ECE 486: Control Systems

Lecture 7A: Summary Of Control Design Issues

Key Takeaways

Models used for control design are often simplified and contain a variety of inaccuracies.

- Uncertain parameters, unmodeled dynamics, nonlinear effects, and implementation effects.

Control design involves trade-offs to satisfy many conflicting objectives.

- Stability, reference tracking, disturbance rejection, actuator effort, noise rejection, and robustness to model uncertainty.

Problem 1

A control system is required to ensure the quadcopter below maintains a desired altitude.

A) Discuss the simplified models that might be used for control design and various sources of inaccuracies.

B) Discuss the various competing objectives that might arise in the design of this system.

$$m\ddot{h} = 4k_T u - mg + F_d$$

DJI Phantom 4Pro
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Solution 1A

A) Discuss the simplified models that might be used for control design and various sources of inaccuracies.

$$m\ddot{h} = 4\underline{K_T} u - mg + \underline{\underline{F_d}}$$

- Aerodynamic forces
- Pitching motion of quad
- All motors are perfectly identical
- Motor dynamics and propeller aero are fast.
- Saturation $u \in [0, 500]$
- Parameter errors (m, K_T)

DJI Phantom 4Pro
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Problem 1B

B) Discuss the various competing objectives that might arise in the design of this system.

- Accuracy of maintaining desired altitude
- Overshoot/Undershoot requirements (might cause a crash or unexpected pilot behavior)
- Rise/Settling Times (Speed of Response)
- Noise (Sensor)
- ~~Fo~~ Disturbance Forces (Reject)
- Remain within motor limits
- Robust to model errors

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Lecture 7B: Open-Loop Control

Key Takeaways

This lecture describes open-loop control.

Open-loop control does not require a sensor and hence it can lead to a cheaper system. It can be effective if:

1. The plant is stable,
2. The disturbances are small, and
3. The model is accurate.

If any of these conditions fails, then open-loop control will either fail to achieve stability (if the plant is unstable) or will not provide accurate tracking.

Problem 2

Consider the following plant:

$$\ddot{y}(t) + 2\dot{y}(t) + 10\,y(t) = 20\,u(t) + 10\,d(t)$$

This system is underdamped with $\omega_n = 3.16\frac{rad}{sec}$, $\zeta = 0.316$, and poles $s_{1,2} = -1 \pm 3j$.

A) What is the model from inputs (r,d) to output y if we use an open-loop controller $u(t) = K_{ol}\, r(t)$?

B) Can the gain K_{ol} be selected so that the control system is overdamped from r to y?

C) Select K_{ol} so that $y(t) \to \bar{r}$ when $r(t) = \bar{r}$ and $d(t) = 0$.

D) Sketch the response y with your gain Kol when $r(t) = 2$ and $d(t) = 1$. What is the impact of the disturbance?

Solution 2A

$$\ddot{y}(t) + 2\dot{y}(t) + 10\,y(t) = 20\,u(t) + 10\,d(t)$$

A) What is the model from inputs (r,d) to output y if we use an open-loop controller $u(t) = K_{ol}\, r(t)$?

$$\ddot{y} + 2\dot{y} + 10y = (20K_{ol})r + 10d$$

Solution 2B

$$\ddot{y}(t) + 2\dot{y}(t) + 10\,y(t) = 20\,u(t) + 10\,d(t)$$

B) Can the gain K_{ol} be selected so that the control system is overdamped from r to y?

