ECE 486: Control Systems

Lecture 13B: Bode Plots

Key Takeaways

A Bode plot for an LTI system $G(s)$ consists of two subplots:

- Magnitude (Gain) vs. frequency and
- Phase vs. frequency.

Such plots are useful to understand the steady-state response of the system $G(s)$ to sinusoids of different frequencies.

Bode Plots

If a stable, LTI system G(s) is forced by $u(t) = \sin(\omega t)$ then:

$$y(t) \to |G(j\omega)| \sin(\omega t + \angle G(j\omega))$$

A Bode Plot is a common tool used to understand a linear system to sinusoids at different frequencies. It consists of two subplots:

- Bode Magnitude (Gain) Plot: Gain vs. Frequency
  - Horizontal axis is ω on a log (base 10) scale in units of rad/sec .
  - Vertical axis is the gain in decibels: $|G(j\omega)|_{dB} = 20\log_{10}|G(j\omega)|$
  - We can convert from dB back to actual gain: $|G(j\omega)| = 10^{|G(j\omega)|_{dB}/20}$

$\vert G(j\omega)\vert$	0.001	0.01	0.1	0.25	0.5	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	2	4	10	100	1000
$\vert G(j\omega)\vert_{dB}$	-60	-40	-20	-12	-6	-3	0	3	6	12	20	40	60

- Bode Phase Plot: Phase vs. Frequency
  - Horizontal axis is ω on a log (base 10) scale in units of rad/sec .
  - Vertical axis is the phase $\angle G(j\omega)$ in degrees.

Example

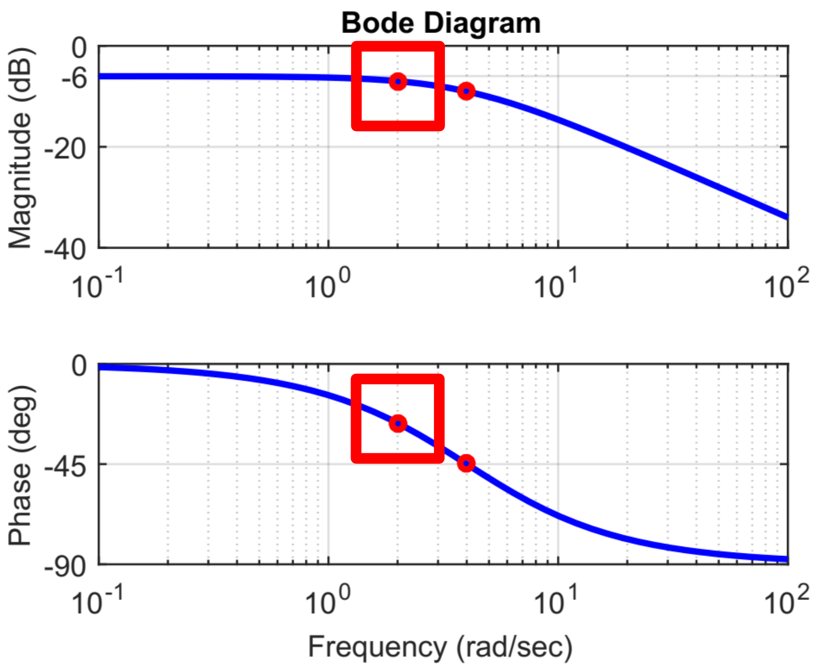
```
>> G = tf(2,[1 4]);
>> bode(G);
```

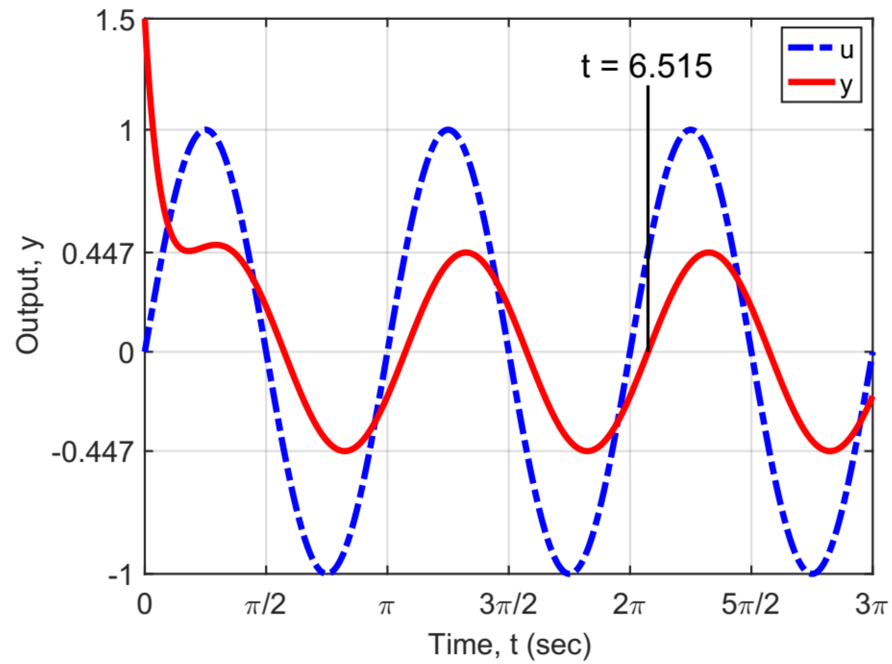
System: $G(s) = \frac{2}{s+4}$

- If $\omega \approx 0$ then $|G(j\omega)| \approx 0.5$ and $\angle G(j\omega) \approx 0^o$ so:

$$\text{If } u(t) = \sin(\omega t) \quad \text{then} \quad y(t) \to 0.5\sin(\omega t)$$

