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- Today's topic: Nyquist stability criterion (more examples); phase and gain margins from Nyquist plots.

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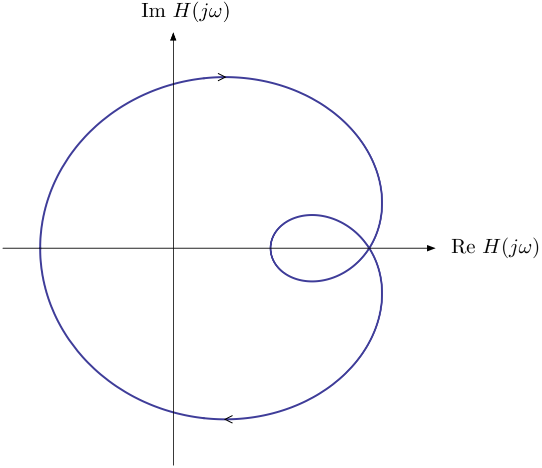
Reading: FPE, Chapter 6

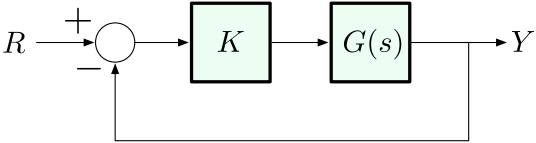
Review: Nyquist Plot

Consider an arbitrary transfer function H.

Nyquist plot: $\operatorname{Im} H(j\omega)$ vs. $\operatorname{Re} H(j\omega)$ as ω varies from $-\infty$ to ∞



Review: Nyquist Stability Criterion

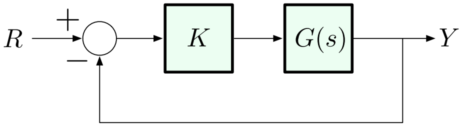


Goal: count the number of RHP poles (if any) of the closed-loop transfer function

$$\frac{KG(s)}{1 + KG(s)}$$

based on frequency-domain characteristics of the plant transfer function $G(s)$

The Nyquist Theorem



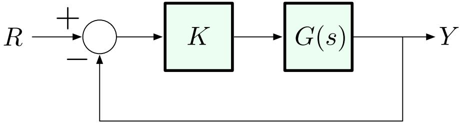
Nyquist Theorem (1928) Assume that $G(s)$ has no poles on the imaginary axis*, and that its Nyquist plot does not pass through the point $-1/K$. Then

$$N = Z - P$$

$$\#(\circlearrowright \text{ of } -1/K \text{ by Nyquist plot of } G(s))$$
$$= \#(\text{RHP closed-loop poles}) - \#(\text{RHP open-loop poles})$$

* Easy to fix: draw an infinitesimally small circular path that goes *around* the pole and stays in RHP

The Nyquist Stability Criterion



$$\underbrace{N}_{\#(\circlearrowright \text{ of } -1/K)} = \underbrace{Z}_{\#(\text{unstable CL poles})} - \underbrace{P}_{\#(\text{unstable OL poles})}$$

$$Z = N + P$$

$$Z = 0 \iff N = -P$$

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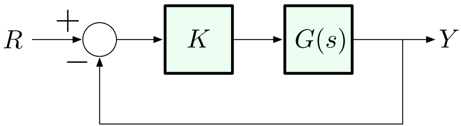

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Nyquist Stability Criterion. Under the assumptions of the Nyquist theorem, the closed-loop system (at a given gain K) is stable *if and only if* the Nyquist plot of $G(s)$ encircles the point $-1/K$ P times *counterclockwise*, where P is the number of unstable (RHP) open-loop poles of $G(s)$.

Applying the Nyquist Criterion

Workflow:

$$\text{Bode } M \text{ and } \phi\text{-plots} \quad \longrightarrow \quad \text{Nyquist plot}$$

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- can work directly with experimental frequency response data (e.g., if we have the Bode plot based on measurements, but do not know the transfer function)

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Advantages of Nyquist over Routh–Hurwitz

- can work directly with experimental frequency response data (e.g., if we have the Bode plot based on measurements, but do not know the transfer function)
- less computational, more geometric (came 55 years after Routh)

Example 1 (From Last Lecture)

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We will now reproduce this answer using the Nyquist criterion.

