ECE 486: Control Systems

Lecture 20A: Frequency Domain Performance

Key Takeaways

Most design requirements can be specified in the frequency domain as bounds:

A) Good reference tracking and disturbance rejection

$$|S(j\omega)| \ll 1 \text{ at low frequencies}$$

B) Good noise rejection

$$|T(j\omega)| \ll 1 \text{ at high frequencies}$$

C) Reasonable control commands

$$|K(j\omega)S(j\omega)| \text{ is bounded}$$

D) Good robustness

$$|S(j\omega)| \leq 2.5 \text{ at all frequencies}$$

Requirements: Closed-Loop Stability + Robustness

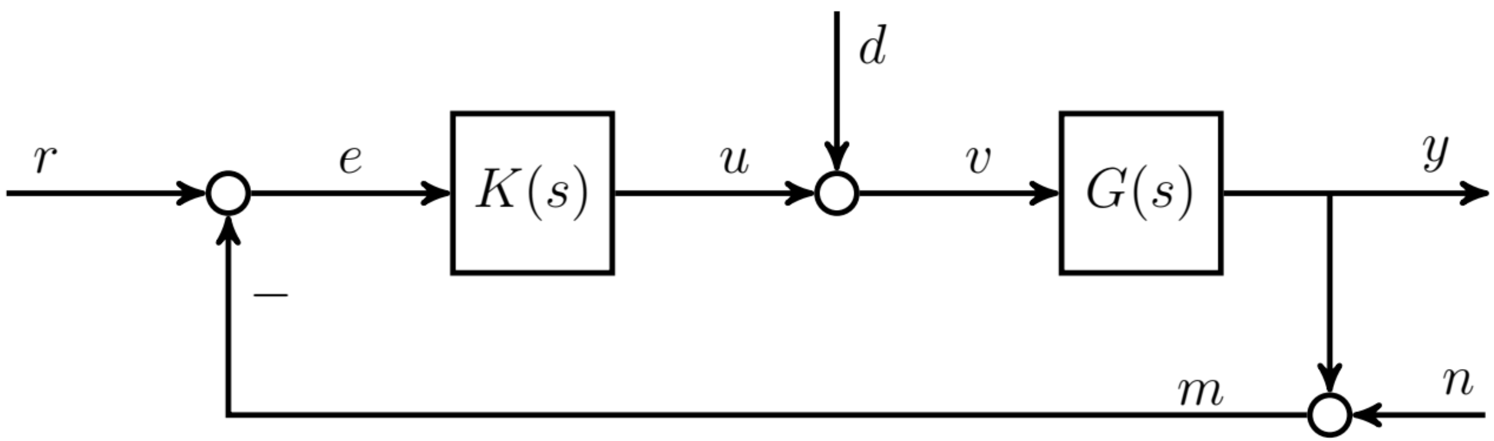
Fact: Closed-loop is stable if and only if all zeros of *1+G(s)K(s)* are in the LHP.

We require:

A) G(s)K(s) has no pole/zero cancellations in the CRHP

B) $S(s) = \frac{1}{1+G(s)K(s)}$ is stable

We also showed previously that $|S(j\omega)| \leq 2.5$ at all frequencies ensures good disk margins.



Requirements: Reference Tracking

Goal: The output *y* should track the reference command *r*.

The transfer function from *r* to *e=r-y* is:

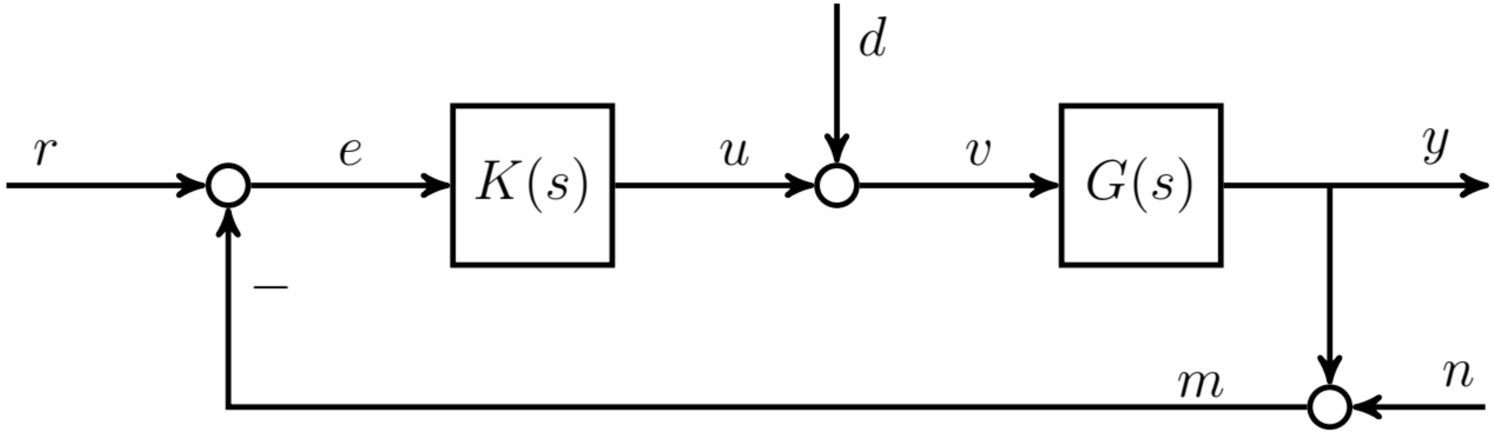
$$S(s) = \frac{1}{1+G(s)K(s)} \quad \text{(Sensitivity)}$$

Consider a sinusoidal reference $r(t) = R_0 \cos(\omega t)$. Then:

$$e(t) \rightarrow |S(j\omega)|R_0 \cos(\, \omega t + \angle S(j\omega)\,)$$

We require $|S(j\omega)| \ll 1$ for good tracking at ω.

If $\omega = 0$ then $r(t) = R_0$ (step) and $e(t) \rightarrow S(j0)R_0$.



Requirements: Disturbance Rejection

Goal: The disturbance d should have small effect on output *y*.

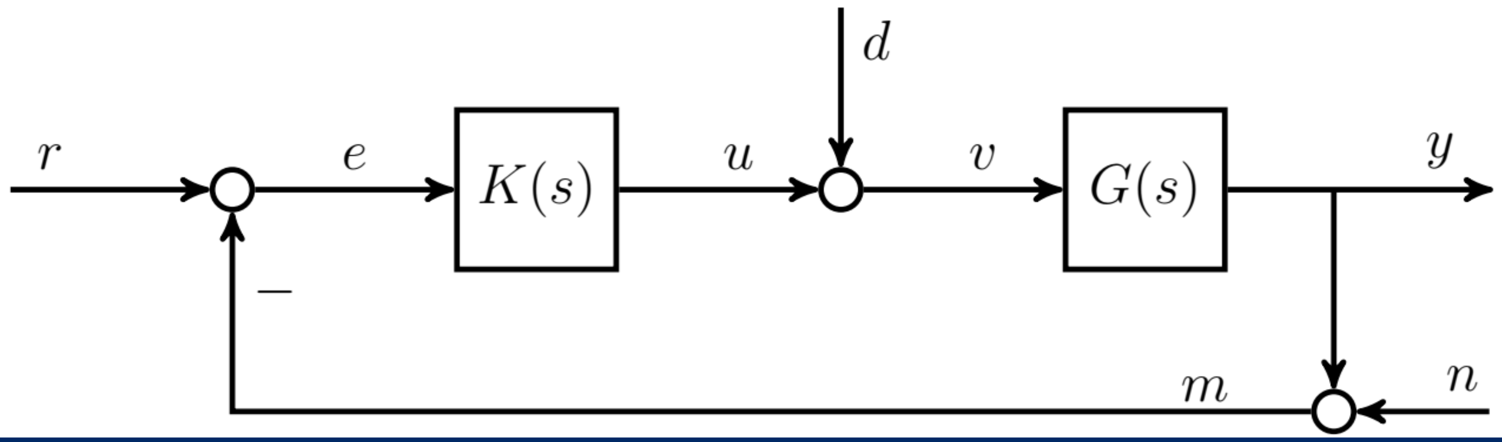
The transfer function from *d* to *y* is G(s) in open loop and *G(s)S(s)* in closed-loop.

Consider a sinusoidal disturbance $d(t) = D_0 \cos(\omega t)$. Then:

(OL) $y(t) \rightarrow |G(j\omega)| D_0 \cos(\,\omega t + \angle G(j\omega\,)$

(CL) $y(t) \rightarrow |G(j\omega)S(j\omega)| D_0 \cos(\,\omega t + \angle G(j\omega)S(j\omega)\,)$

We require $|S(j\omega)| \ll 1$ for good disturbance rejection at ω.



Requirements: Noise Rejection

Goal: The noise n should have small effect on output *y*.

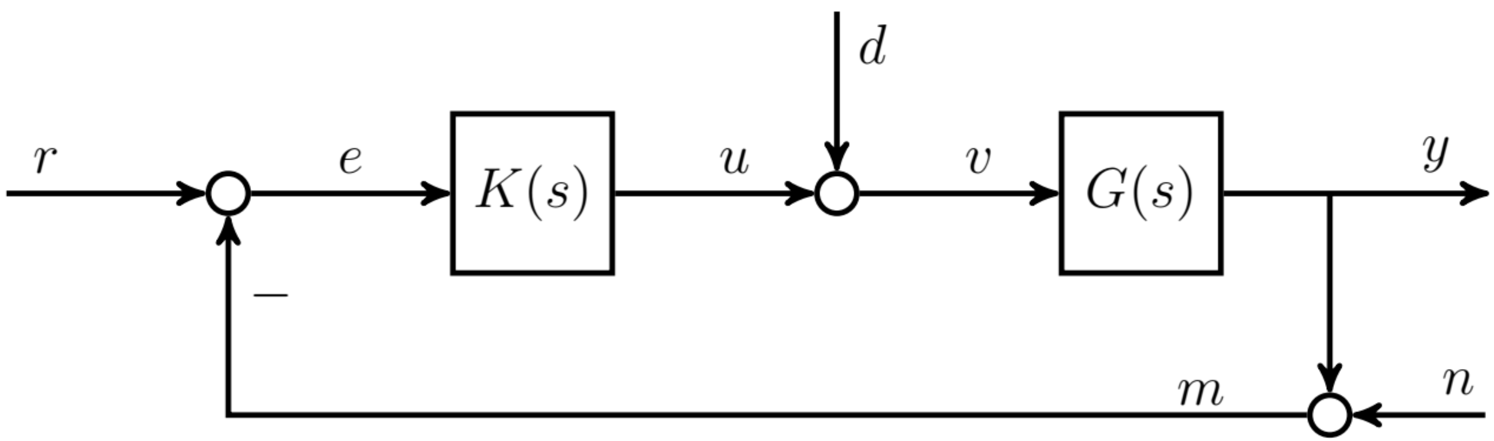
The transfer function from *n* to *y* is –*T(s)* where:

$$T(s) = \frac{G(s)K(s)}{1+G(s)K(s)} \quad \text{(Complementary Sensitivity)}$$

Consider a sinusoidal noise $n(t) = N_0 \cos(\omega t)$. Then:

$$y(t) \to -|T(j\omega)| N_0 \cos(\,\omega t + \angle T(j\omega)\,)$$

We require $|T(j\omega)| \ll 1$ for good noise rejection at ω.



Requirements: Control Effort

Goal: The control *u* should remain within allowable limits.

The transfer function from *r* to *u* is *K(s)S(s)*.

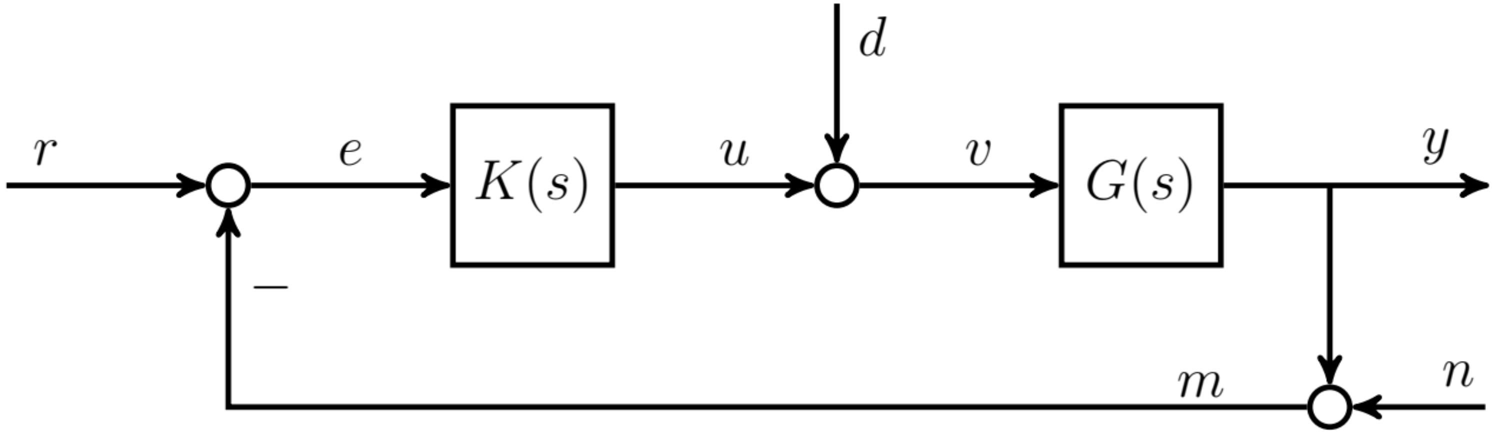
Consider a sinusoidal reference $r(t) = R_0 \cos(\omega t)$. Then:

$$u(t) \to |K(j\omega)S(j\omega)| R_0 \cos(\, \omega t + \angle K(j\omega)S(j\omega) \,)$$

To remain within saturation limits $|u(t)| \leq u_{max}$,

$$|K(j\omega)S(j\omega)| R_0 \leq u_{max} \Rightarrow |K(j\omega)S(j\omega)| \leq \frac{u_{max}}{R_0}$$

We also need to ensure that *n* does not cause large *u*.



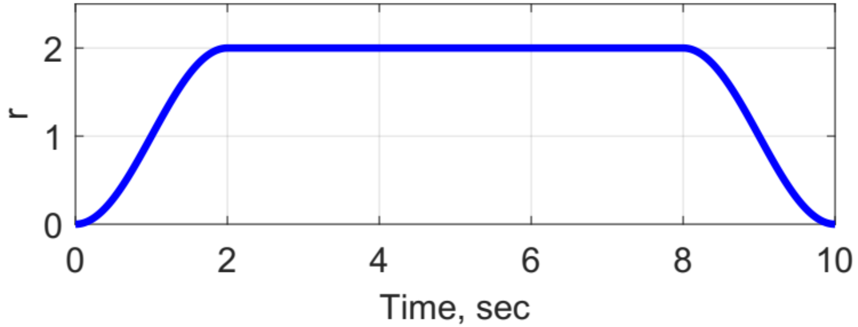
Design Requirements: *S(s)* vs. *T(s)*

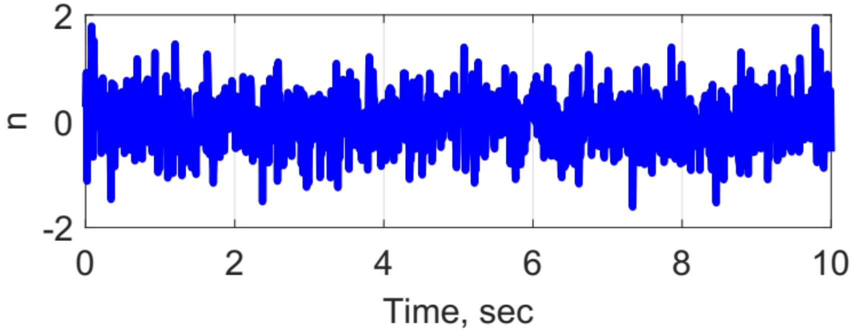
Reference tracking and disturbance rejection: $|S(j\omega)| \ll 1$

Noise rejection: $|T(j\omega)| \ll 1$

However S(s)+T(s)=1 so we can't have both $|S(j\omega)| \ll 1$ and $|T(j\omega)| \ll 1$ at the same frequency. This conflict is resolved by splitting the requirements by frequency:

$|S(j\omega)| \ll 1$ at low frequencies and $|T(j\omega)| \ll 1$ at high frequencies.