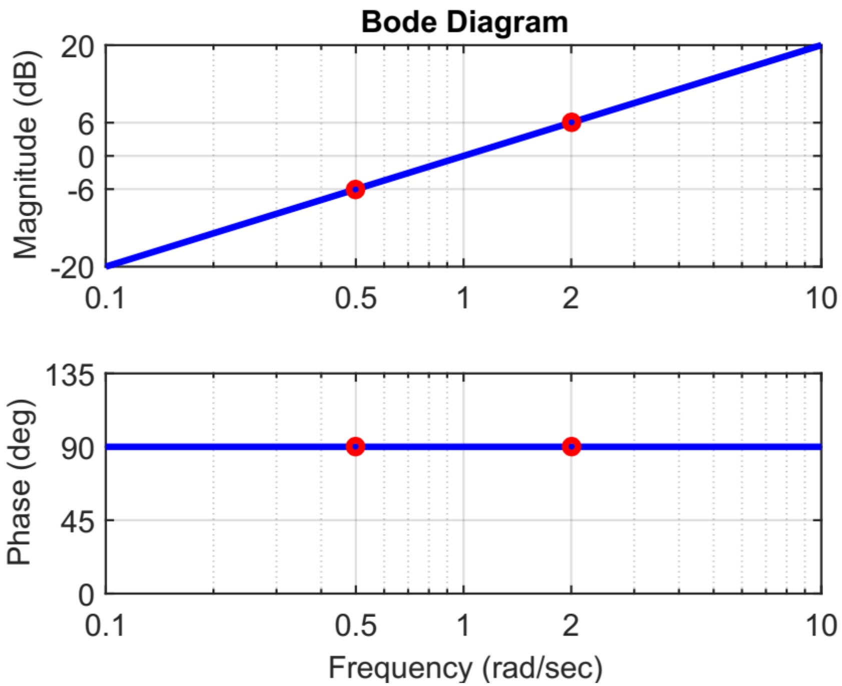
- If $\omega \to \infty$ then $|G(j\omega)| \to 0$ and $\angle G(j\omega) \approx -90^o$
- If $\omega = 2\frac{rad}{sec}$ then $|G(j\omega)| = 0.45$ and $\angle G(j\omega) = -27^o$

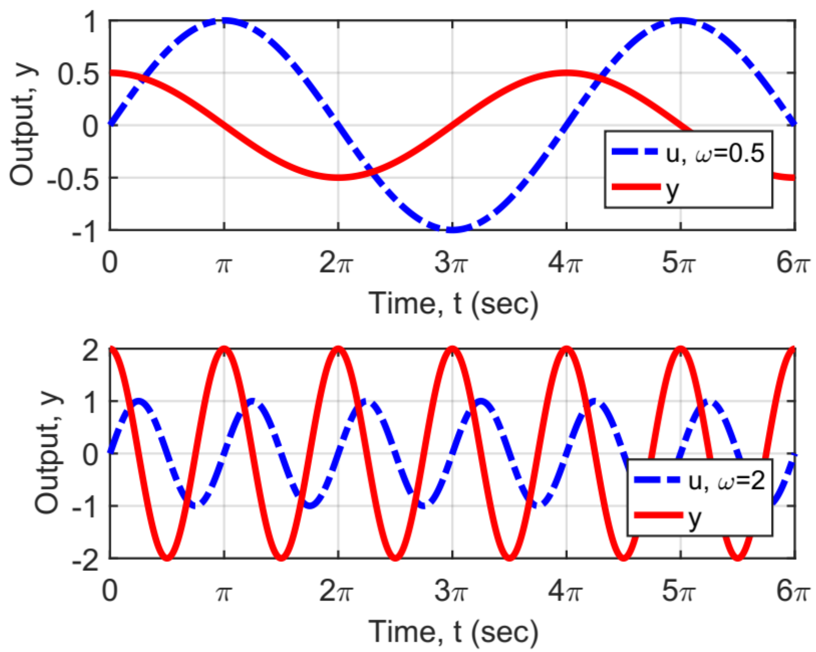




Bode Plot: Differentiator

- Differentiator: $y(t) = \dot{u}(t)$ $\quad G(s) = \frac{s}{1} = s$
  - If $u(t) = \sin(\omega t)$ then $y(t) = \omega \cos(\omega t) = \omega \sin(\omega t + \frac{\pi}{2})$
  - This agrees with $|G(j\omega)| = \omega$ and $\angle G(j\omega) = \frac{\pi}{2} rad = 90^o$
- Properties:
  - Differentiator amplifies higher frequencies and output leads input.
  - Slope of magnitude plot is +20dB/decade