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— Nyquist plots are always *symmetric w.r.t. the real axis*!!

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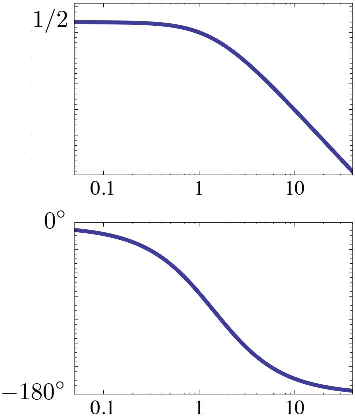

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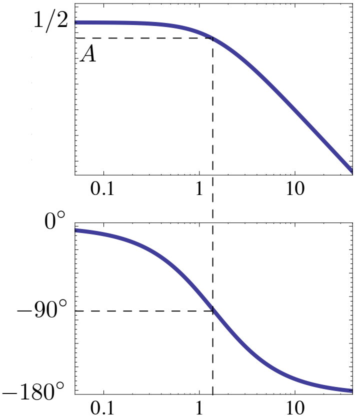

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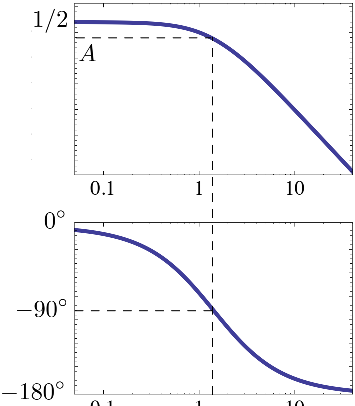

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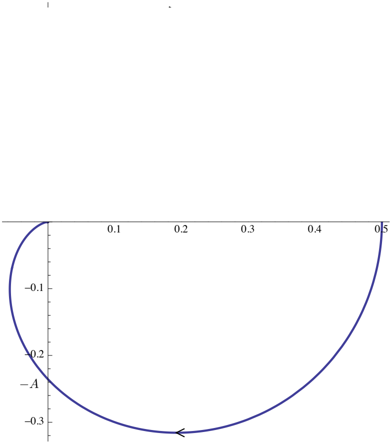

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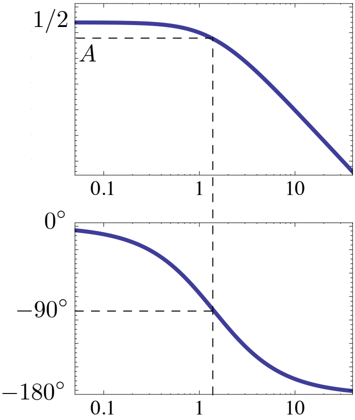

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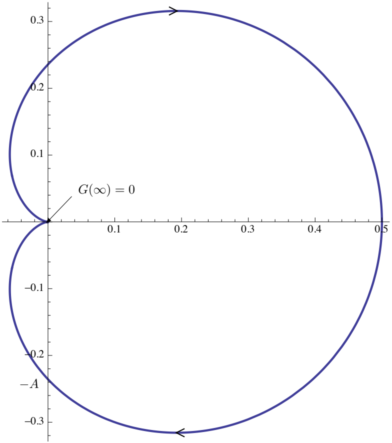

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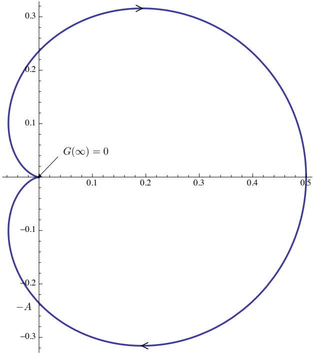

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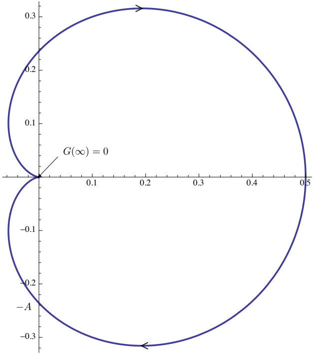