Basic Frequency Domain Trade-offs

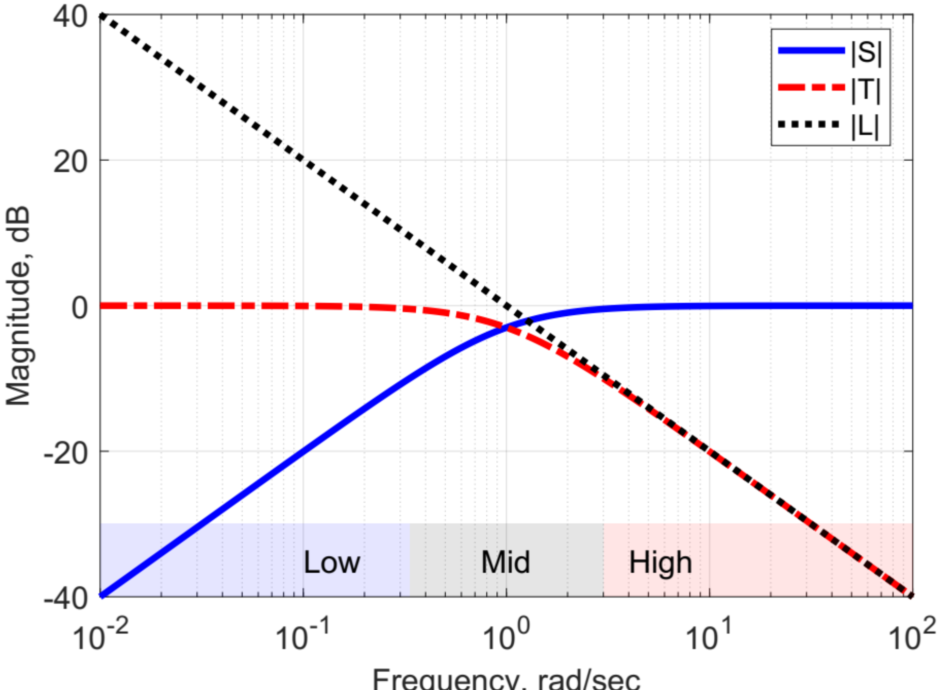
Plant: $\dot{y}(t) = u(t)$ with $G(s) = \frac{1}{s}$

Controller: $u(t) = K_p e(t)$ with $K(s) = K_p$

Loop: $L(s) = G(s)K(s) = \frac{K_p}{s}$

Sensitivity: $S(s) = \frac{1}{1+G(s)K(s)} = \frac{s}{s+K_p}$

Complementary Sensitivity: $T(s) = \frac{G(s)K(s)}{1+G(s)K(s)} = \frac{K_p}{s+K_p}$



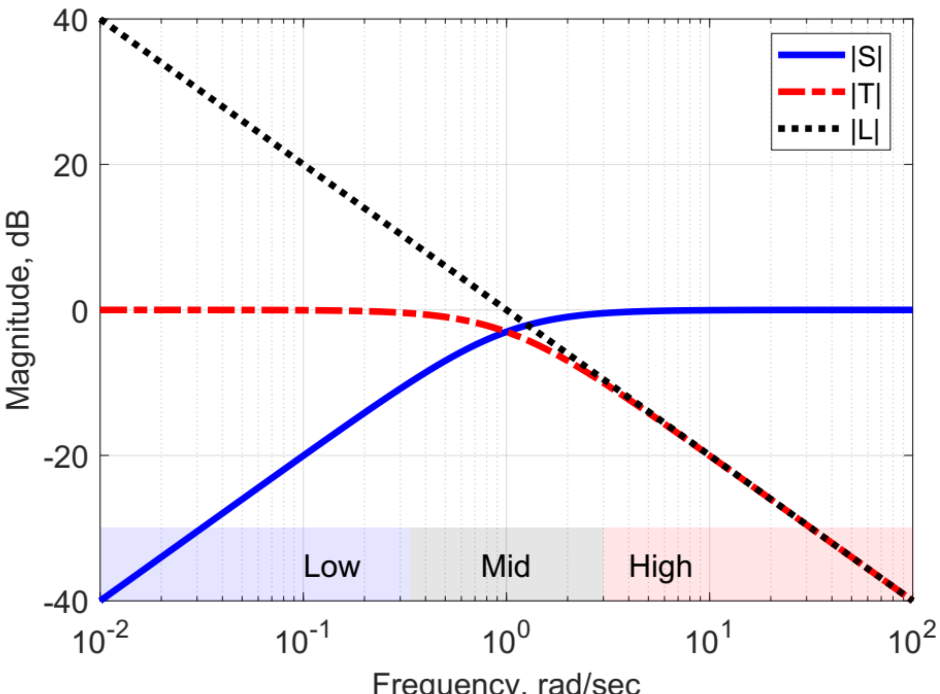
Bode magnitude plots for $K_p = 1$.

Basic Frequency Domain Trade-offs

Low Frequencies: Good reference tracking and disturbance rejection but poor noise rejection.

High Frequencies: Good noise rejection but poor reference tracking and disturbance rejection.

Middle Frequencies: Loop bandwidth ω_L is where $|L(j\omega_L)| = 1$.



Loop bandwidth:

$$L(s) = \frac{K_p}{s} \Rightarrow \omega_L = K_p$$

Closed-loop time constant:

$$S(s) = \frac{s}{s+K_p} \Rightarrow \tau = \frac{1}{K_p}$$

Higher bandwidths correspond to faster response.

Basic Frequency Domain Trade-offs

Low Frequencies: Good reference tracking and disturbance rejection but poor noise rejection.

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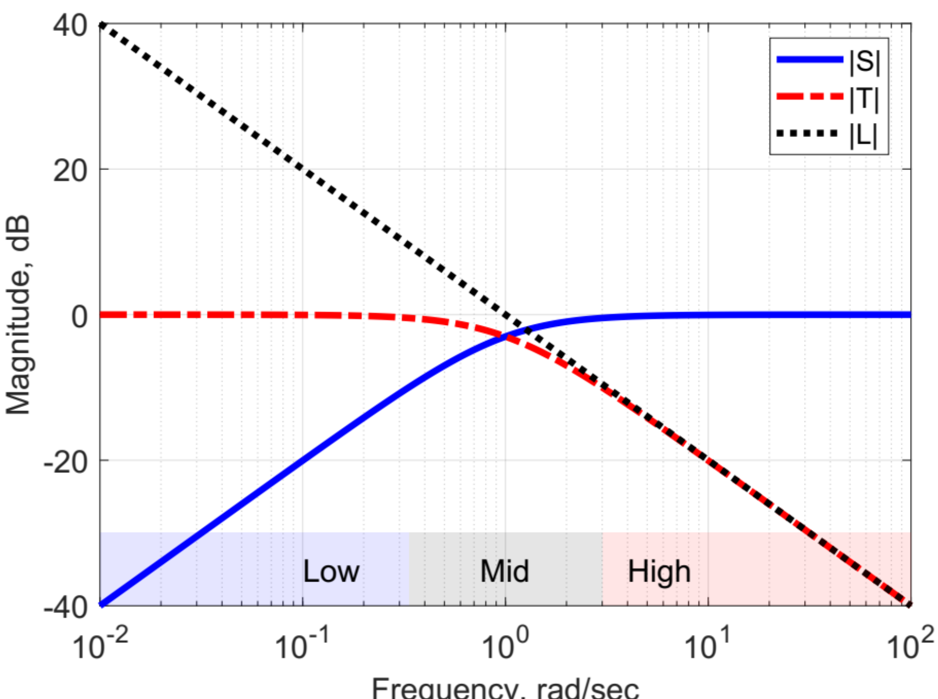

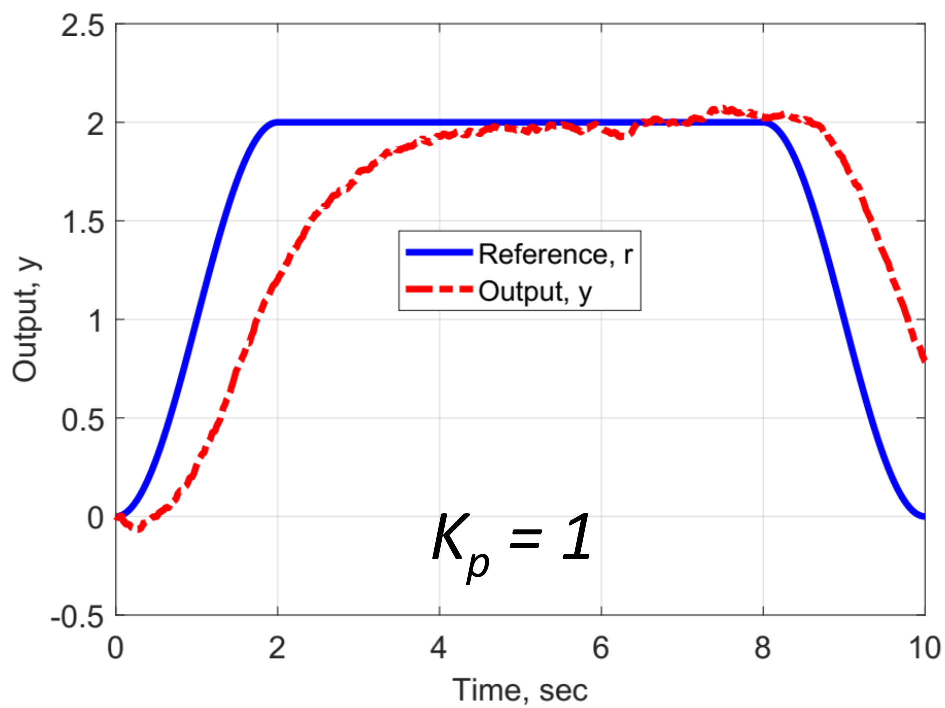

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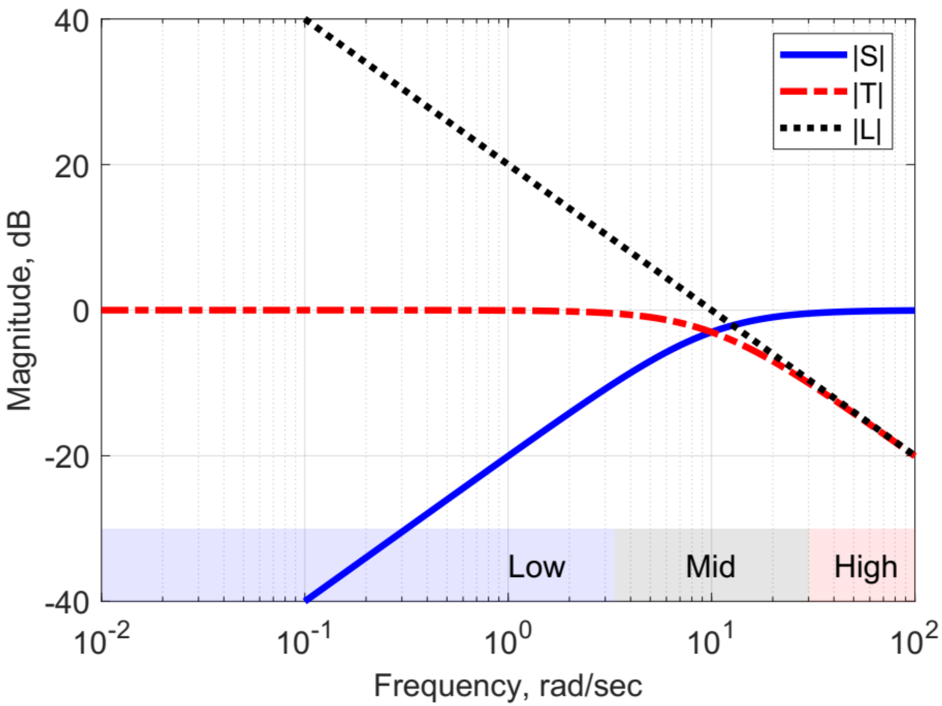
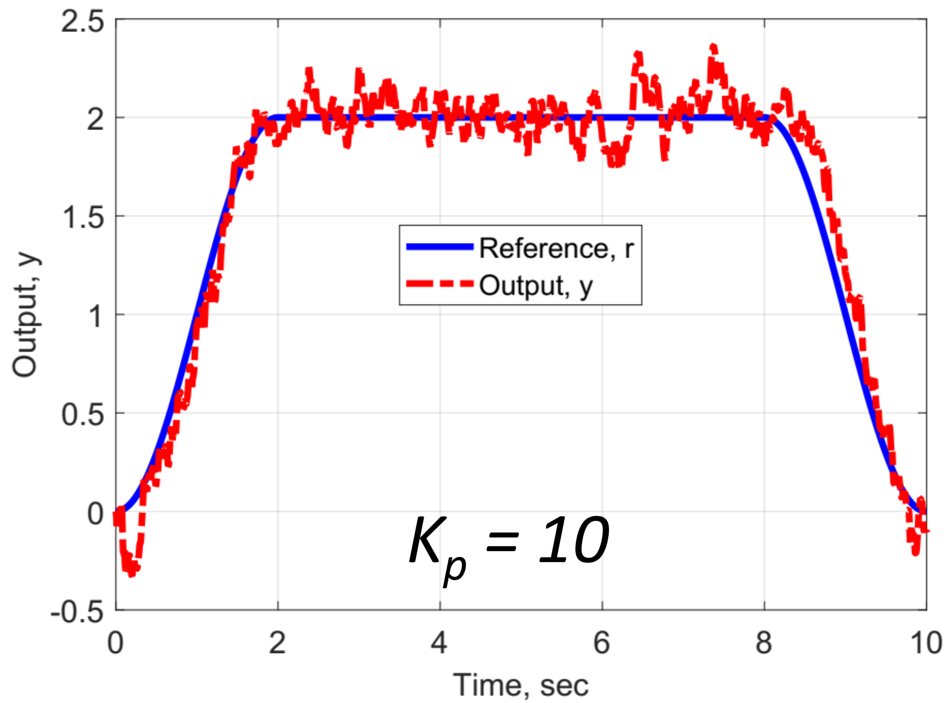


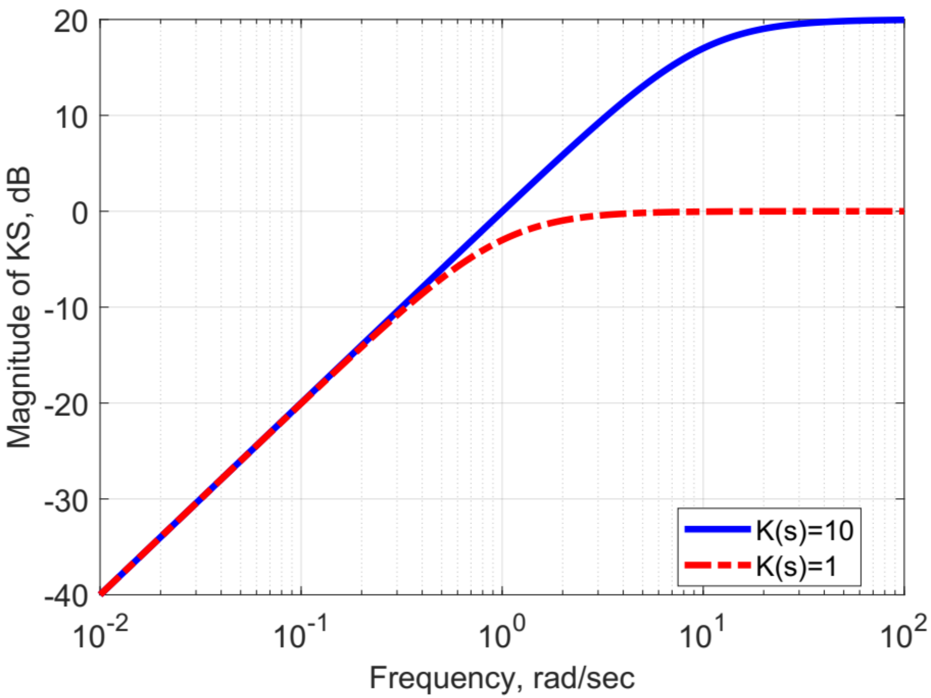


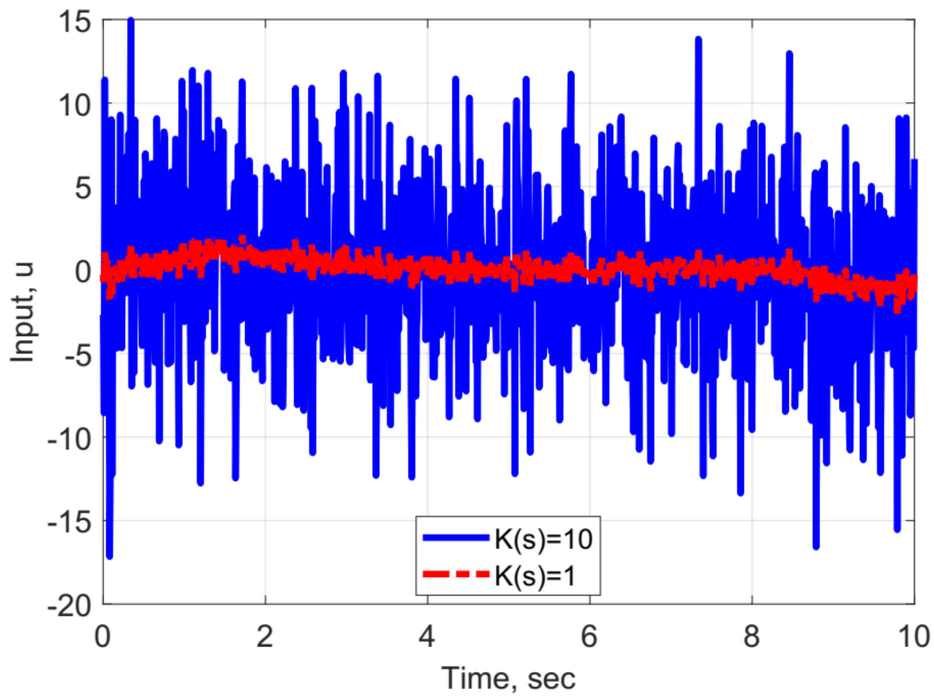
Control Effort

Plant: $\dot{y}(t) = u(t)$ with $G(s) = \frac{1}{s}$

Controller: $u(t) = K_p e(t)$ with $K(s) = K_p$

Closed-loop *r* to *u*: $K(s)S(s) = \frac{K_p s}{s+K_p}$





ECE 486: Control Systems

Lecture 20B: Introduction to Loopshaping

Key Takeaways

Loopshaping is a design method that focuses on the loop L(s).

We build the controller from components targeting low, middle, and high frequencies.

Low Frequencies: Good reference tracking / disturbance rejection.

$$|S(j\omega)| \ll 1 \Leftrightarrow |L(j\omega)| \gg 1$$

High Frequencies: Good noise rejection.

$$|T(j\omega)| \ll 1 \Leftrightarrow |L(j\omega)| \ll 1$$

Middle Frequencies (Crossover Region):

Speed of Response: Loop bandwidth ω_L such that $|L(j\omega_L)| = 1$

Stability/Robustness: Transition with a shallow slope.

Speed of Response: Bandwidth

For first- and second-order systems we used settling time and/or rise time as measures of the speed of response.

For higher-order systems, an alternative frequency domain notion for speed of response is useful: bandwidth.