$u = K_{ol}\, r$

$$\ddot{y} + 2\dot{y} + 10y = (20 K_{ol})\, r + 10d$$

$$G(s) = \frac{20 K_{ol}}{s^2 + 2s + 10} \longrightarrow -1 \pm 3j$$

Underdamped

No, it is underdamped

for any choice of K_{ol}

Solution 2C

$$\ddot{y}(t) + 2\dot{y}(t) + 10\,y(t) = 20\,u(t) + 10\,d(t)$$

C) Select K_{ol} so that $y(t) \rightarrow \bar{r}$ when $r(t) = \bar{r}$ and $d(t) = 0$.

$$\underbrace{\underline{\ddot{y} + 2\dot{y}} + 10y = (20K_{ol})\,r}\; + \cancel{10d}^{=0}$$

Steady-state $(y \rightarrow \bar{y})$

$$10\bar{y} = (20K_{ol})\bar{r}$$

$$\parallel$$

$$10\bar{r}$$

$$\boxed{K_{ol} = \frac{10}{20} = 0.5}$$

Solution 2D

$$\ddot{y}(t) + 2\dot{y}(t) + 10\,y(t) = 20\,u(t) + 10\,d(t)$$

D) Sketch the response y with your gain Kol when $r(t) = 2$ and $d(t) = 1$. What is the impact of the disturbance?

$K_{ol} = 1/2$

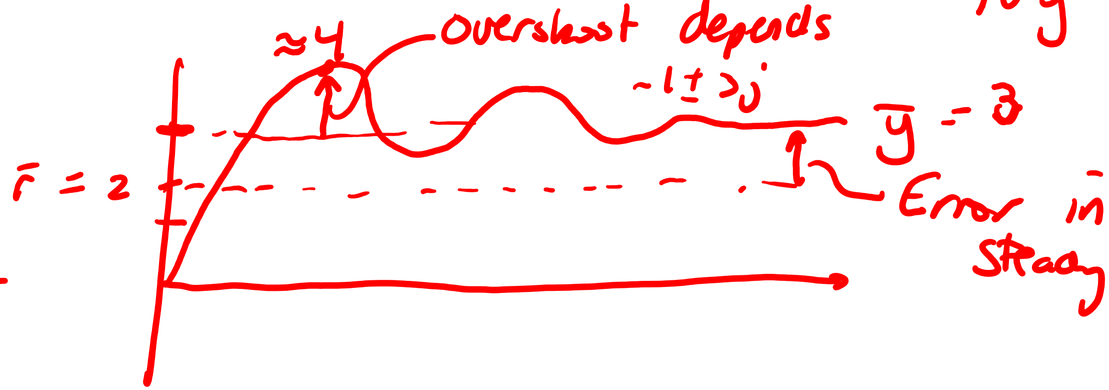
$\ddot{y} + 2\dot{y} + 10y = 10\underset{2}{\underset{\shortparallel}{r}} + 10\underset{1}{\underset{\shortparallel}{d}} = \underline{30}$ -

$r=2$ and $d=1$

$s = -1 \pm 3j$

$\zeta = 0.316$

$10\bar{y} = 30$



$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = \cancel{35}\ 0.35$

$= \dfrac{y_p - \bar{y}}{\bar{y}} = 0.35 \longrightarrow y_p = \bar{y} + 0.35\,\bar{y} = (1.35)\cdot 3 \approx 4$

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Lecture 7C: Proportional (P) Control

Key Takeaways

This lecture describes closed-loop proportional control. The controller sets the plant input proportional to the error.

Closed-loop control can achieve higher performance but requires a sensor.

$$u = \underbrace{K_p}_{\uparrow}(r - y)$$

Larger proportional gains:

(i) improve reference tracking and disturbance rejection

(ii) increase the closed-loop speed of response

but:

(i) require larger control inputs

(ii) can excite unmodeled dynamics

Problem 3

Consider the following plant:

$$2\dot{y}(t) + 3\,y(t) = -4\,u(t) + \,d(t)$$

First-order

A) What is the model from inputs (r,d) to output y if we use an proportional controller $u(t) = K_p\,(r(t) - y(t))$?

B) Select K_p so that the steady-state error $\bar{e} = \bar{r} - \bar{y}$ is less than 0.1 when $r(t) = \bar{r} = 2$ and $d(t) = 1$.

C) Sketch the response y with your gain K_p when $r(t) = 2$ and $d(t) = 1$. What is the time constant of the closed-loop?