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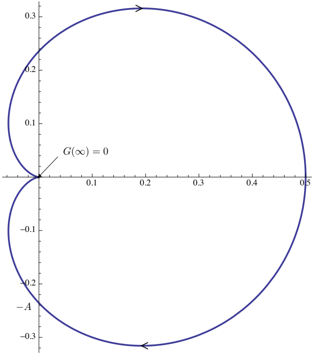

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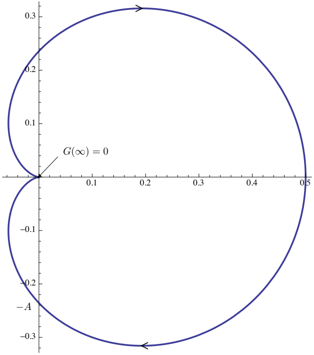

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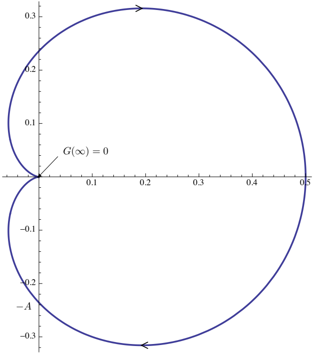

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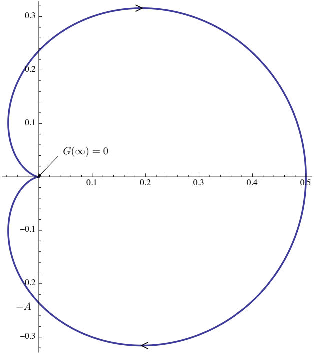

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  closed-loop stable for $K > -2$

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Let's see how to spot this using the Nyquist criterion ...

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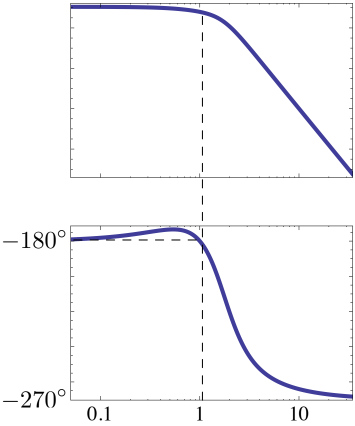

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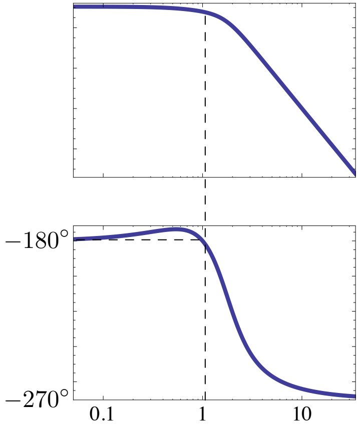

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$\omega = 0 \quad M = 1/3,\ \phi = -180^\circ$

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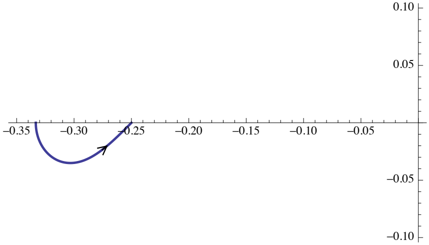

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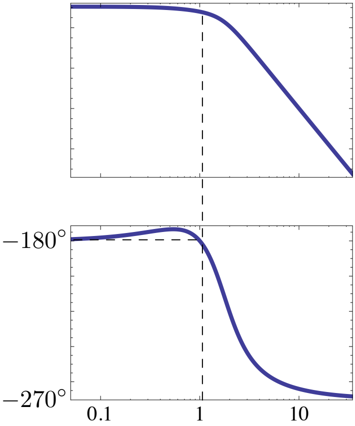

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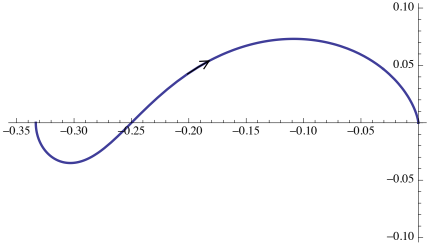