1. **Loop Bandwidth, $\boldsymbol{\omega_L}$**: Smallest frequency with $|L(j\omega_L)| = 1$.
2. **Sensitivity Bandwidth, $\boldsymbol{\omega_S}$**: Highest frequency such that

$$|S(j\omega)| \le \frac{1}{\sqrt{2}} = -3dB \text{ for all } \omega \le \omega_S$$

3. **Complementary Sensitivity Bandwidth, $\boldsymbol{\omega_T}$**: Lowest frequency such that

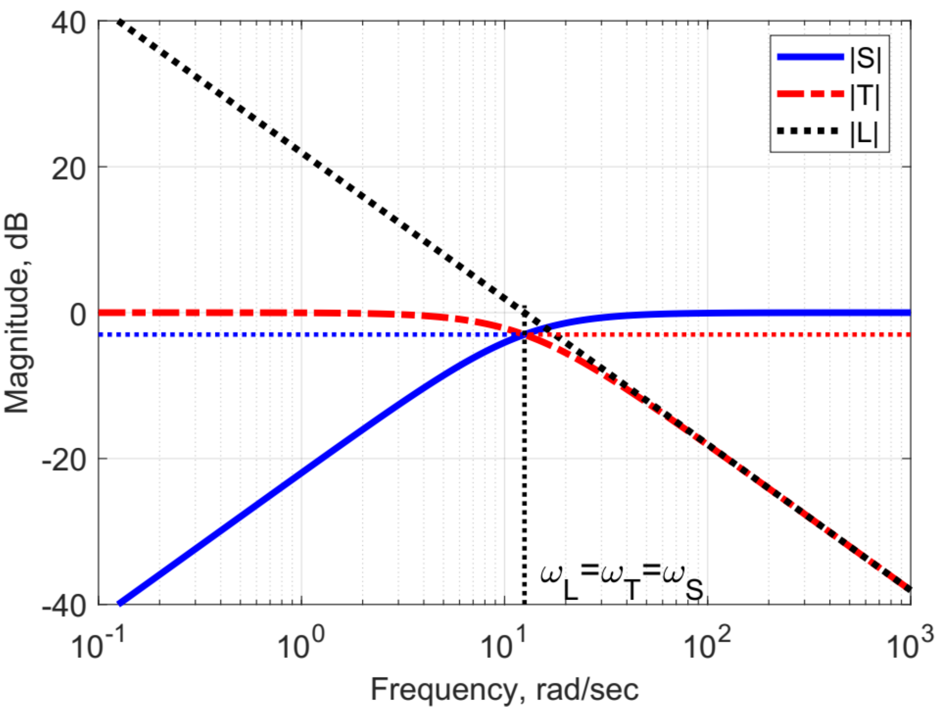
$$|T(j\omega)| \le \frac{1}{\sqrt{2}} = -3dB \text{ for all } \omega \ge \omega_T$$

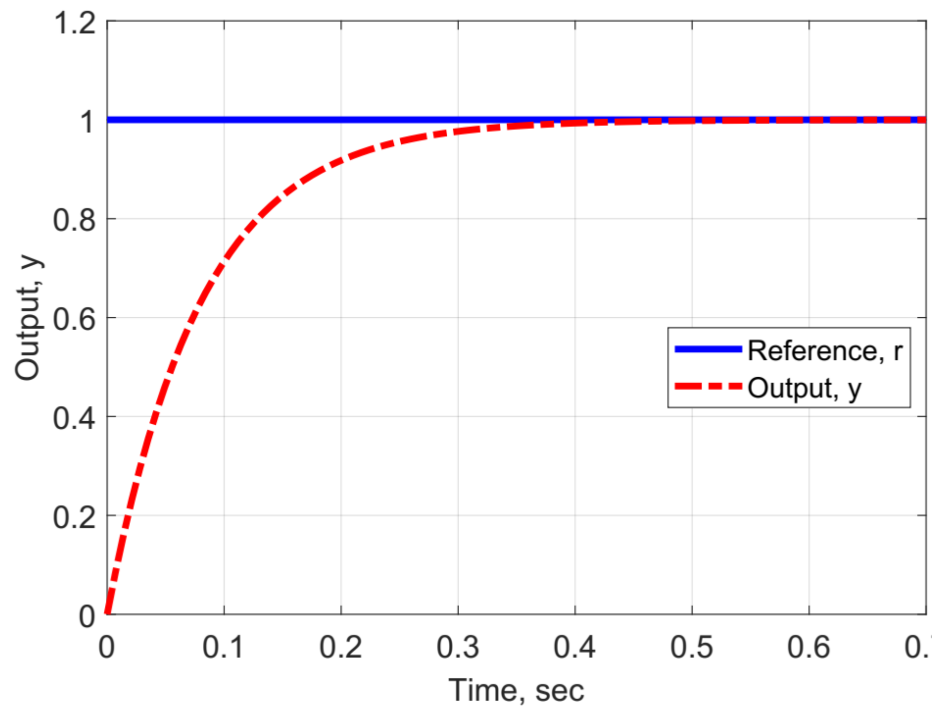
Speed of Response: Bandwidth

Example: $G(s) = \frac{1}{s}$ and $K(s) = 12.5$

Bandwidths: $\omega_L = \omega_T = \omega_S = 12.5\frac{rad}{sec}$

Note that $S(s) = \frac{s}{s+12.5} \Rightarrow$ Time Constant $\tau = \frac{1}{12.5}sec = \frac{1}{\omega_L}$



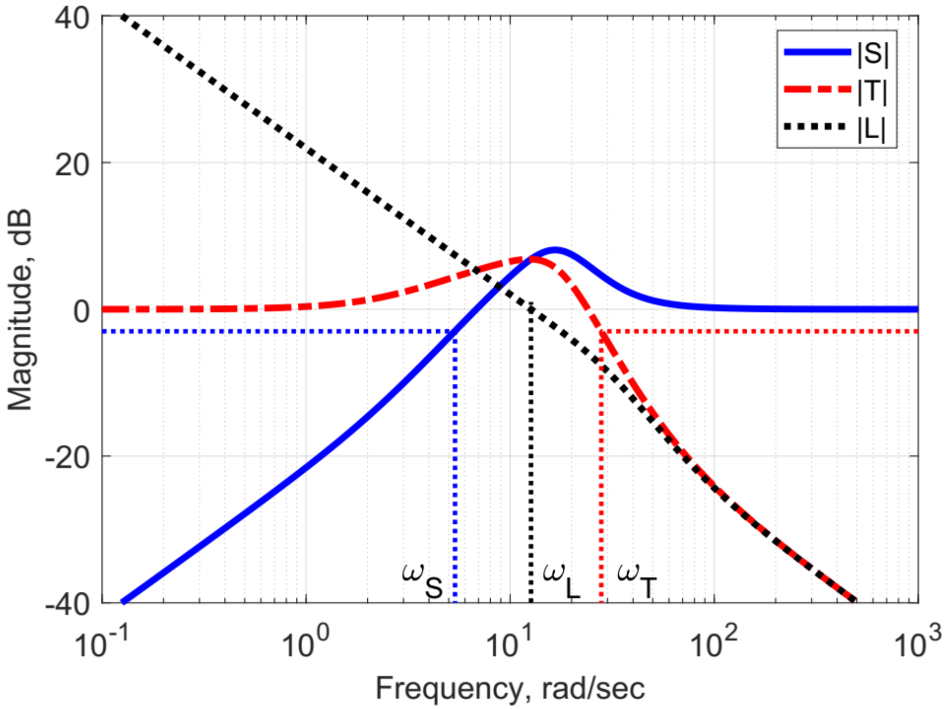


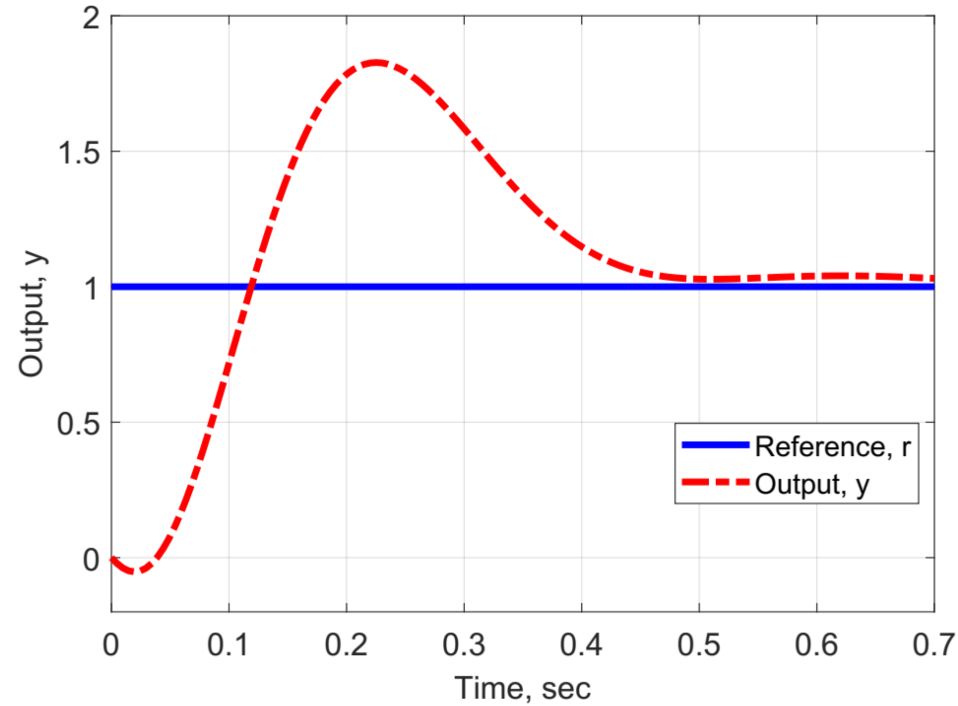
Speed of Response: Bandwidth

Example: $G(s) = \frac{-0.5s^2+1250}{s^3+47s^2+850s-3000}$ and $K(s) = \frac{10s+30}{s}$

Bandwidths: $\omega_S = 5\frac{rad}{sec}, \omega_L = 12.5\frac{rad}{sec}, \omega_T = 28\frac{rad}{sec}$

Settling Time is $\approx 0.6sec = \frac{3}{\omega_S}$





Bode Gain-Phase Relation

Loopshaping focuses on $|L(j\omega)|$ with less emphasis on $\angle L(j\omega)$.

Fact: Assume *L(s)* has all poles and zeros in the LHP. Then:

$$\angle L(j\omega_0) \approx \angle L(0) + \frac{90^o}{20dB} \times \left.\frac{d|L(j\omega)|_{dB}}{d\log_{10}\omega}\right|_{\omega=\omega_0}$$

Comments:

1. The approximation is accurate if the slope is roughly constant for $\omega \in [\frac{\omega_0}{\sqrt{10}}, \sqrt{10}\omega_0]$.

2. The approximation arises from an exact formula by Bode.

3. The phase change from $\omega = 0$ to ω_0 is $\pm 90^o$ for every $\pm 20\frac{dB}{decade}$ of slope.

Bode Gain-Phase Relation

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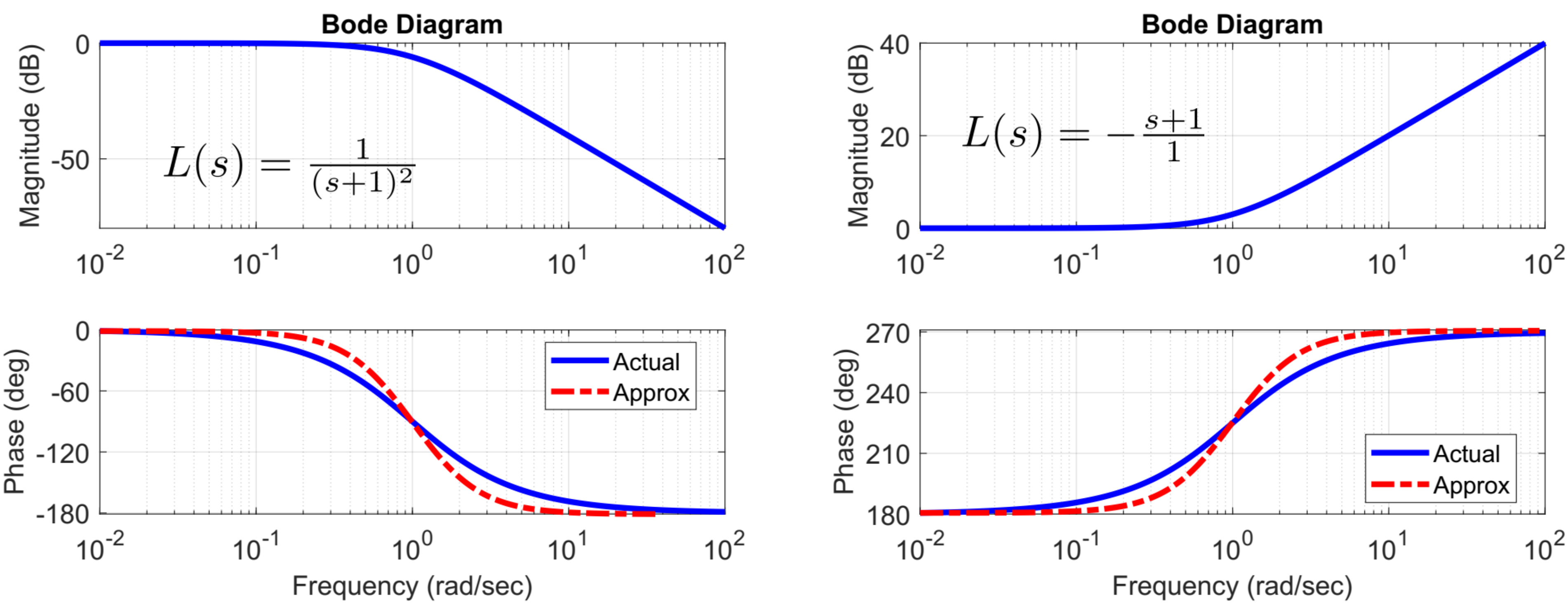


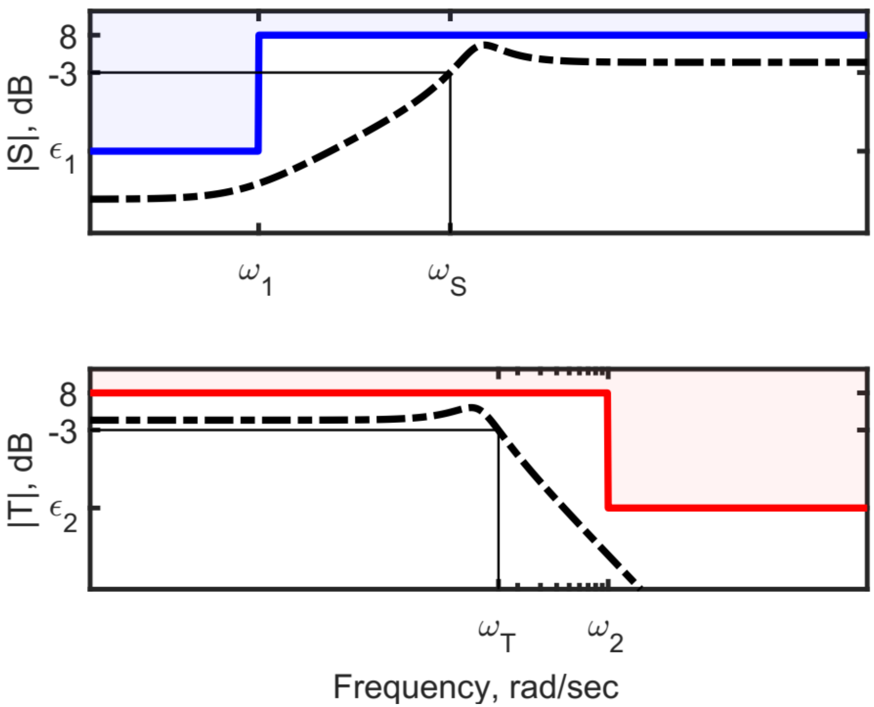
Requirements on the Loop *L(s)*

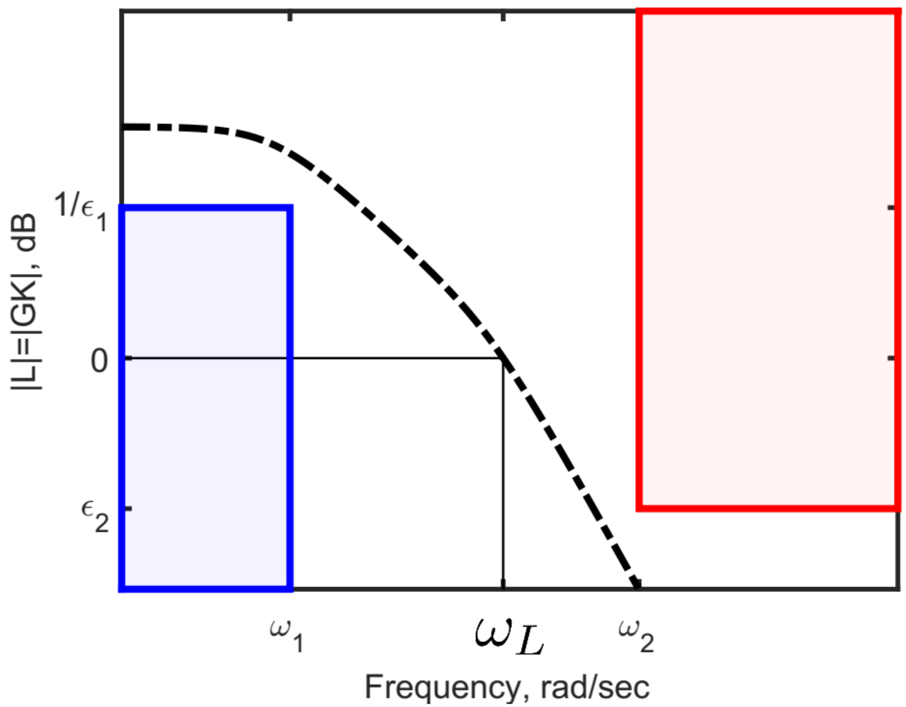
Recall $L(s) = G(s)K(s), S(s) = \frac{1}{1+L(s)}, T(s) = \frac{L(s)}{1+L(s)}$

Low Frequencies: $|S(j\omega)| \ll 1 \Leftrightarrow |L(j\omega)| \gg 1$

Note: $|L(j\omega)| \gg 1 \Leftrightarrow |K(j\omega)S(j\omega)| \approx \frac{1}{|G(j\omega)|}$

