ECE 486: Control Systems

Lecture 12A: Root Locus Rules DEF

Problem 1

Consider the following functions.

$$L = \frac{1}{s^2+2s+10}, \qquad L = \frac{s-3}{s^2+2s+10}, \qquad L = \frac{s+4}{s^5+1}$$

Sketch the root loci by hand by applying rules A-F.

Problem 1A

Consider the following functions.

$$L = \frac{1}{s^2+2s+10}$$

Sketch the root loci by hand by applying rules A-F.

A : # of branches = 2

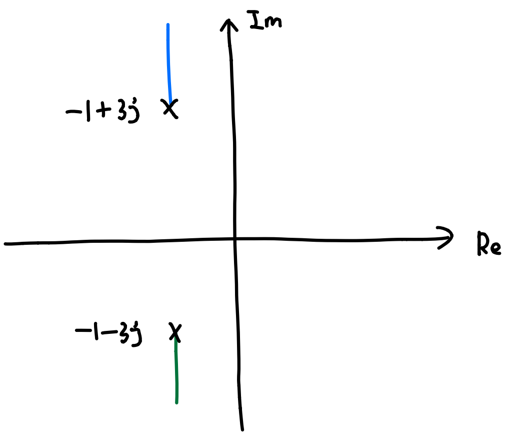
B : branches start at $-1 \pm 3j$

C : branches end at ∞

D : real locus : none

E : exit angle : $90°$, $270°$

F : $j\omega$– crossing : none



Problem 1B

Consider the following functions.

$$L = \frac{s-3}{s^2+2s+10}$$

Re

Sketch the root loci by hand by applying rules A-F.

A: # of branches = 2

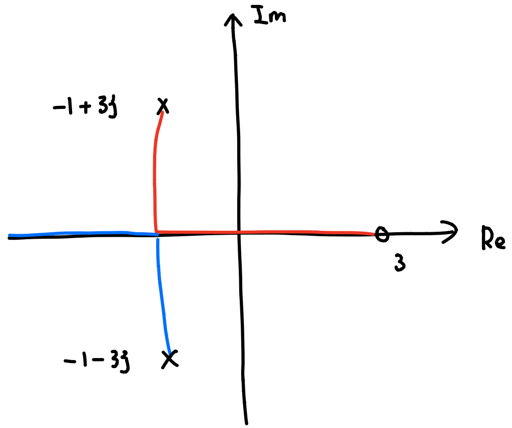
B: branches start at $-1 \pm 3j$

C: branches end at 3, $\pm\infty$

D: real locus: $(-\infty, 3)$

E: exit angle: 180°

F: $j\omega$- crossing: $\omega = 0$



Problem 1C

Consider the following functions.

$$L = \frac{s+4}{s^5+1}$$

Sketch the root loci by hand by applying rules A-F.

A: # of branches = 5

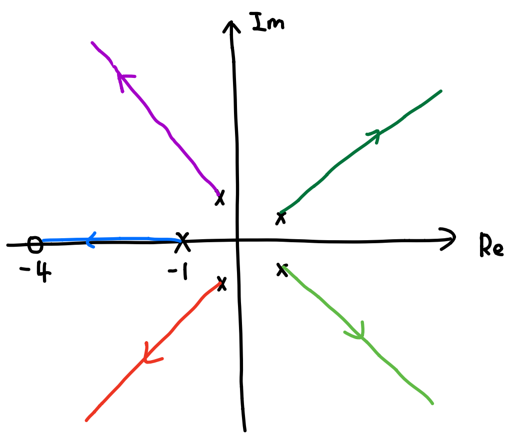
B: branches start at -1, $-0.309 \pm 0.9511j$, $0.809 \pm 0.5878j$

C: branches end at -4, $\pm\infty$

D: real locus: $(-4, -1)$

E: exit angle: $\frac{\pi k}{4}$, $k = 1, 3, 5, 7$

F: $j\omega$- crossing: None



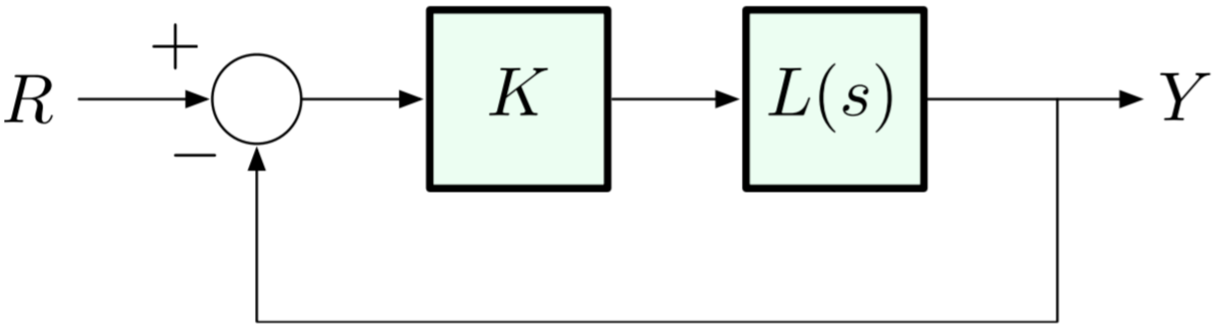
Solution 1-Extra Space

ECE 486: Control Systems

Lecture 12B: Case Study on Root Locus Design

Problem 2

Suppose the following block diagram.



(a) If L has 5 LHP poles, 2 RHP poles, and 7 LHP zeros, is the closed-loop system stable for very large K>0?

(b) If L has 4 LHP poles, and 2 LHP zeros, is the closed-loop system stable for very large K>0?

Problem 2A

(a) If L has 5 LHP poles, 2 RHP poles, and 7 LHP zeros, is the closed-loop system stable for very large K>0?

YES. Root locus branches start at poles and end at zeros. In this case, we have equal number of poles and zeros, all branches will end at zero. Since we have LHP zeros, a large enough K can make the closed-loop system stable.

Problem 2B

(b) If L has 4 LHP poles, and 2 LHP zeros, is the closed-loop system stable for very large K>0?

Although all poles and zeros are at LHP, it does not necessary mean that all values of K will keep the closed-loop system stable. Since we have more poles than zeros, the asymptotes should be considerd. If there are $j\omega$- crossings, then some asymptotes may escape into RHP with large gain, causing the system to be unstable.

Solution 2-Extra Space