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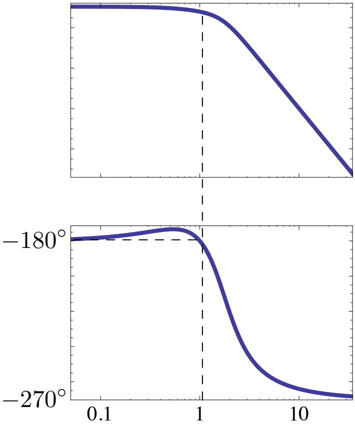

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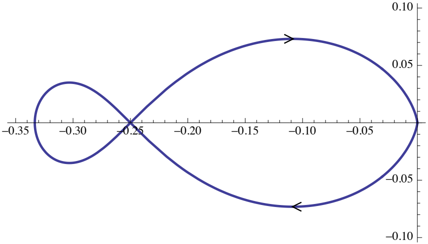

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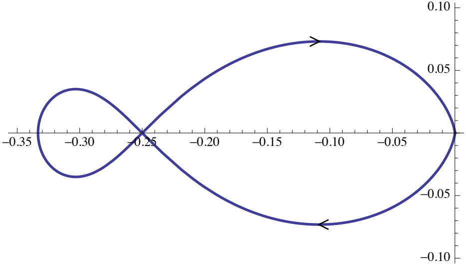

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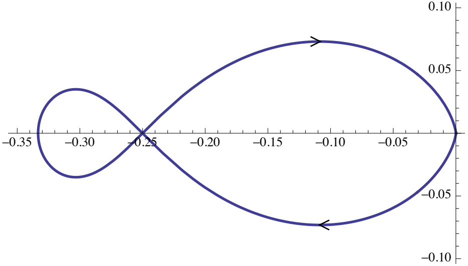

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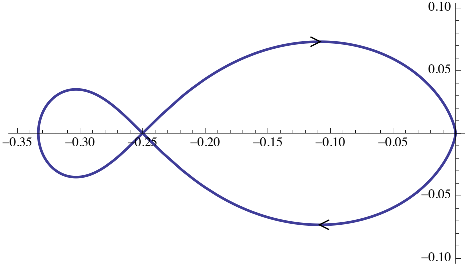

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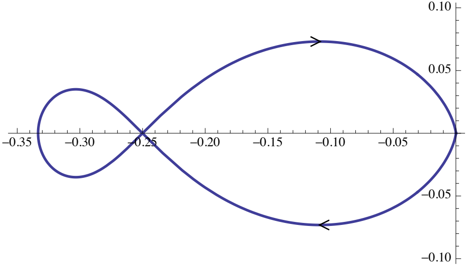

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only $-1/3 < -1/K < -1/4$

$\Longrightarrow 3 < K < 4$

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Closed-loop stability range for $G(s) = \dfrac{1}{(s-1)(s^2+2s+3)}$ is $3 < K < 4$ (using either Routh or Nyquist).

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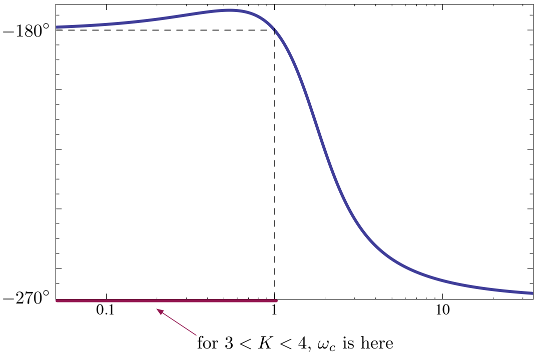

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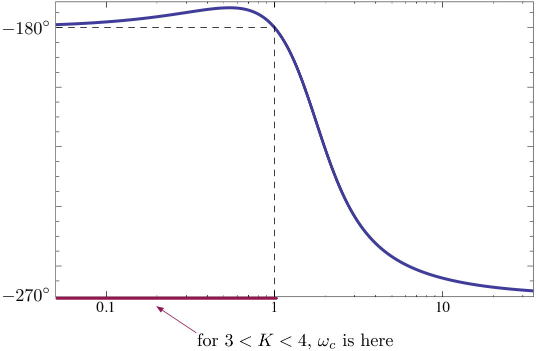

So, in this case, stability $\Longleftrightarrow$ PM > 0 (typical case).