High Frequencies: $|T(j\omega)| \ll 1 \Leftrightarrow |L(j\omega)| \ll 1$



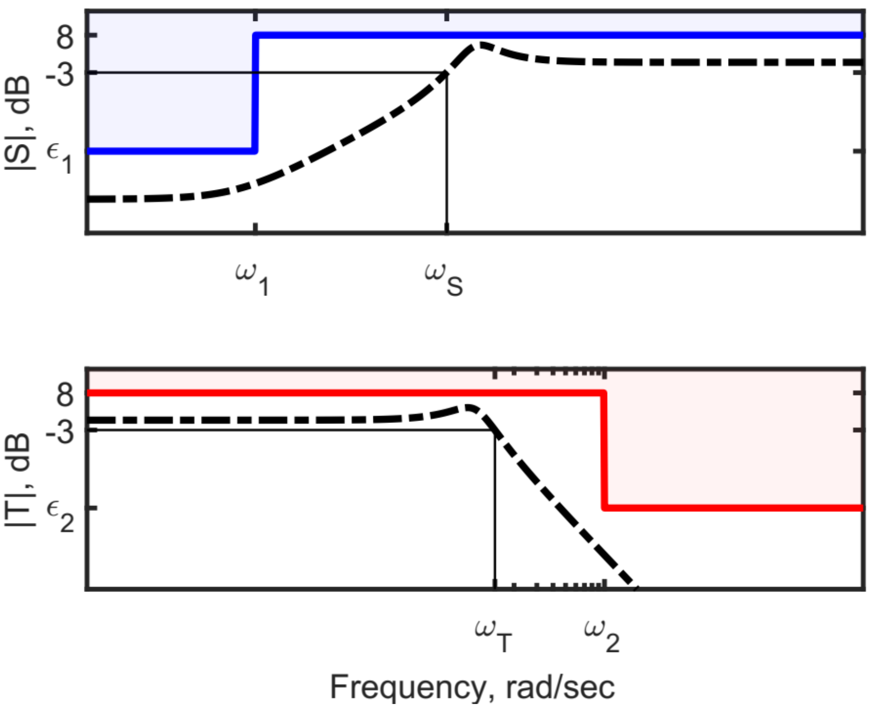


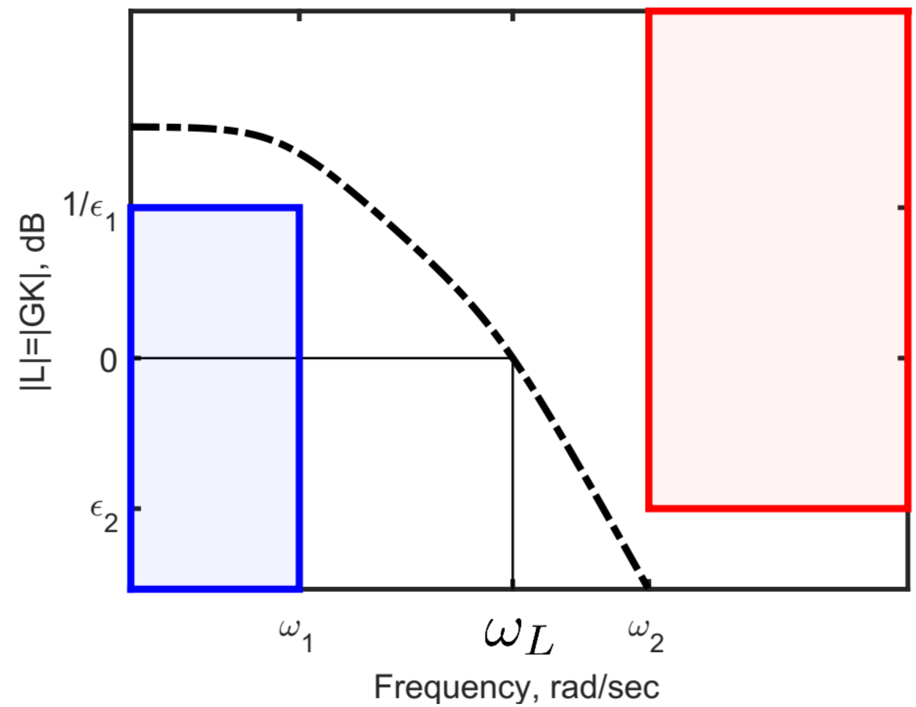
Requirements on the Loop *L(s)*

Middle Frequencies (Crossover Region): The slope near ω_L should not be too steep to ensure stability and robustness.

-A slope of $\approx -40\frac{dB}{dec}$ means $\angle L(j\omega) \approx -180^o$ and closed-loop will be unstable and/or have poor phase margins.

-Slope should not be steeper than $\approx -30\frac{dB}{dec}$ to ensure 45° margin.





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Lecture 20C: Controller Components For Loopshaping

Key Takeaways

Loopshaping builds controllers from the following components:

A) Proportional Gain: A gain (> 1) increases the loop magnitude at all frequencies. This increases bandwidth and reduces steady state error but degrades noise rejection.

B) Integral Boost: Increases the low frequency gain but leaves the high frequencies unchanged. This gives zero steady state error but has negligible effect on bandwidth and noise sensitivity.

C) High Frequency Roll-off: Decreases the high frequency gain but leaves the low frequencies unchanged. This improves noise rejection but has negligible effect on bandwidth and steady-state error.

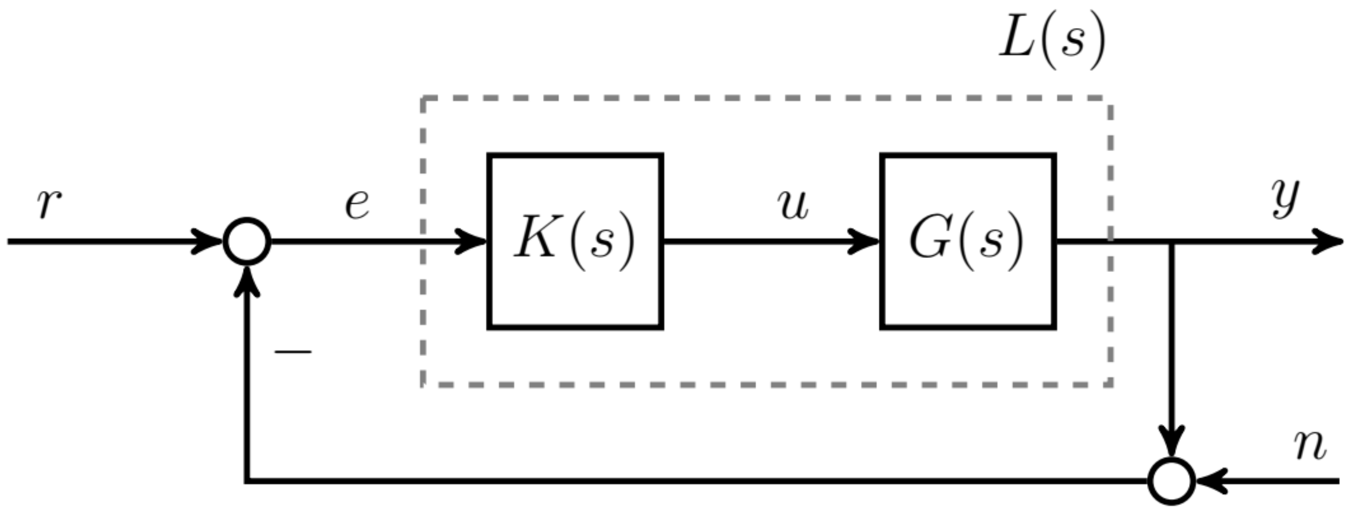
D) Lead: Makes the slope more shallow near the crossover frequency. This improves robustness but it slightly degrades both the low frequency tracking and high frequency noise rejection.

Example

Plant: $\dot{y}(t) + 2y(t) = 5u(t)$ and $G(s) = \frac{5}{s+2}$

Control: $K(s) = 1$

$$L(s) = G(s)K(s) = G(s), \quad S(s) = \frac{1}{1+G(s)} = \frac{s+2}{s+7}, \quad T(s) = \frac{G(s)}{1+G(s)} = \frac{5}{s+7}$$



Example

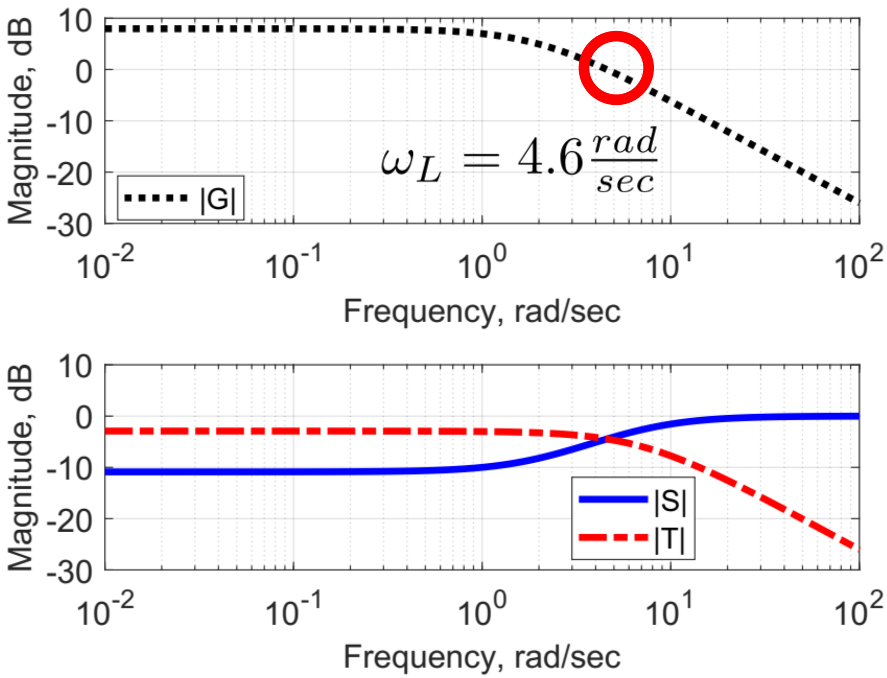
Plant: $\dot{y}(t) + 2y(t) = 5u(t)$ and $G(s) = \frac{5}{s+2}$

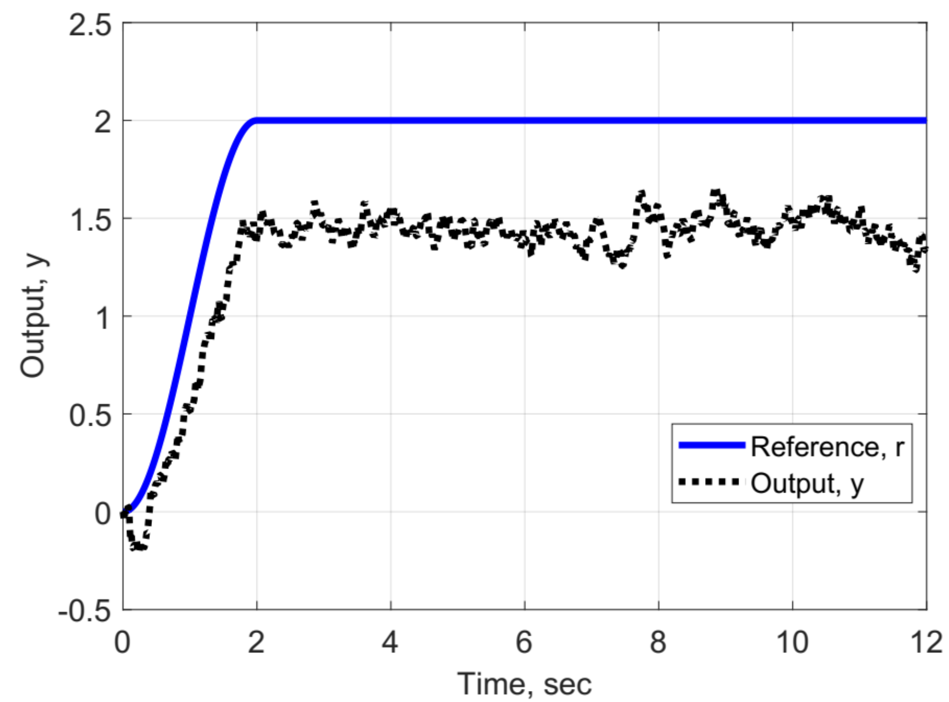
Control: $K(s) = 1$

$$L(s) = G(s)K(s) = G(s), \quad S(s) = \frac{1}{1+G(s)} = \frac{s+2}{s+7}, \quad T(s) = \frac{G(s)}{1+G(s)} = \frac{5}{s+7}$$

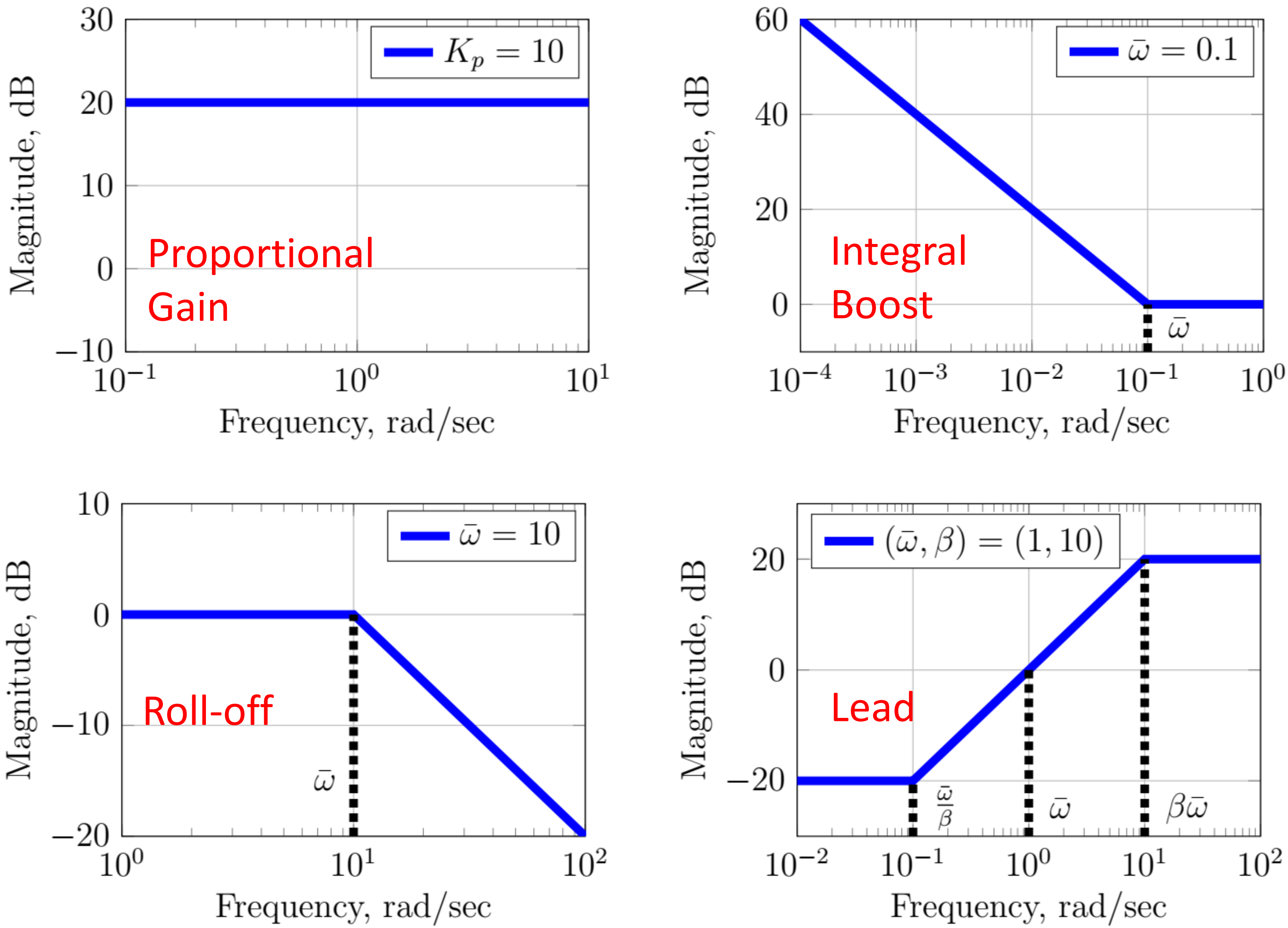
$$\dot{e}(t) + 7e(t) = \dot{r}(t) + 2r(t) \quad \Rightarrow \text{If } r(t) \rightarrow \bar{r} \text{ then } e(t) \rightarrow S(0)\bar{r}$$

$$\Rightarrow r(t) \rightarrow 2 \text{ then } e(t) \rightarrow \frac{2}{7} \cdot 2 \approx 0.57$$





Controller Components



Proportional Gain

Proportional Gain: $K(s) = K_p$

Recall the following fact for Bode magnitudes in dB:

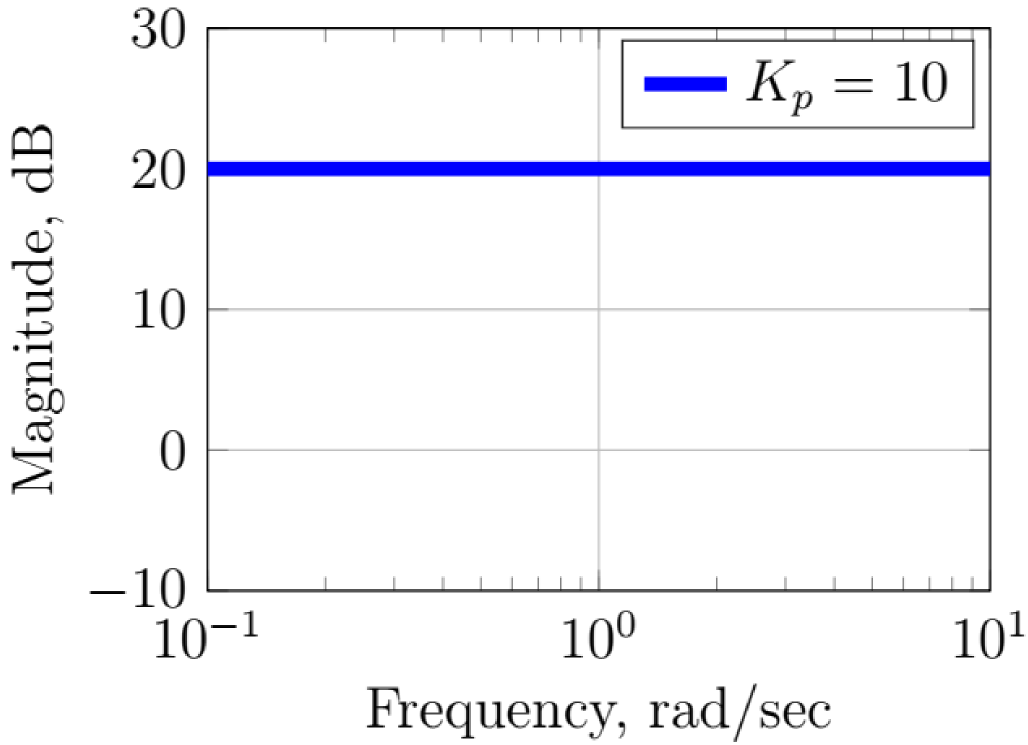
$$20\log_{10}|G(j\omega)K(j\omega)| = 20\log_{10}|G(j\omega)| + 20\log_{10}|K(j\omega)|$$

Properties:

-If Kp>1 then gain shifts entire loop mag. up.

