ECE 486: Control Systems

Lecture 9A: PI Tuning for First-Order Systems

Key Takeaways

This lecture describes a method to tune PID controllers using pole placement.

For first-order systems, the approach is to:

- Use PI control and
- Select the gains to place the two closed-loop poles at desired locations.

The choice of natural frequency (time constant) is critical.

Problem 1

Consider the plant with the following transfer function:

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$

The poles of this system are at s=-1, -10±j.

A) What is the dominant pole approximation $G_a(s)$ for this plant?

B) Would you recommend using a PI, PD, or PID Controller?

C) Choose the controller gains so that the closed-loop with $G_a(s)$ has poles repeated at $s=-1$.

D) Where are the poles for the closed-loop with your controller and the actual plant $G(s)$? [Use numerical tools to solve.]

Solution 1A

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$

A) What is the dominant pole approximation $G_a(s)$ for this plant?

Solution 1B

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$

B) Would you recommend using a PI, PD, or PID Controller?

Solution 1C

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$

C) Choose the controller gains so that the closed-loop with $G_a(s)$ has poles repeated at $s=-1$.

Solution 1D

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$

D) Where are the poles for the closed-loop with your controller and the actual plant *G(s)*? [Use numerical tools to solve.]

Solution 1-Extra Space

Problem 2

Again consider the plant with the following transfer function:

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$

The poles of this system are at s=-1, -10±j.

A) Rechoose your controller gains so that the closed-loop with $G_a(s)$ has poles repeated at ***s=-2.***

B) Where are the poles for the closed-loop with your controller and the actual plant *G(s)*? [Use numerical tools to solve.]

C) What is the impact, if any, of the neglected poles?

Solution 2A

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$

A) Rechoose your controller gains so that the closed-loop with $G_a(s)$ has poles repeated at ***s=-2*.**

Solution 2B and 2C

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$

B) Where are the poles for the closed-loop with your controller and the actual plant *G(s)*? [Use numerical tools to solve.]

C) What is the impact, if any, of the neglected poles?

Solution 2-Extra Space

ECE 486: Control Systems

Lecture 9B: PID Tuning for Second-Order Systems

Key Takeaways

This lecture describes a method to tune PID controllers using pole placement.

For second-order systems, the approach is to:

- Use PID control and
- Select the gains to place the three closed-loop poles at desired locations.
- A PI controller (without the D-term) should be used if the plant has sufficient damping.

The choice of natural frequency (time constant) is critical.

Problem 3

Consider the plant with the following transfer function:

$$G(s) = \frac{20}{s^2 - 6s + 10}$$

A) What is the closed-loop ODE from reference *r* to output *y* if you use a PID controller? $u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau + K_d \dot{e}(t)$

B) Choose the controller gains so that the closed-loop has poles repeated at $s=-3$. Hint: $(s+3)^3 = s^3 + 9\,s^2 + 27\,s + 27$

C) What is the impact of implementing the derivative term as $K_d\,\dot{e}$ versus the rate feedback form $-K_d\dot{y}$?

Solution 3A

$$G(s) = \frac{20}{s^2 - 6s + 10}$$

A) What is the closed-loop ODE from reference *r* to output *y* if you use a PID controller? $u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau + K_d \dot{e}(t)$

Solution 3B

$$G(s) = \frac{20}{s^2 - 6s + 10}$$

B) Choose the controller gains so that the closed-loop has poles repeated at $s=-3$. Hint: $(s+3)^3 = s^3 + 9\,s^2 + 27\,s + 27$

Solution 3C

$$G(s) = \frac{20}{s^2 - 6s + 10}$$

C) What is the impact of implementing the derivative term as $K_d\ \dot{e}$ versus the rate feedback form $-K_d\dot{y}$?

Solution 3-Extra Space