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$\therefore$ 2 RHP poles

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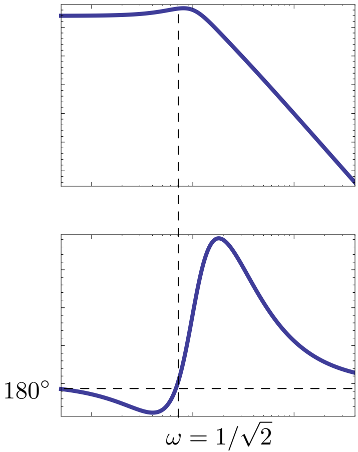

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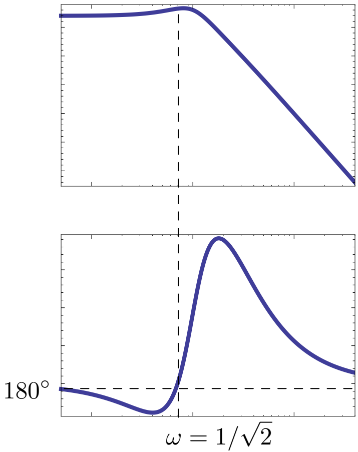

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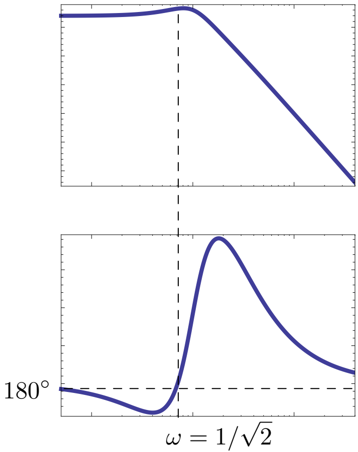

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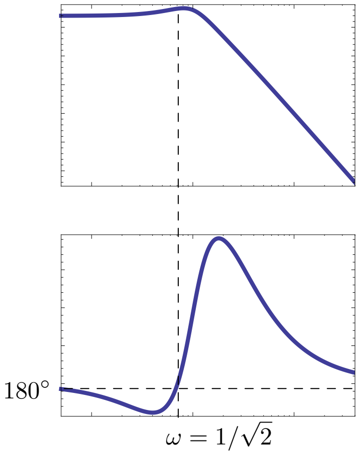

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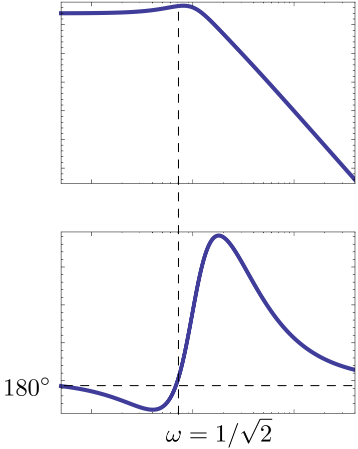

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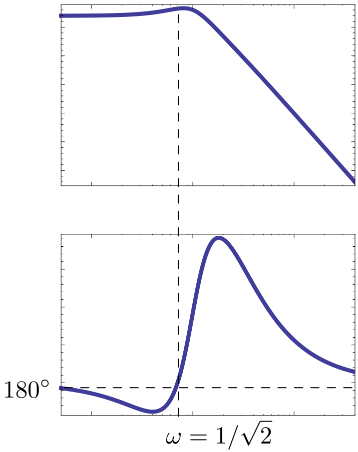