-If Kp<1 then gain shifts entire loop mag. down.

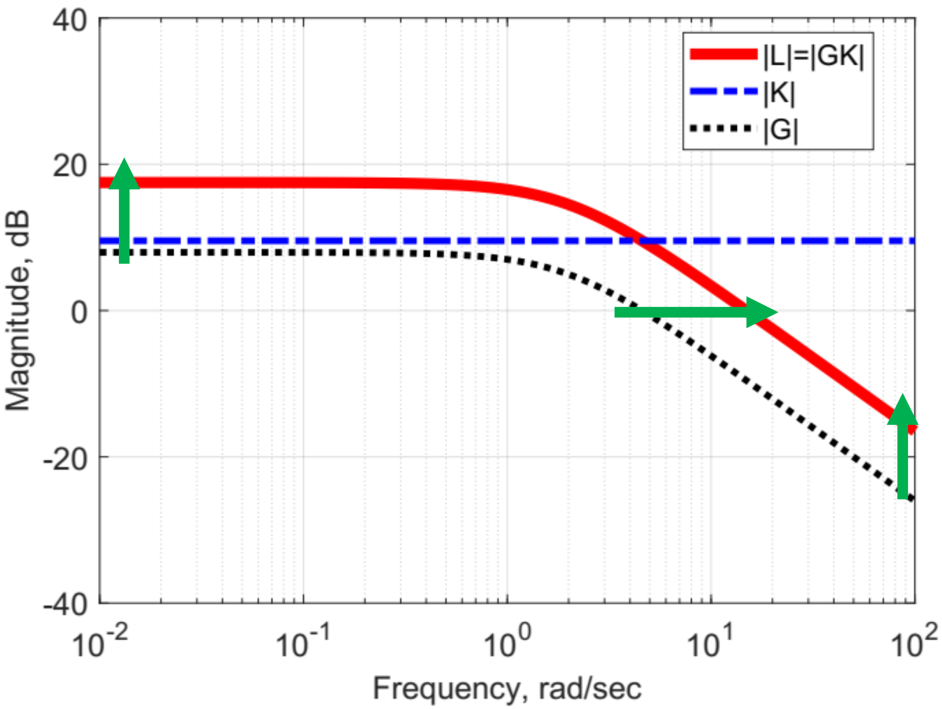
Proportional gain is used to set the loop bandwidth (crossover frequency).

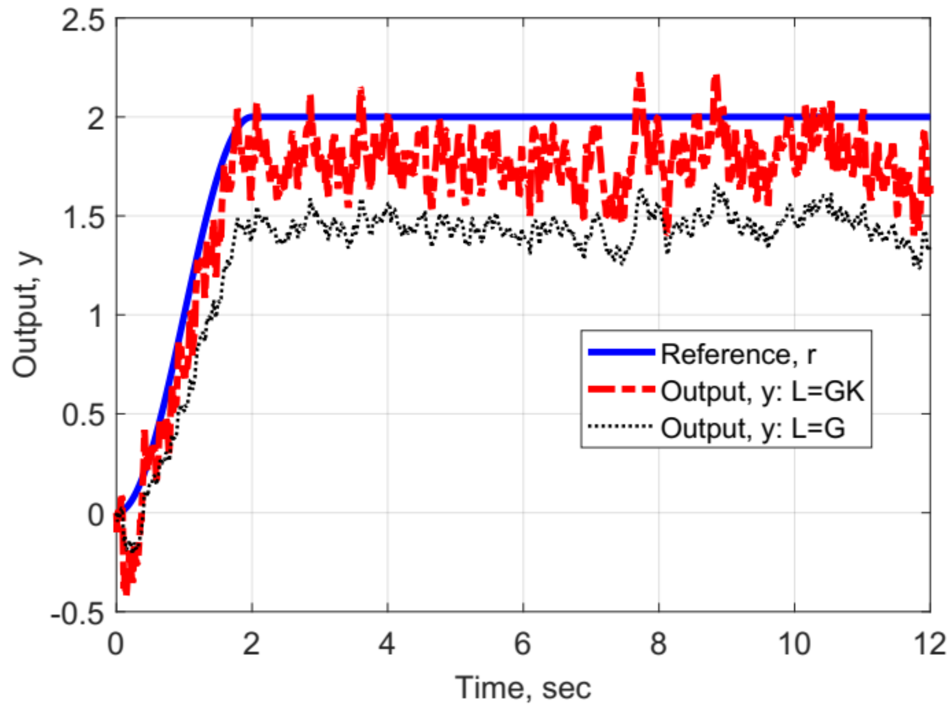


Effect of Proportional Gain

Plant: $\dot{y}(t) + 2y(t) = 5u(t)$ and $G(s) = \frac{5}{s+2}$

Control: $K(s) = 3 = 9.5dB$





Integral Boost

Integral Boost: $K(s) = \frac{s+\overline{\omega}}{s}$

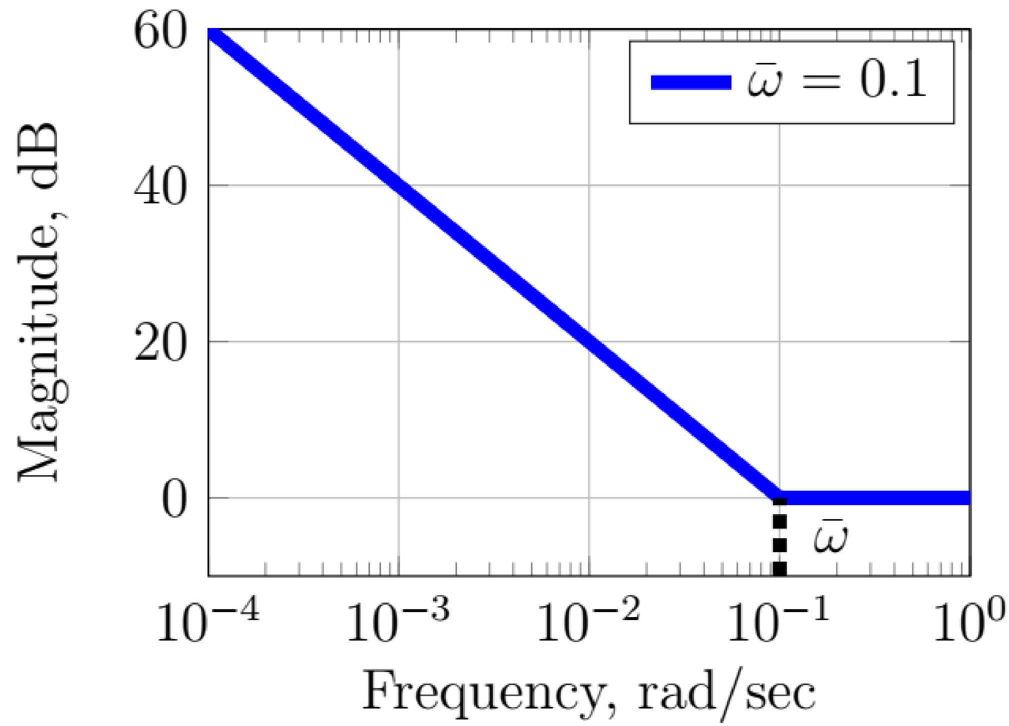
Properties:

-Corner frequency $\overline{\omega}$, high frequency gain $|K(j\omega)| = 1$, and low frequency slope of $-20\frac{dB}{dec}$.

-Corresponds to PI control:

$$\dot{u}(t) = \dot{e}(t) + \bar{\omega}e(t)$$

$$\Rightarrow u(t) = e(t) + \bar{\omega}\int_0^t e(\tau)\,d\tau$$



Integral boost is used to increase low frequency gain and ensure zero steady-state error.

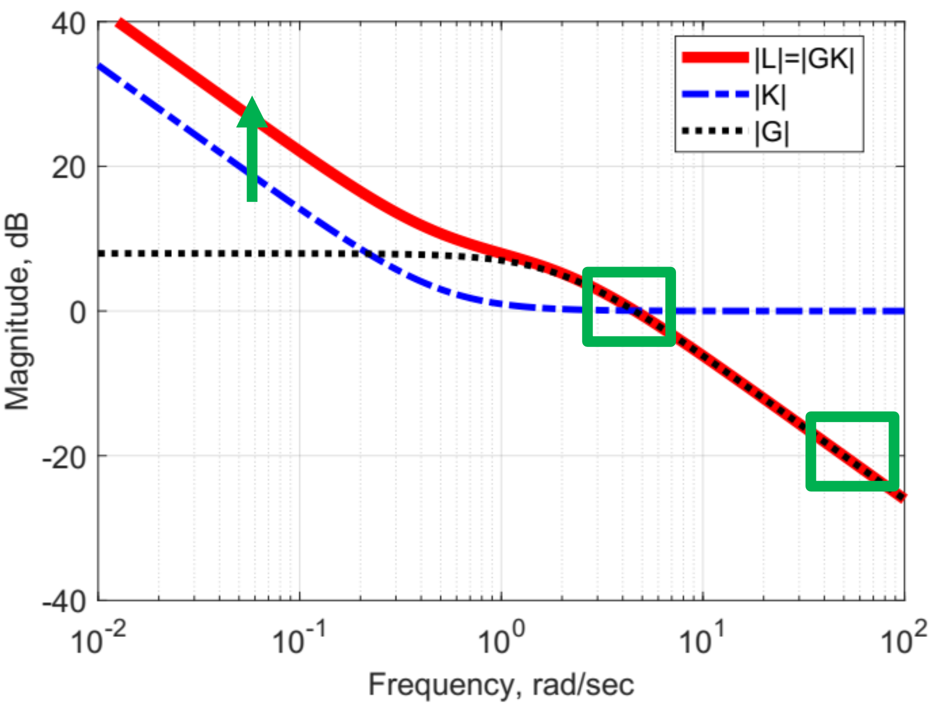
Effect of Integral Boost

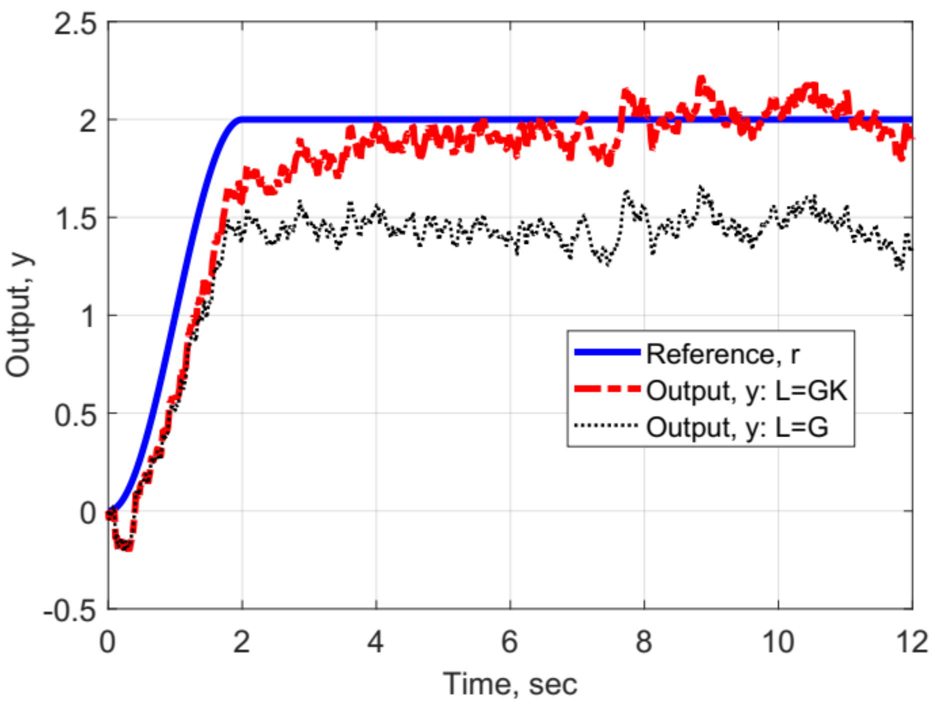
Plant: $\dot{y}(t) + 2y(t) = 5u(t)$ and $G(s) = \frac{5}{s+2}$

Control: $K(s) = \frac{s+\bar{\omega}}{s}$ with $\bar{\omega} = 3\frac{rad}{sec}$

$$|K(0)| = \infty \Rightarrow |L(0)| = \infty \quad \Rightarrow |S(0)| = \left|\frac{1}{1+L(0)}\right| = 0$$

Integral control ensures zero error in steady-state





High Frequency Roll-off

Roll-off: $K(s) = \frac{\bar{\omega}}{s+\bar{\omega}}$

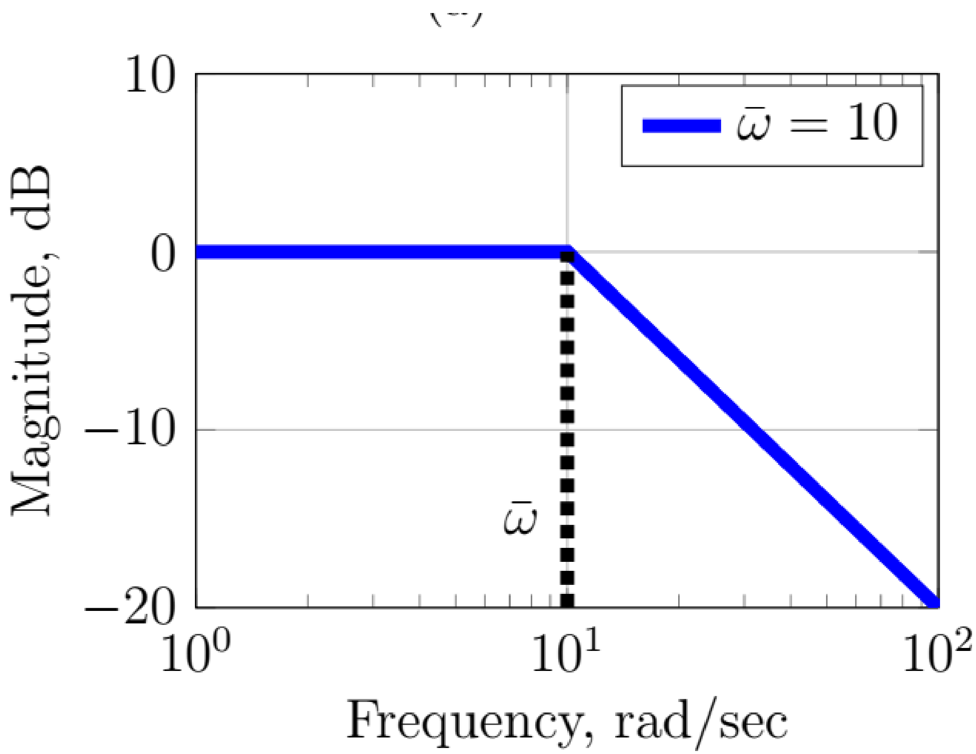
Properties:

-Corner frequency $\bar{\omega}$, low frequency gain $|K(j\omega)| = 1$, and high frequency slope of $-20\frac{dB}{dec}$.

-Corresponds to the ODE:

$$\dot{u}(t) + \bar{\omega}u(t) = \bar{\omega}e(t)$$

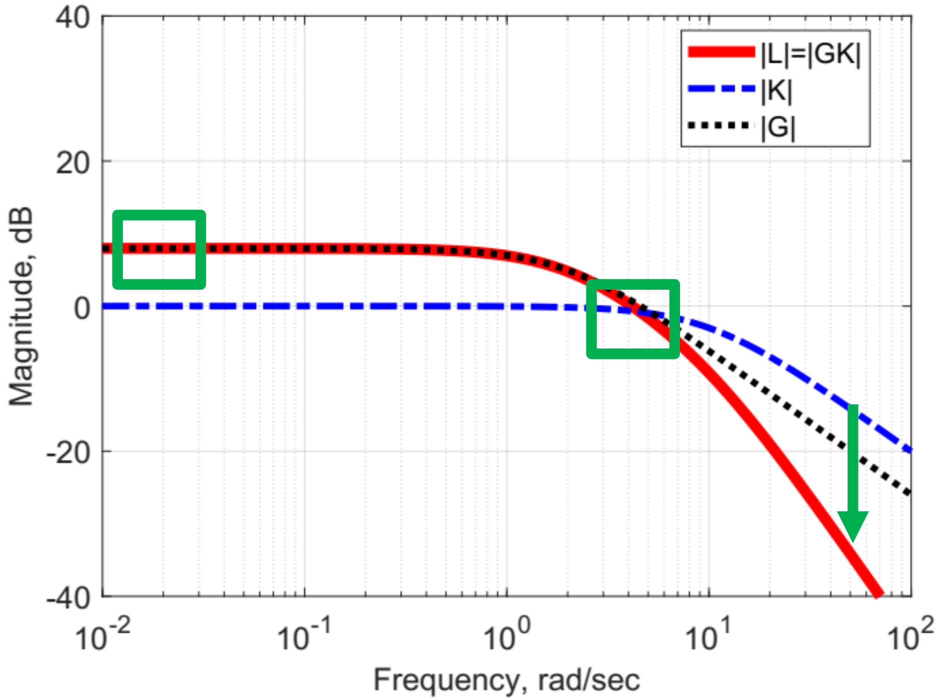
Roll-off is used to decrease high frequency gain and attenuate sensor noise.

