that ensures DCM operation under all conditions.

Solution

- As stated, the answer is $L_{crit} \rightarrow \infty$, because the minimum load is 0 W.
- We will need to add a ballast load to support a finite inductance.
- There is no single answer, but let us pick 2 W as the ballast load to give a valid L with only 1% extra power loss.

Solution

- Now we have an input inductor current equal to P_{out}/V_{in}, with a P_{out} range of 2 W to 202 W and a V_{in} range of 8 V to 25 V.
- We have $D_2 = V_{in}/V_{out}$.
- If the inductor matches L_{crit}, the current ripple is twice the average, and $\Delta i_L = 2P_{out}/V_{in}$.
- With the transistor on, $V_{in} = L\ \Delta i_L/\Delta t$.

Solution

- Since Δi_L is known, we can solve to get $V_{in}D_1T/L_{crit} = \Delta i_L = 2P_{out}/V_{in}$, and $L_{crit} = V_{in}^2D_1T/(2P_{out})$.
- We need the value that works in all cases – the largest. But V_{in} and D_1 are not independent: $D_1 = 1 – D_2 = 1 – V_{in}/V_{out}$.
- $L_{crit} = V_{in}^2(1-V_{in}/V_{out})T/(2P_{out})$.
- Highest value at 2 W out and at 33 V, but our input only extends to 25 V.

Solution

- The end result is 1.56 mH. We should set L > 1.6 mH to avoid DCM with a 2 W ballast load.
- Now, what if we want to ensure DCM instead. Need the lowest value, then set L lower.
- Occurs at 202 W load and 8 V in.
- L < 2.66 uH in this case.

Summary

- Critical inductance: the value just big enough to maintain $i_L > 0$. This avoids DCM.
- Critical capacitance: the value just big enough to maintain $v_C > 0$. Again avoids DCM.
- Concepts apply to all converters.

Rectifier

- Consider L_{crit} in an m-pulse rectifier.
- When the load is just series R-L, *the critical inductance is zero* under some conditions.
- If we add load filter capacitance, we have to compute L_{crit}.
- The ripple is no longer triangular, but we still know what is happening.

Rectifier

- If $L = L_{crit}$, we know that the current minimum is exactly zero.
- The current also returns to zero at the end of each period.
- The average current must be consistent with the load