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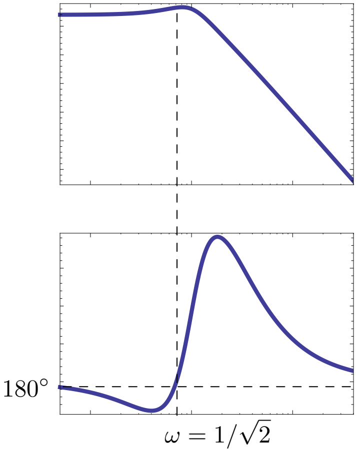

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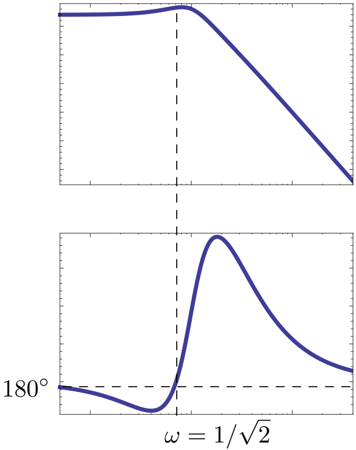

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(need to guess this, e.g., by mouseclicking in Matlab)

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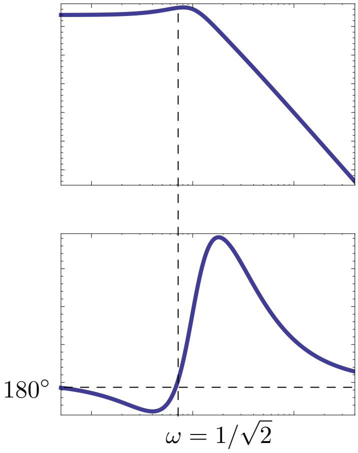

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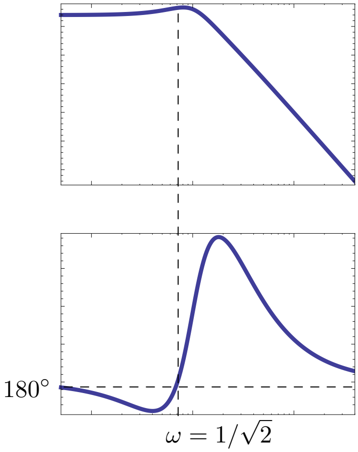
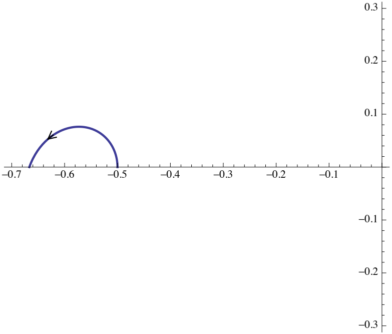


Nyquist plot:

$\omega = 0 \quad M = 1/2,\ \phi = 180°$

$\omega = 1/\sqrt{2} \quad M = 2/3,\ \phi = 180°$



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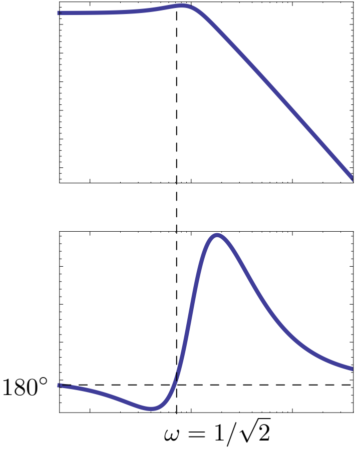

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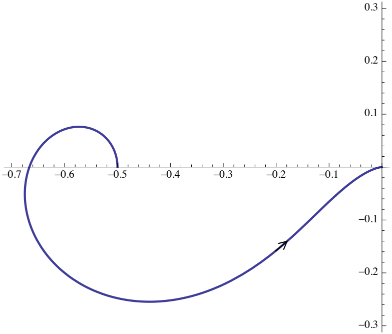

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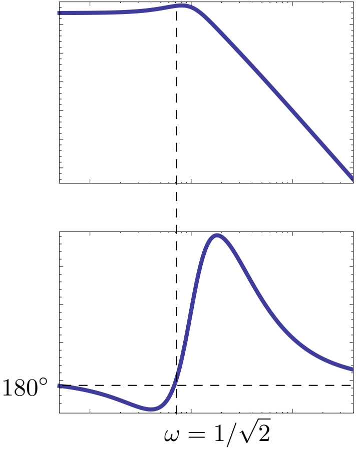