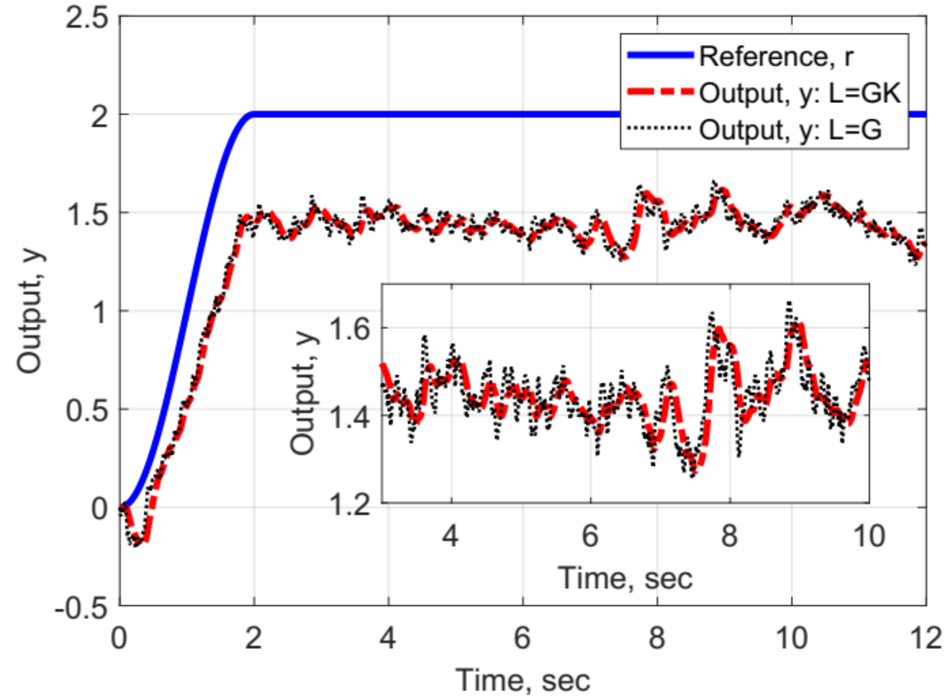


Effect of Roll-off

Plant: $\dot{y}(t) + 2y(t) = 5u(t)$ and $G(s) = \frac{5}{s+2}$

Control: $K(s) = \frac{\bar{\omega}}{s+\bar{\omega}}$ with $\bar{\omega} = 10\frac{rad}{sec}$





Lead

Lead: $K(s) = \frac{\beta s+\overline{\omega}}{s+\beta\overline{\omega}}$

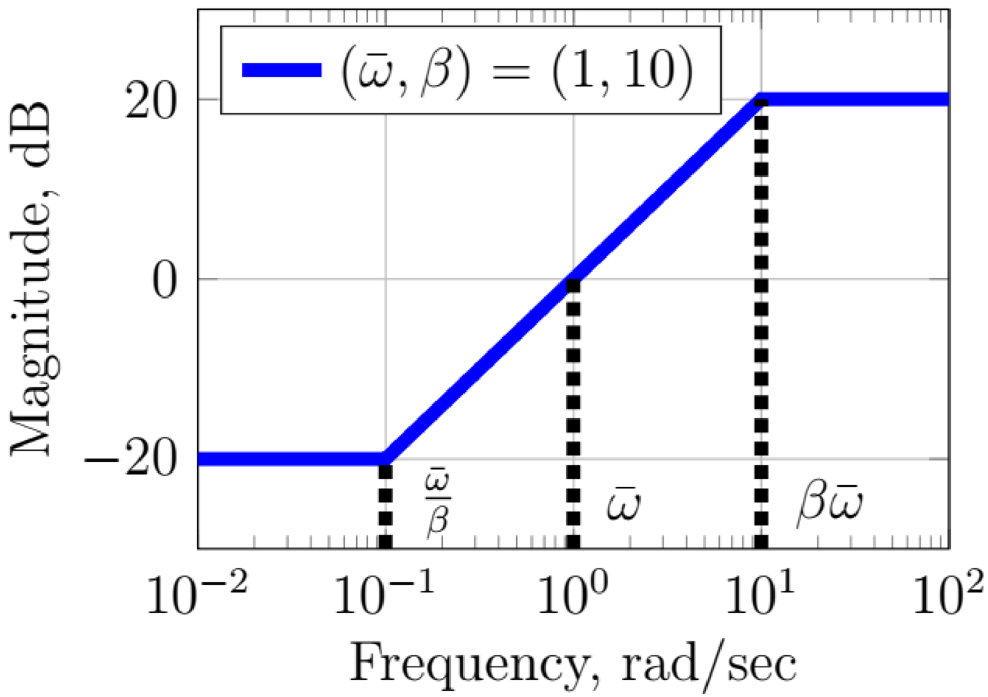
Properties:

-Zero at $-\frac{\overline{\omega}}{\beta}$ and pole at $-\beta\overline{\omega}$,

-Low frequency gain $\frac{1}{\beta}$ and high frequency gain β

-Positive slope at $\overline{\omega}$

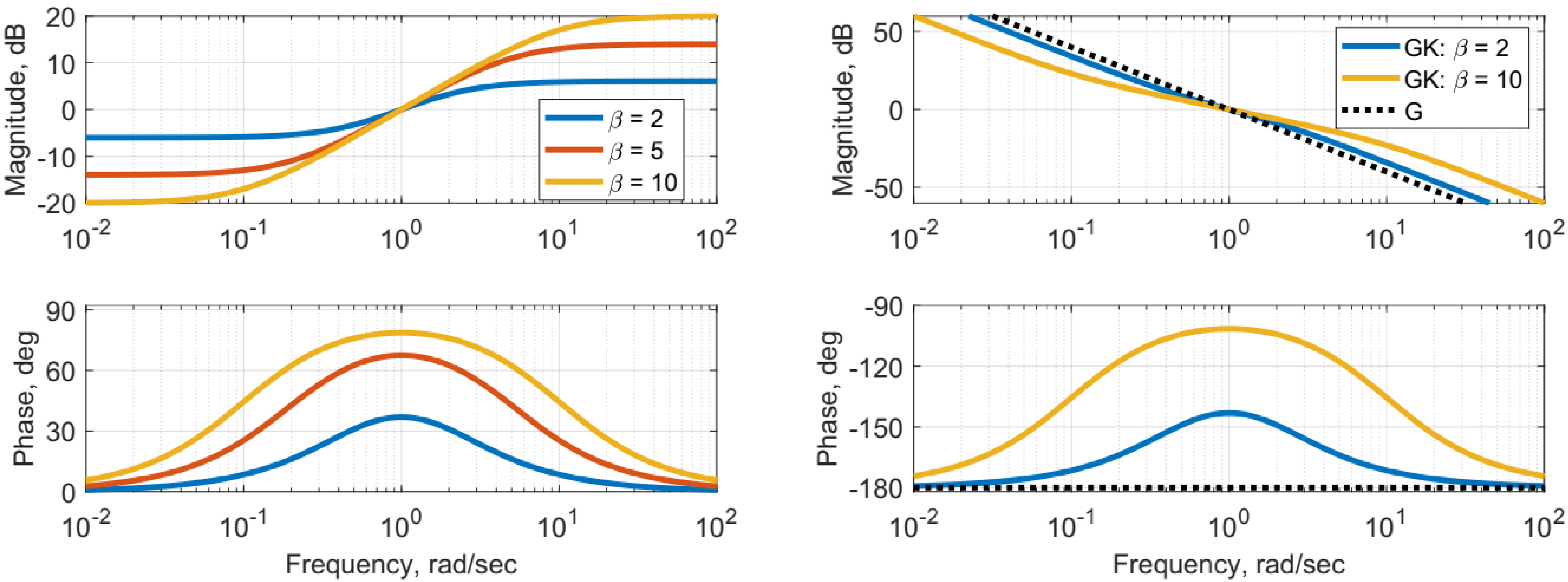
Lead is used to make the slope shallower and hence ensure stability and robustness.



Effect of Lead

Plant: $\ddot{y}(t) = u(t)$ and $G(s) = \frac{1}{s^2}$

Control: $K(s) = \frac{\beta s + \bar{\omega}}{s + \beta \bar{\omega}}$ with $\bar{\omega} = 1 \frac{rad}{sec}$



ECE 486: Control Systems

Lecture 20D: Loopshaping Design Process

Key Takeaways

The basic steps of the loopshaping process are:

1) Use a proportional gain to set the desired crossover frequency. This sets the bandwidth / speed of response.

2) Use an integral boost to increase $|L(j\omega)|$ at low frequencies. This improves the reference tracking and disturbance rejection.

3) Use a roll-off to reduce $|L(j\omega)|$ at high frequencies. This improves the noise rejection.

4) Add lead control (if needed) to modify the slope of $|L(j\omega)|$ near the crossover. This is used for closed-loop stability and robustness.

This approach can be used on higher-order plants using controllers that are, in general, more complex than a PID controller.

Basic Design Process

Key design parameter: Desired loop crossover ω_c

1. Proportional Gain: Select $K_p = \pm \frac{1}{|G(j\omega_c)|}$

Loop $L_1 = G\ K_p$ has the desired crossover, $|L(j\omega_c)| = 1$.

2. Integral Boost: Select $K_i(s) = \frac{s+\omega_i}{s}$ with $\omega_i \leq \omega_c$

Loop $L_2 = G\ K_p\ K_i$ has improved low frequency tracking.

Good initial choice $\omega_i = \omega_c/3$ so that $|K_i(j\omega)| \approx 1$ for $\omega \geq \omega_c$.

3. Roll-off: Select $K_r(s) = \frac{\omega_r}{s+\omega_r}$ with $\omega_r \geq \omega_c$

Loop $L_3 = G\ K_p\ K_i K_r$ has improved noise rejection / robustness.

Good initial choice $\omega_r = 3\omega_c$ so that $|K_r(j\omega)| \approx 1$ for $\omega \leq \omega_c$.

4. Lead (If needed): Select $K_l(s) = \frac{\beta s+\omega_c}{s+\beta\omega_c}$ with $\beta \approx 3 - 10$

Loop $L_4 = G\ K_p\ K_i K_r K_l$ has improved stability margins

Example 1: First-Order System

Design a loopshaping controller for $G(s) = -\frac{0.25}{s+0.5}$

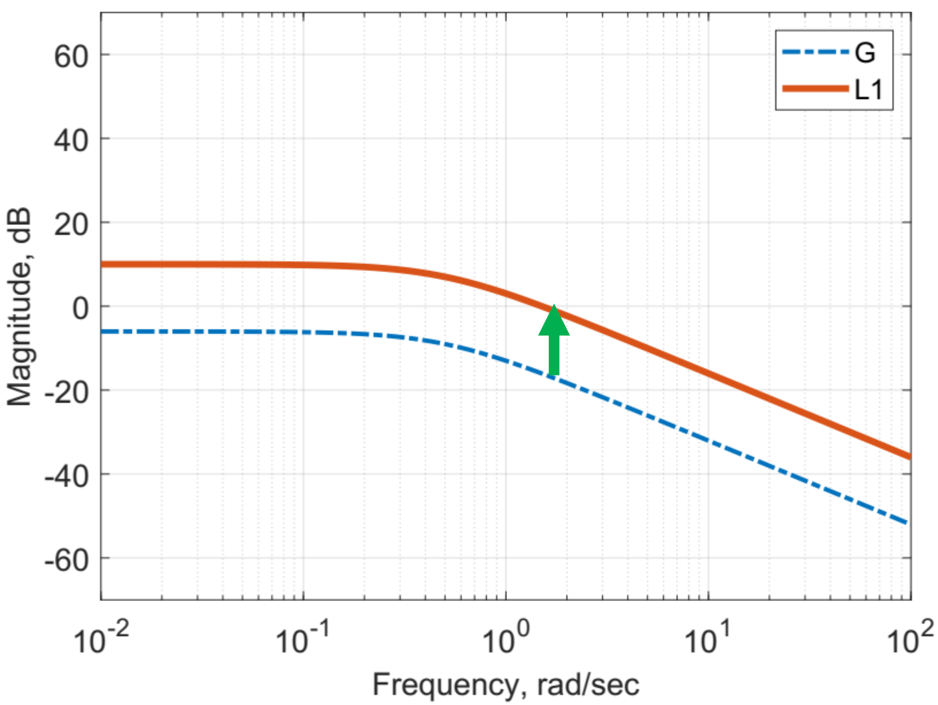
Desired crossover at $\omega_c = 1.5\, rad/sec$

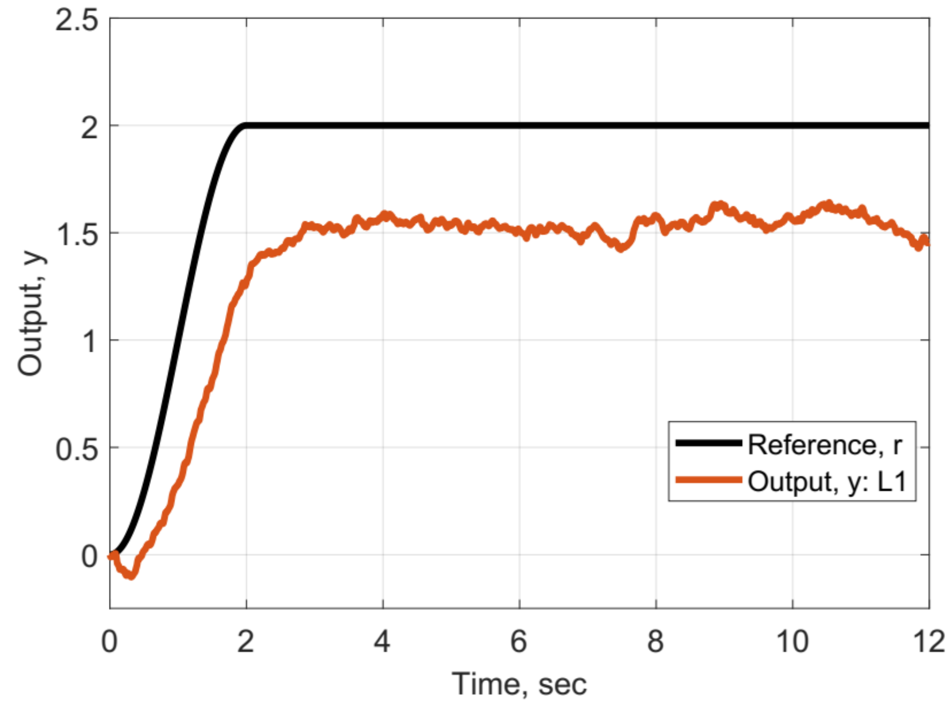
Step 1: Proportional Gain

Plant $G(s) = -\frac{0.25}{s+0.5}$ and desired crossover at $\omega_c = 1.5\,\frac{rad}{sec}$

$K_p = -\frac{1}{|G(j\omega_c)|} = -6.32$ (Note $K_p<0$ because $G(0)<0$).

$L_1 = G\,K_p$



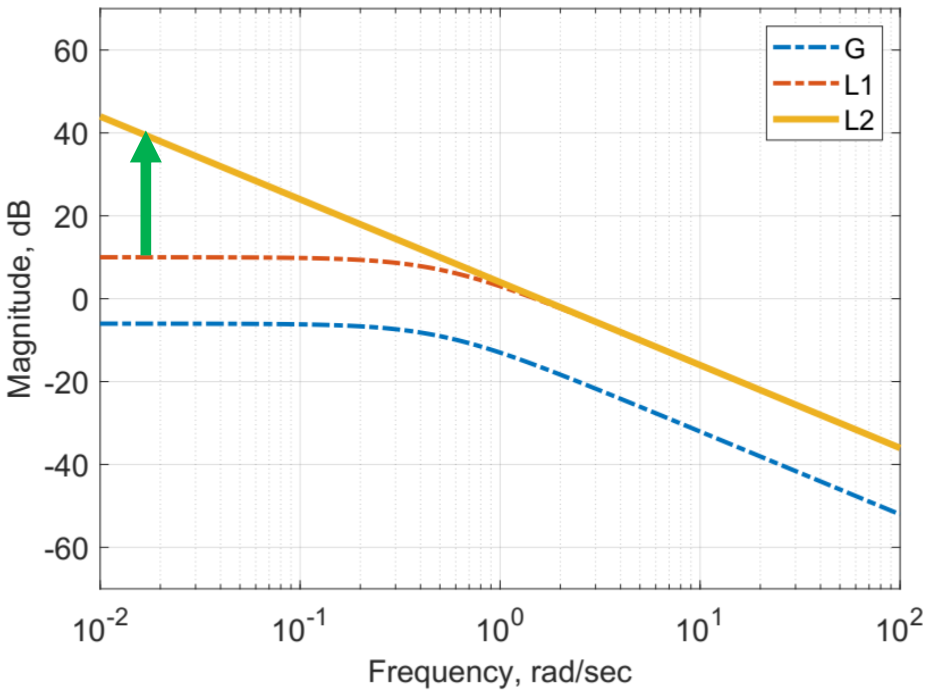


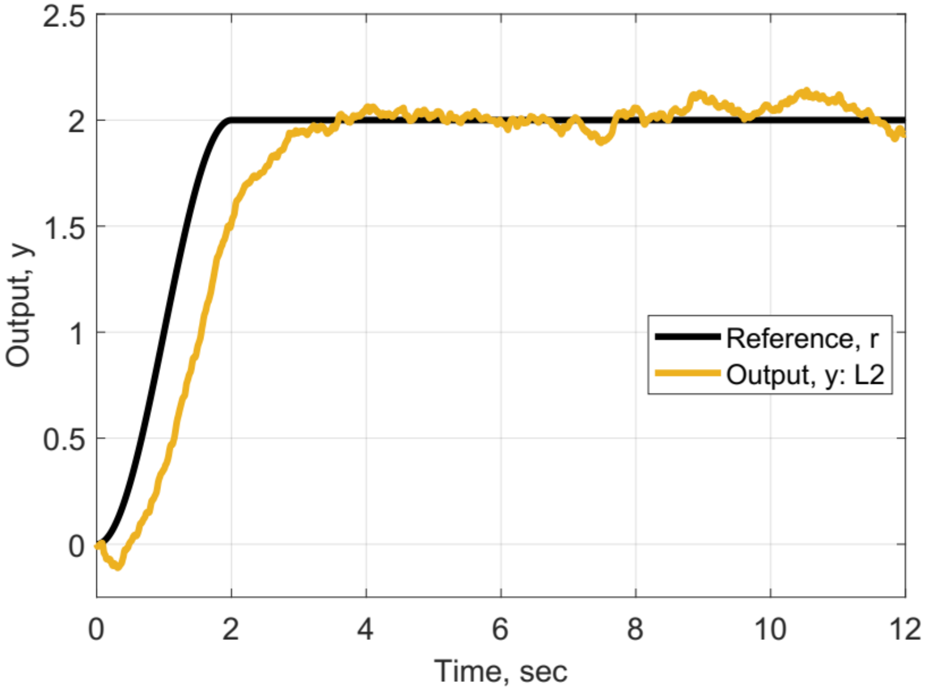
Step 2: Integral Boost

Plant $G(s) = -\frac{0.25}{s+0.5}$ and desired crossover at $\omega_c = 1.5\,\frac{rad}{sec}$