Solution 3A

$$2\dot{y}(t) + 3\,y(t) = -4\,u(t) + \;d(t)$$

A) What is the model from inputs (r,d) to output y if we use an proportional controller $u(t) = K_p\,(r(t) - y(t))$?

$$2\dot{y} + 3y = -\underline{4\,K_p}(r - y) \quad + d$$

$$\boxed{2\dot{y} + (3 - 4K_p)y = -4K_p\,r + d}$$

Solution 3B

$$2\dot{y}(t) + 3\,y(t) = \boxed{-}4\,u(t) + d(t)$$

B) Select K_p so that the steady-state error $\bar{e} = \bar{r} - \bar{y}$ is less than 0.1 when $r(t) = \bar{r} = 2$ and $d(t) = 1$.

$$2\dot{y} + \underbrace{(3 - 4K_p)}\,y = -4K_p r + d$$

Closed-loop is Stable if $3 - 4K_p > 0 \Leftrightarrow 4K_p < 3$

$$\boxed{K_p < 3/4}$$

$r \to y$

$$G(s) = \frac{-4K_p}{2s + \underbrace{(3-4K_p)}_{>0}}$$

In steady-state $(3 - 4K_p)\bar{y} = -4K_p\bar{r} + \bar{d}$

$$[1.9,\ 2.1] \approx \bar{y} = \underbrace{\left(\frac{-4K_p}{3-4K_p}\right)}_{\to 1 \text{ as } K_p \to -\infty} 2 + \underbrace{\left(\frac{1}{3-4K_p}\right)}_{\to 0 \text{ as } K_p \to -\infty} \cdot 1$$

$$\boxed{K_p = -100}$$

$$\bar{y} = (0.9926)\,2 + (0.0025)\,1 = \underline{1.988}$$

Solution 3C

$$2\dot{y}(t) + 3\,y(t) = -4\,u(t) + \,d(t)$$

$y(0)=0$

C) Sketch the response y with your gain K_p when $r(t) = 2$ and $d(t) = 1$. What is the time constant of the closed-loop?

$K_p = -100$

$2\dot{y} + (3 - 4K_p)\,y = -4K_p\, r + d$

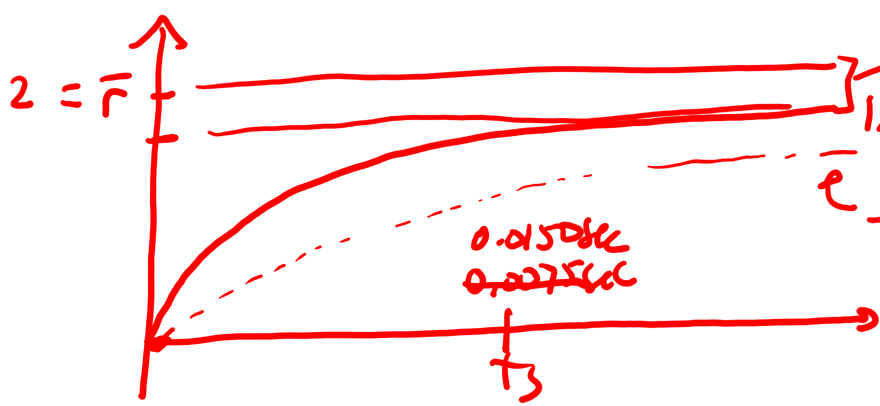
$2\dot{y} + (403)\,y = 400 r + d$

$\dot{y} + \frac{403}{2} y = 200 r + \frac{d}{2}$

$\tau = 2/403$ sec = ~~0.0025 sec~~ 0.0050

$t_s = 3\tau$ = ~~0.0075 sec~~ 0.0150

$\bar{e} = \bar{r} - \bar{y} = 0.012$



t

e smaller K_p will ↑ $\bar{e}$ and ↑ T_s

(t u)