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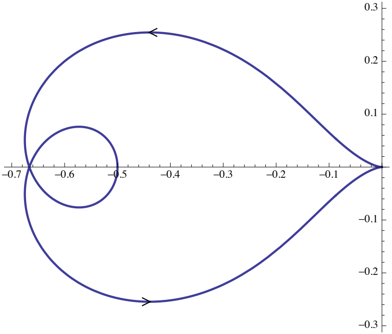

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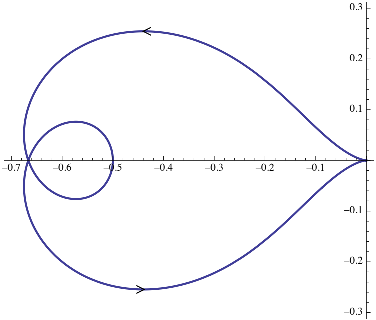

$$\#(\circlearrowright \text{ of } -1/K)$$
$$= \#(\text{RHP CL poles})$$
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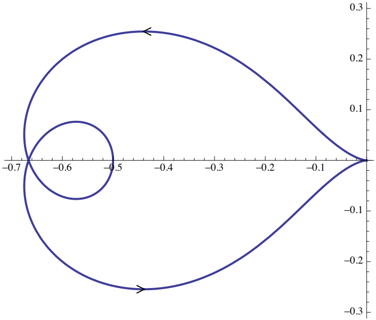

$K \in \mathbb{R}$ is stabilizing if and only if

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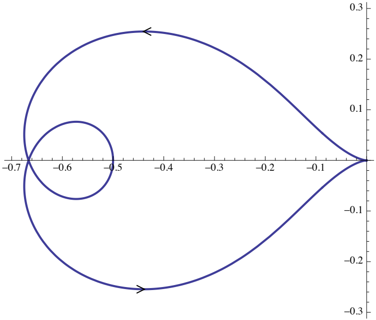

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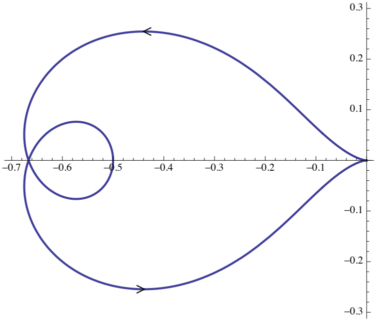

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only $-2/3 < -1/K < -1/2$

$$\Longrightarrow \frac{3}{2} < K < 2$$

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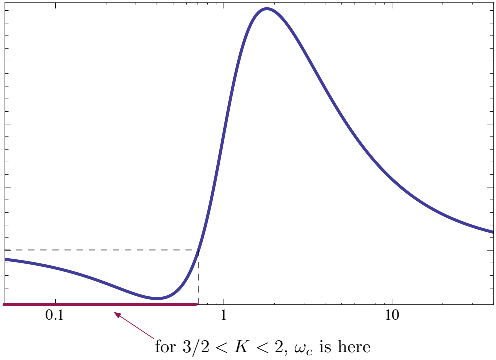

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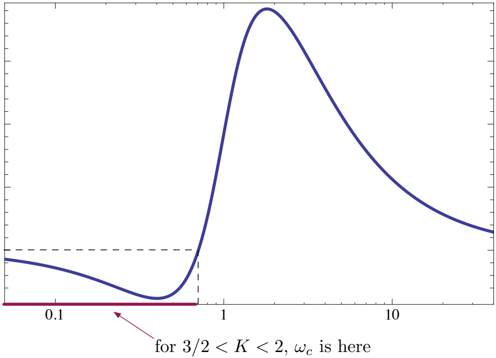