$K_i = \frac{s+\omega_i}{s}$ with $\omega_i = \frac{\omega_c}{3} = 0.5\,\frac{rad}{sec}$

$L_2 = G\ K_p\ K_i$



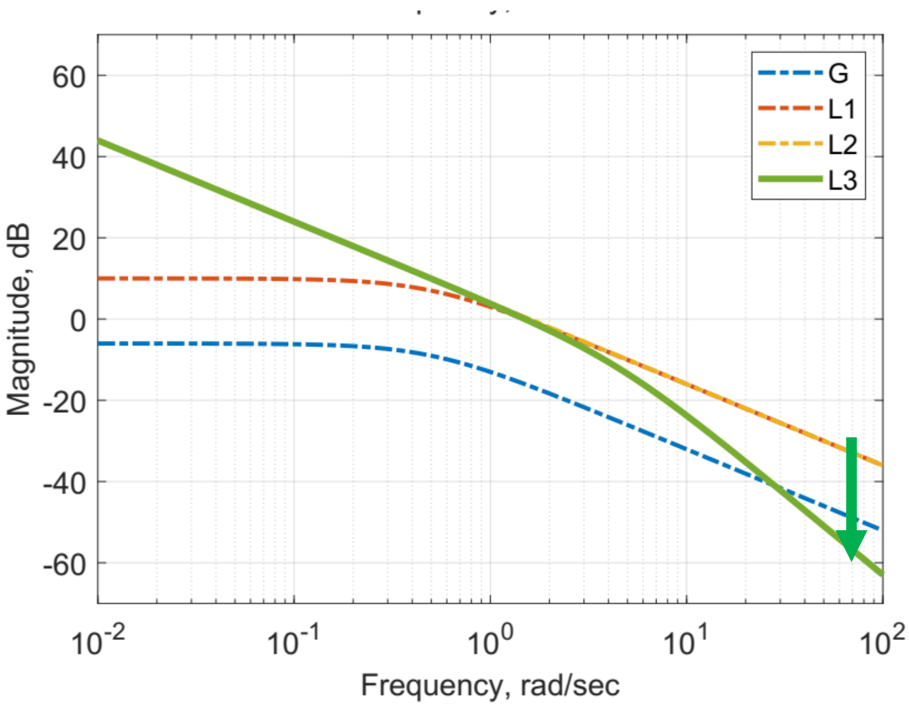


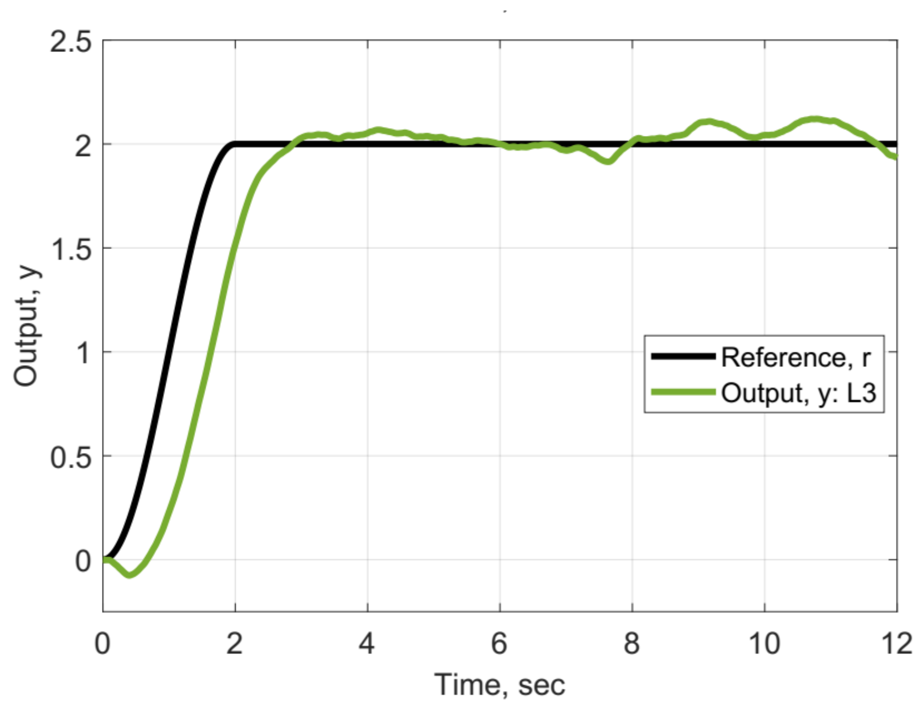
Step 3: Roll-off

Plant $G(s) = -\frac{0.25}{s+0.5}$ and desired crossover at $\omega_c = 1.5\,\frac{rad}{sec}$

$K_r = \frac{\omega_r}{s+\omega_r}$ with $\omega_r = 3\omega_c = 4.5\,\frac{rad}{sec}$

$L_3 = G\ K_p\ K_i\ K_r$





Step 4: Lead

Plant $G(s) = -\frac{0.25}{s+0.5}$ and desired crossover at $\omega_c = 1.5\,\frac{rad}{sec}$

Loop L_3 = G K_p K_i K_r has a "shallow" slope near crossover.

The closed-loop is stable with [0,∞) gain margins and $\pm 72^o$ phase margins.

No lead control is required.

Final Controller: $$K(s) = K_p K_i(s) K_r(s) = -\frac{28.5s + 14.2}{s^2 + 4.5s}$$

$$\ddot{u}(t) + 4.5\dot{u}(t) = -28.5\dot{e}(t) - 14.2e(t)$$

Example 1: Matlab Code

```
>> G = -tf(0.25,[1 0.5]);            % Plant
>> wc = 1.5;                         % Desired crossover, rad/sec

>> Kp = -1/abs(evalfr(G, 1j*wc));    % Proportional Gain
>> wi = wc/3;                        % Boost frequency, rad/sec
>> Ki = tf([1 wi],[1 0]);            % Integral Boost
>> wr = 3*wc;                        % Roll-off frequency, rad/sec
>> Kr = tf(wr,[1 wr]);               % Roll-off
>> K = Kp*Ki*Kr;                     % Final Controller
>> L3 = G*K;                         % Final loop

>> S = feedback(1,L3);               % Closed-loop sensitivity
>> isstable(S)                       % Verify closed-loop stability
>> allmargin(L3)                     % Classical margins
```

Example 2: Higher-Order System

Design a loopshaping controller for

$$G(s) = \frac{4}{s^2} \frac{400}{s^2 + 0.08s + 400} \frac{15}{s + 15}$$

Desired crossover at $\omega_c = 2.0\, rad/sec$

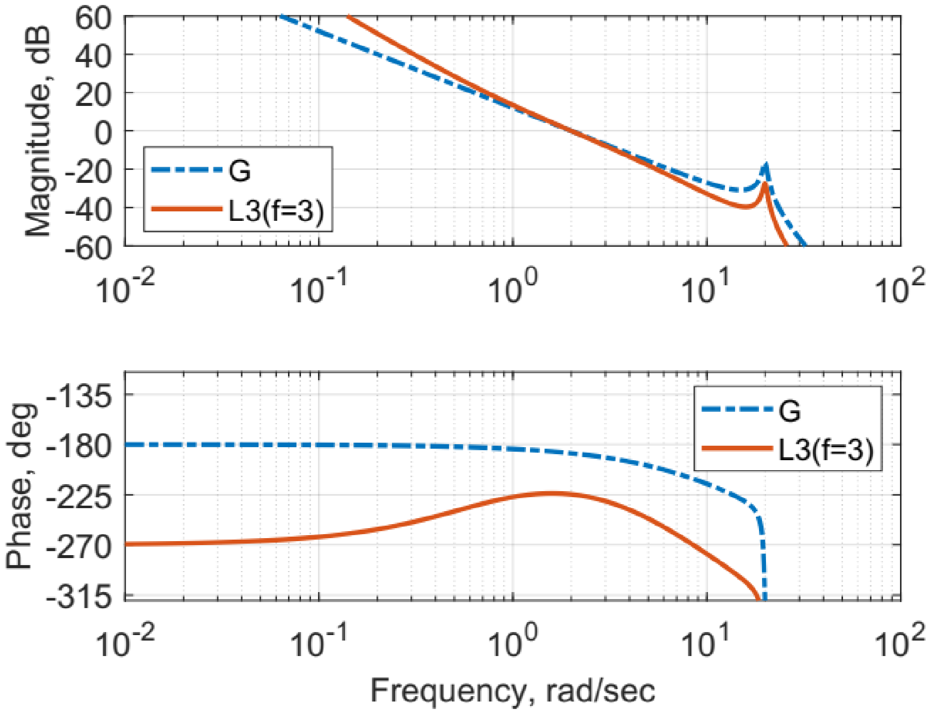
Step 1) Gain: Select $K_p = \frac{1}{|G(j\omega_c)|} \approx 1$

Step 2) Boost: $K_i = \frac{s+\omega_i}{s}$

with $\omega_i = \frac{\omega_c}{3}$

Step 3) Rolloff: $K_r = \frac{\omega_r}{s+\omega_r}$

with $\omega_r = 3\omega_c$



Example 2: Higher-Order System

Design a loopshaping controller for

$$G(s) = \frac{4}{s^2} \frac{400}{s^2 + 0.08s + 400} \frac{15}{s + 15}$$

Desired crossover at $\omega_c = 2.0\, rad/sec$

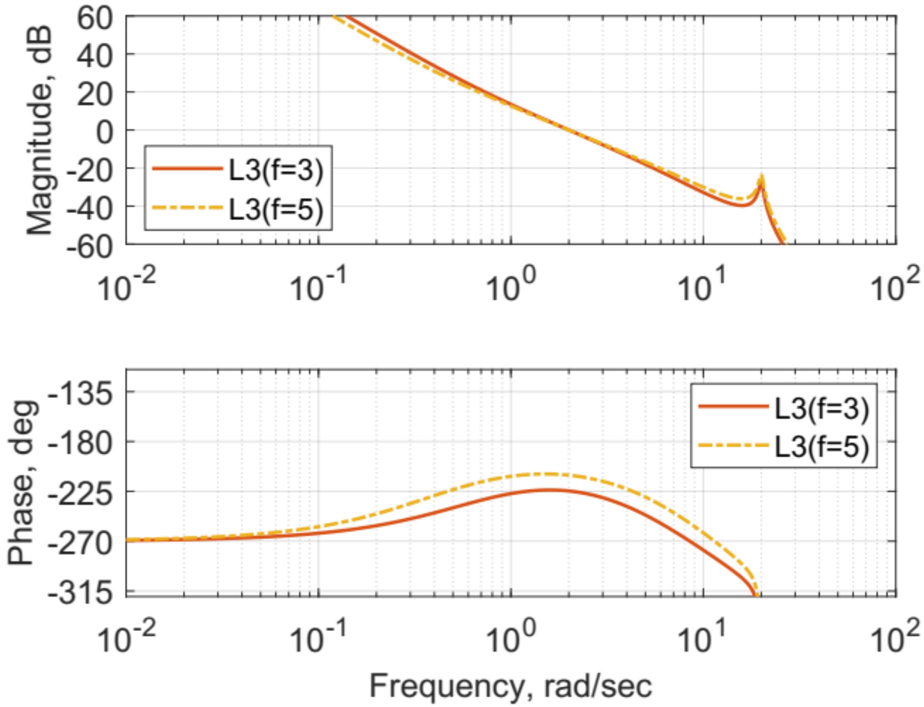
Step 1) Gain: Select $K_p = \frac{1}{|G(j\omega_c)|} \approx 1$

Step 2) Boost: $K_i = \frac{s+\omega_i}{s}$

with $\omega_i = \frac{\omega_c}{5}$

Step 3) Rolloff: $K_r = \frac{\omega_r}{s+\omega_r}$

with $\omega_r = 5\omega_c$



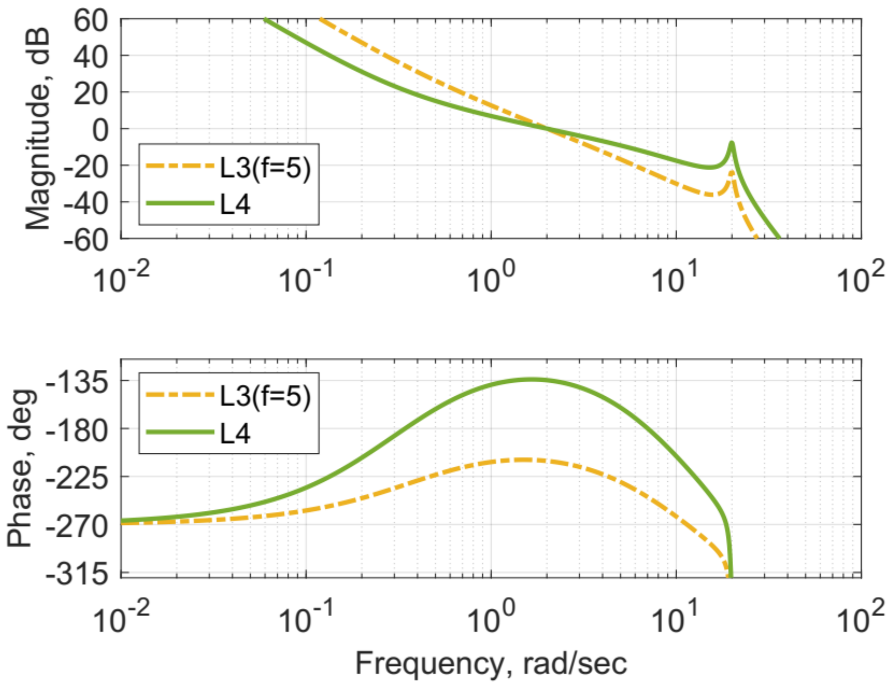
Step 4: Lead

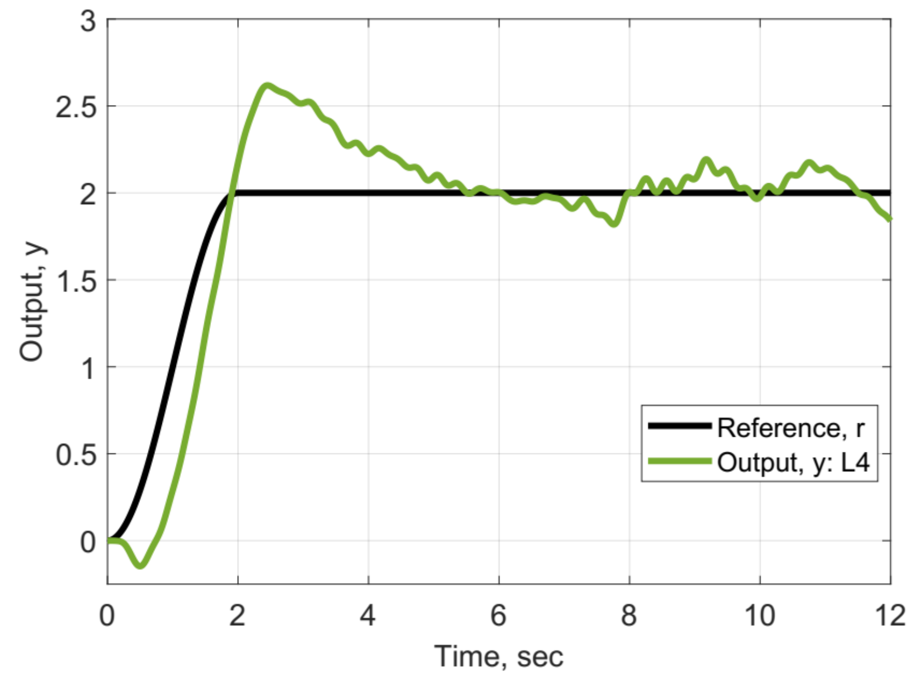
Loop $L_3 = G\,K_p\,K_i\,K_r$ has a "steep" slope near crossover.

Closed-loop is unstable with L_3 so lead control is needed.

$K_l(s) = \frac{\beta s+\omega_c}{s+\beta\omega_c}$ with $\beta = 8$

$L_4 = G\,K_p\,K_i\,K_r\,K_l$ → Closed-loop is stable 45^o of phase margin.





Example 2: Matlab Code

```
>> G1 = tf(1,[1 0 0]);
>> H = 4*tf(400,[1 2*0.02*20 400])*tf(15,[1 15]);
>> G = G1*H;                                  % Plant
>> wc = 2.0;                                  % Desired crossover, rad/sec

>> Kp = 1/abs(evalfr(G, 1j*wc));              % Proportional Gain
>> wi = wc/5;                                 % Boost frequency, rad/sec
>> Ki = tf([1 wi],[1 0]);                     % Integral Boost
>> wr = 5*wc;                                 % Roll-off frequency, rad/sec
>> Kr = tf(wr,[1 wr]);                        % Roll-off
>> wl = wc;                                   % Lead frequency, rad/sec
>> beta = 8;                                  % Lead parameter
>> Kl = tf([beta wl],[1 beta*wl]);            % Lead
>> K = Kp*Ki*Kr*Kl;                           % Final Controller
>> L4 = G*K;                                  % Final loop

>> S = feedback(1,L4);                        % Closed-loop sensitivity
>> isstable(S)                                % Verify closed-loop stability
>> allmargin(L4)                              % Classical margins
```

PID vs. Loopshaping

PID with approximate derivative:

$$K(s) = K_p + \frac{K_i}{s} + K_d \frac{\alpha s}{s+\alpha}$$
$$= \frac{(K_p + K_d\alpha)s^2 + (K_p\alpha + K_i)s + K_i\alpha}{s^2 + \alpha s}$$

Loopshaping with proportional, integral boost, and lead:

$$K(s) = K_p \cdot \frac{s + \bar{\omega}_I}{s} \cdot \frac{\beta s + \omega_c}{s + \beta\omega_c}$$
$$= \frac{(K_p\beta)s^2 + K_p(\omega_c + \beta\omega_I)s + K_p\omega_I\omega_c}{s^2 + (\beta\omega_c)s}$$

These are different parameterizations for the same class of controllers.