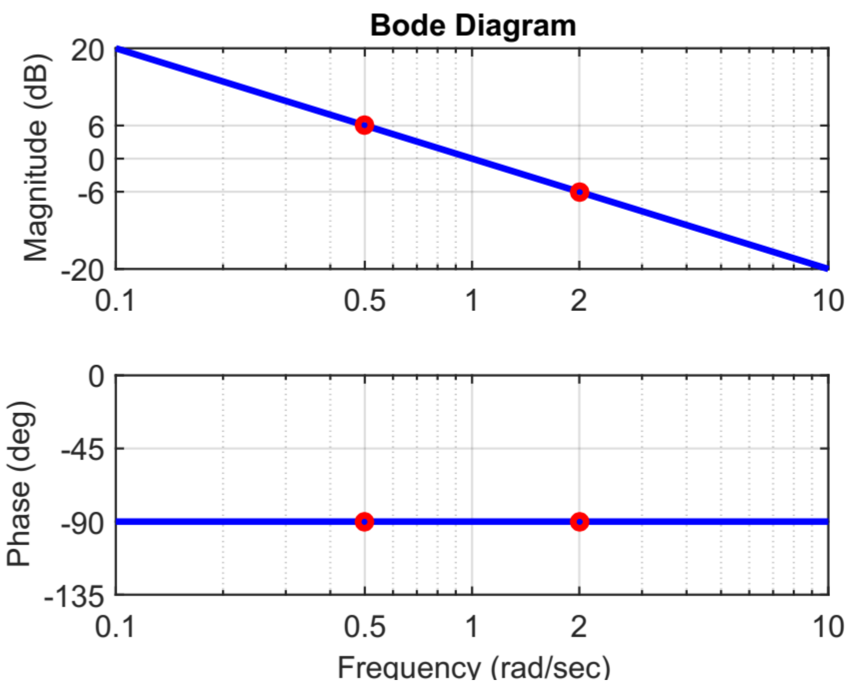


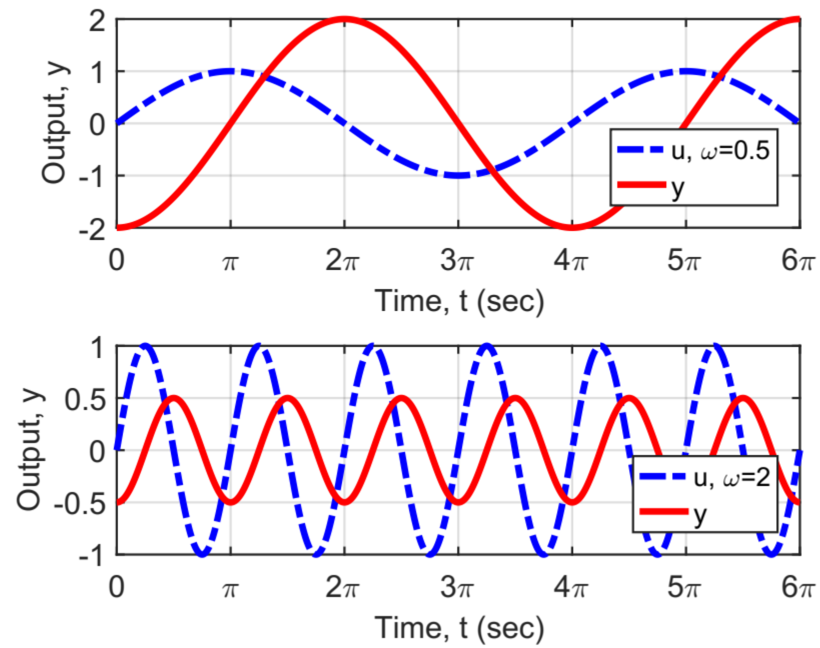


Bode Plot: Integrator

- Integrator: $\dot{y}(t) = u(t)$ $G(s) = \frac{1}{s}$
  - If $u(t) = \sin(\omega t)$ then $y(t) = -\frac{1}{\omega}\cos(\omega t)$

    [Neglecting a constant term and effect of initial conditions]





Bode Plot: Integrator

- Integrator: $\dot{y}(t) = u(t)$ $G(s) = \frac{1}{s}$
  - If $u(t) = \sin(\omega t)$ then $y(t) = \frac{1}{\omega}\sin\left(\omega t - \frac{\pi}{2}\right)$
  - This agrees with $|G(j\omega)| = \frac{1}{\omega}$ and $\angle G(j\omega) = -\frac{\pi}{2} rad = -90^o$
- Properties:
  - Integrator amplifies lower frequencies and output lags input.
  - Slope of magnitude plot is -20dB/decade.