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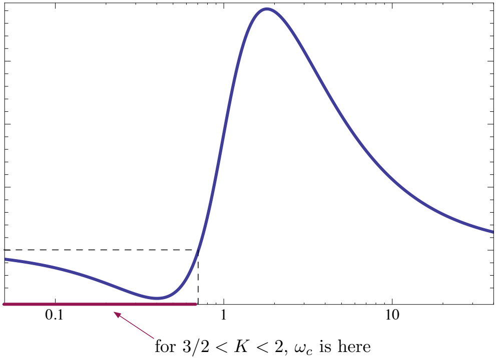
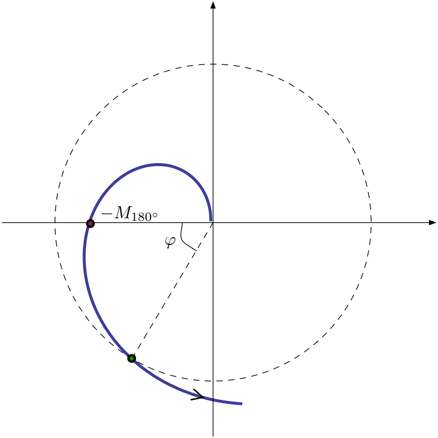


So, in this case, stability $\Longleftrightarrow$ PM < 0 (atypical case; Nyquist criterion is the only way to resolve this ambiguity of Bode plots).

Stability Margins

How do we determine stability margins (GM & PM) from the Nyquist plot?

GM & PM are defined relative to a given K, so consider Nyquist plot of $KG(s)$ (we only draw the $\omega > 0$ portion):



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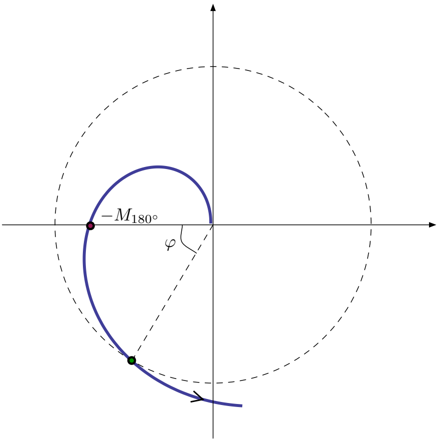

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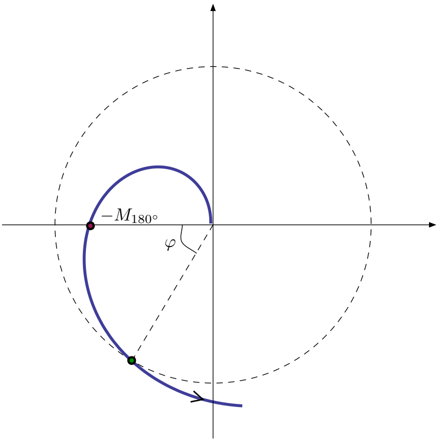

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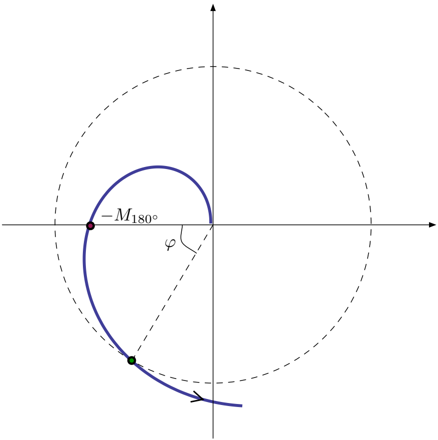

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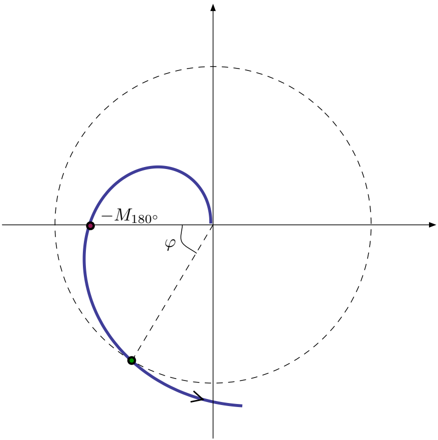

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- $\text{PM} = \varphi$

  — the phase difference from 180° when $M = 1$