Loopshaping can be viewed as a generalization of PID that enables

- Additional controller components (rolloff, notches, etc)
- Closer connection to frequency-domain trade-offs
- Extensions to multivariable systems.

ECE 486: Control Systems

Lecture 20E: Loopshaping Design Theorems

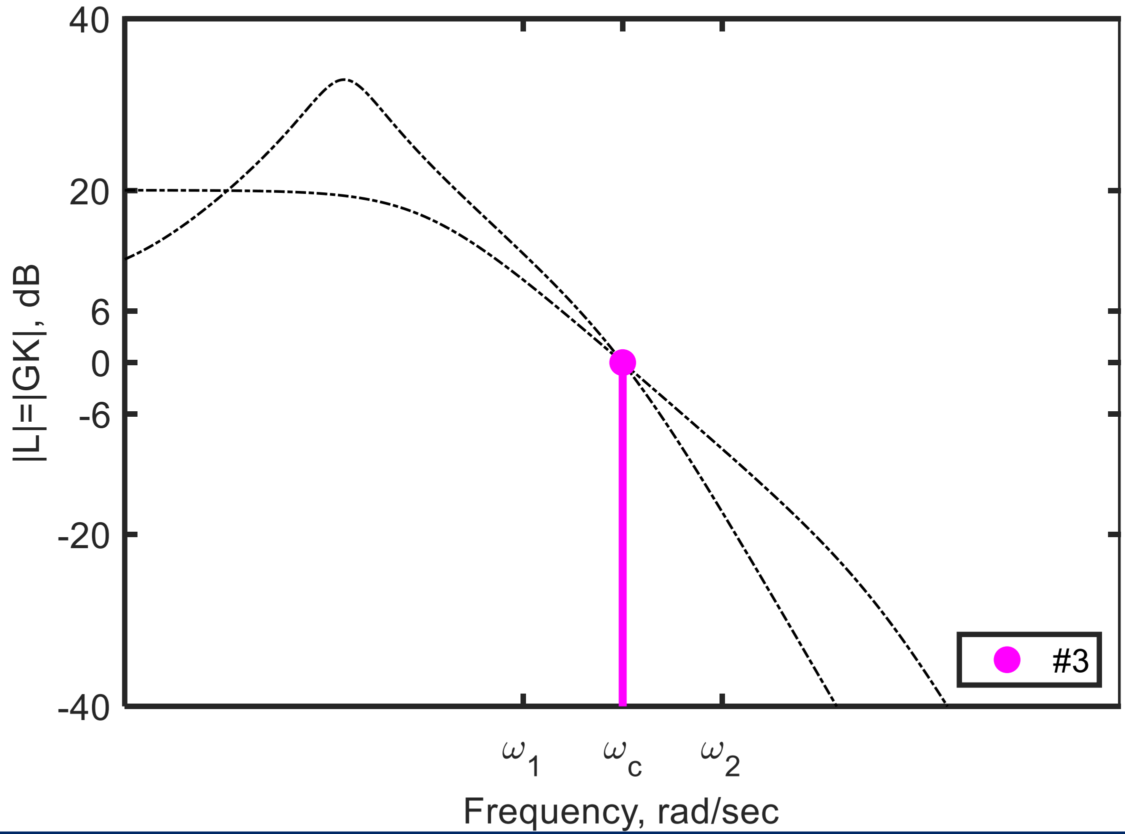
Key Takeaways

This lecture presents two important "theorems" regarding the loopshaping design process.

Under mild conditions, the loopshaping design process will yield a stable closed-loop with good stability margins.

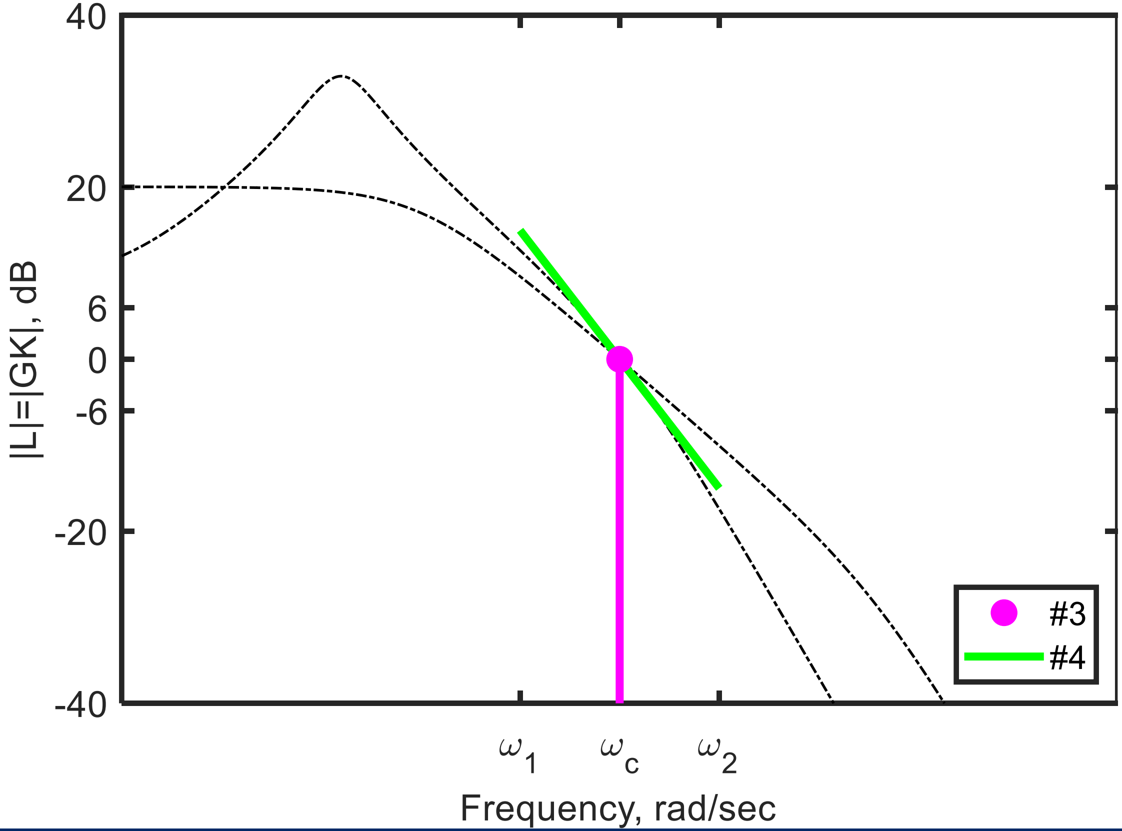
Basic Assumptions on *L(s)=G(s)K(s)*

1. *L(s)* has all poles and zeros in the LHP.
2. *L(0)>0*
3. One crossover ω_c



Basic Assumptions on *L(s)=G(s)K(s)*

1. *L(s)* has all poles and zeros in the LHP.
2. *L(0)>0*
3. One crossover ω_c
4. Shallow slope ($\geq -30\frac{dB}{dec}$) for one decade around ω_c



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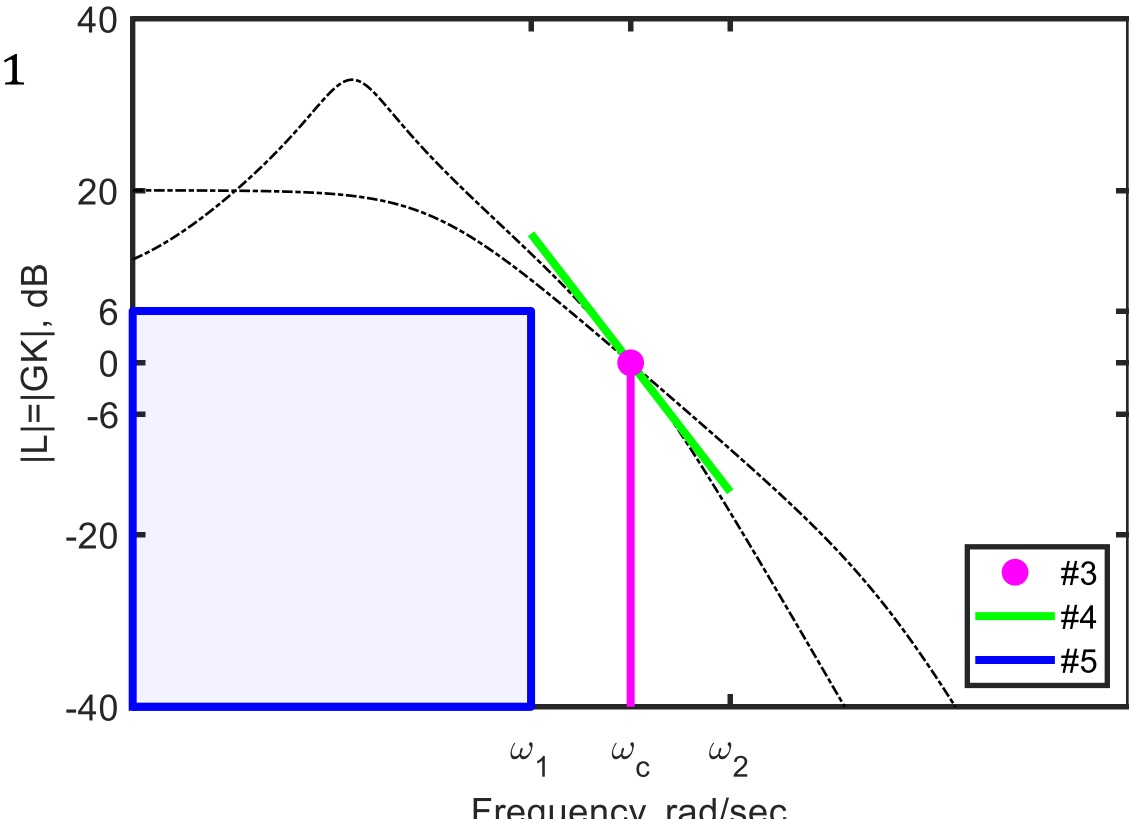
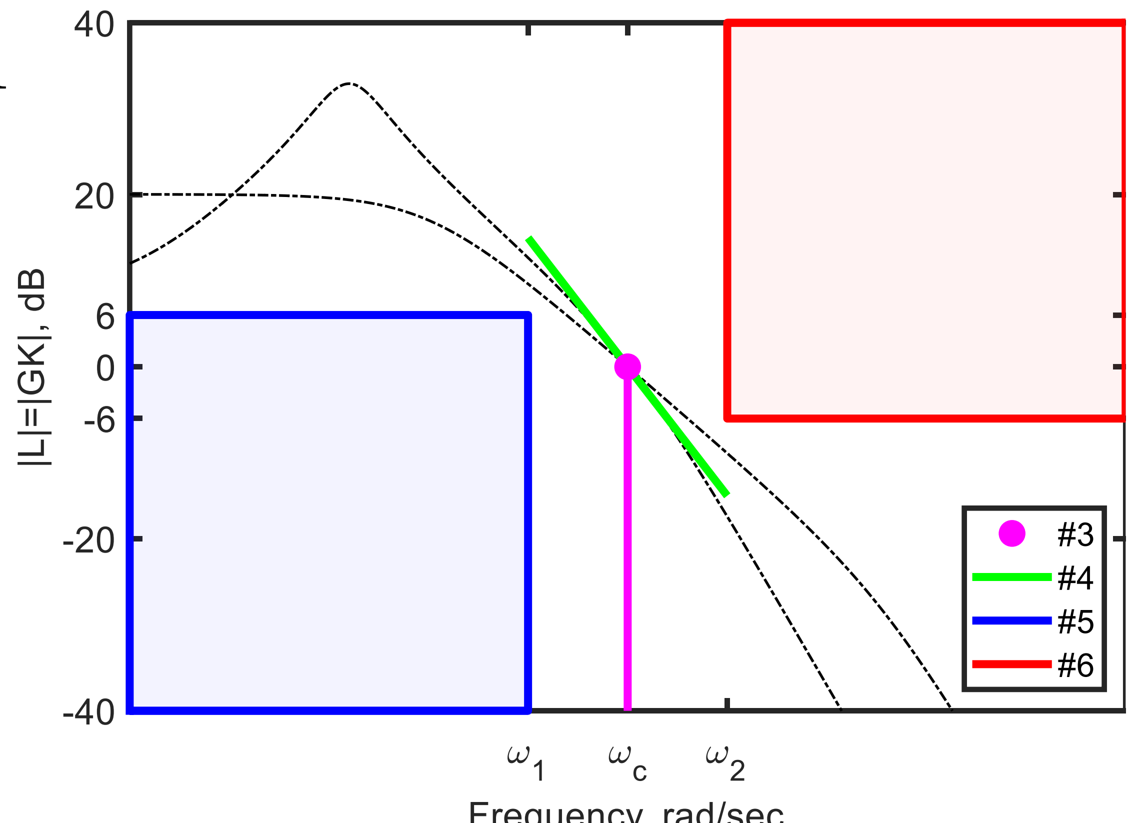
4. Shallow slope ($\geq -30\frac{dB}{dec}$) for one decade around ω_c
5. $|L(j\omega)| \geq 2$ for $\omega \leq \omega_1$



Basic Assumptions on *L(s)=G(s)K(s)*

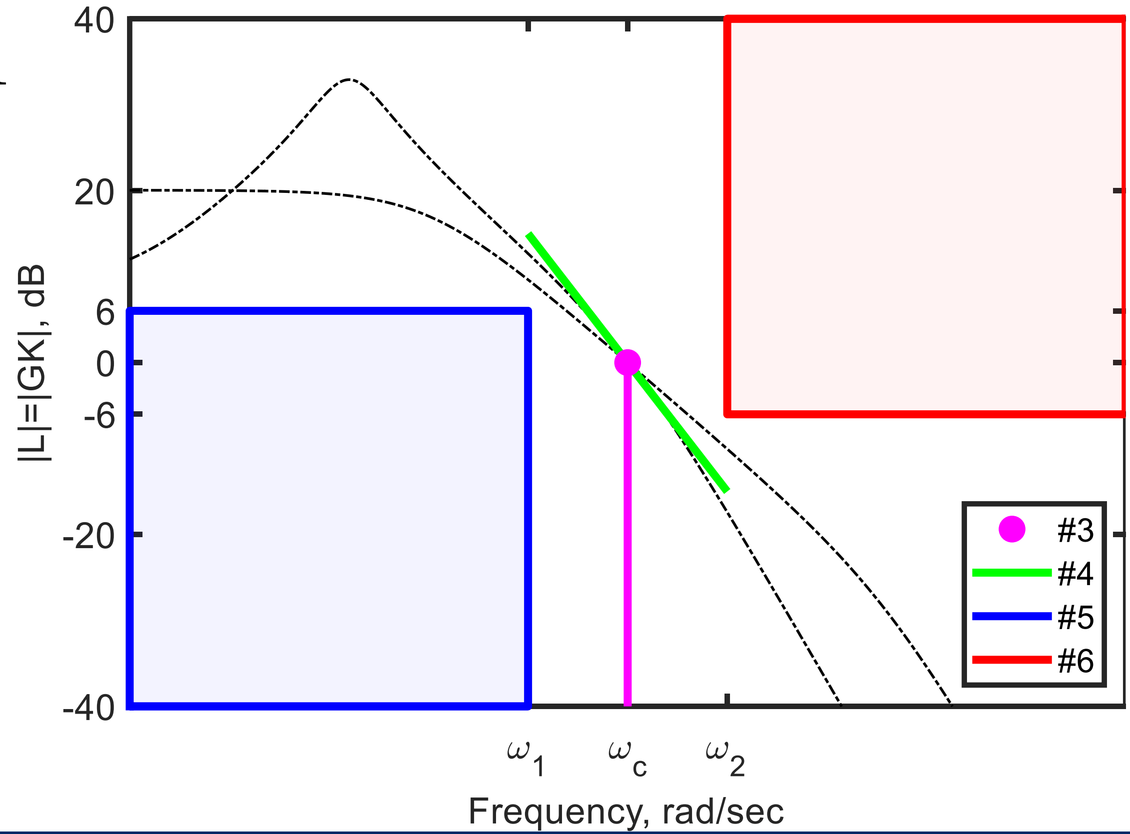
1. *L(s)* has all poles and zeros in the LHP.
2. *L(0)>0*
3. One crossover ω_c
4. Shallow slope ($\geq -30\frac{dB}{dec}$) for one decade around ω_c
5. $|L(j\omega)| \geq 2$ for $\omega \leq \omega_1$
6. $|L(j\omega)| \leq \frac{1}{2}$ for $\omega \geq \omega_2$



Loopshaping Design Theorem

1. *L(s)* has all poles and zeros in the LHP.
2. *L(0)>0*
3. One crossover ω_c
4. Shallow slope ($\geq -30\frac{dB}{dec}$) for one decade around ω_c
5. $|L(j\omega)| \geq 2$ for $\omega \leq \omega_1$
6. $|L(j\omega)| \leq \frac{1}{2}$ for $\omega \geq \omega_2$

If L(s) satisfies 1-6 then the closed-loop is stable with approximate gain, phase, and disk margins ≥ ±6dB, ≥ ±45°, and d_{min} ≥ 0.5



Loopshaping Design Theorem With Integrators

1. $L(s)=\frac{1}{s^k}H(s)$ where H(s) has all poles and zeros in the LHP.
2. *H(0)>0*
3. One crossover ω_c
4. Shallow slope ($\geq -30\frac{dB}{dec}$) for one decade around ω_c
5. $|L(j\omega)| \geq 2$ for $\omega \leq \omega_1$
6. $|L(j\omega)| \leq \frac{1}{2}$ for $\omega \geq \omega_2$

If L(s) satisfies 1-6 then the closed-loop is stable with approximate gain, phase, and disk margins ≥ ±6dB, ≥ ±45°, and d_{min} ≥ 0.5