ECE 486: Control Systems

Lecture 13C: Bode Plots for First-Order Systems

Key Takeaways

This lecture focuses on Bode plots for first order systems.

The Bode plot for $G(s) = \frac{b_0}{s+a_0}$ has the following key features:

- The pole defines a corner frequency ($\omega = |a_0|$) for the system.
- The magnitude is flat at low frequencies and rolls off at −20dB per decade at high frequencies.
- The phase transitions by ±90° near the corner frequency with precise details depending on the signs of (b_0, a_0).

The Bode plot for $G(s) = \frac{s+b_0}{a_0}$ has the similar features except:

- The zero defines a corner frequency ($\omega = |b_0|$) for the system.
- The magnitude rolls up at +20dB per decade at high frequencies.

First-Order Systems

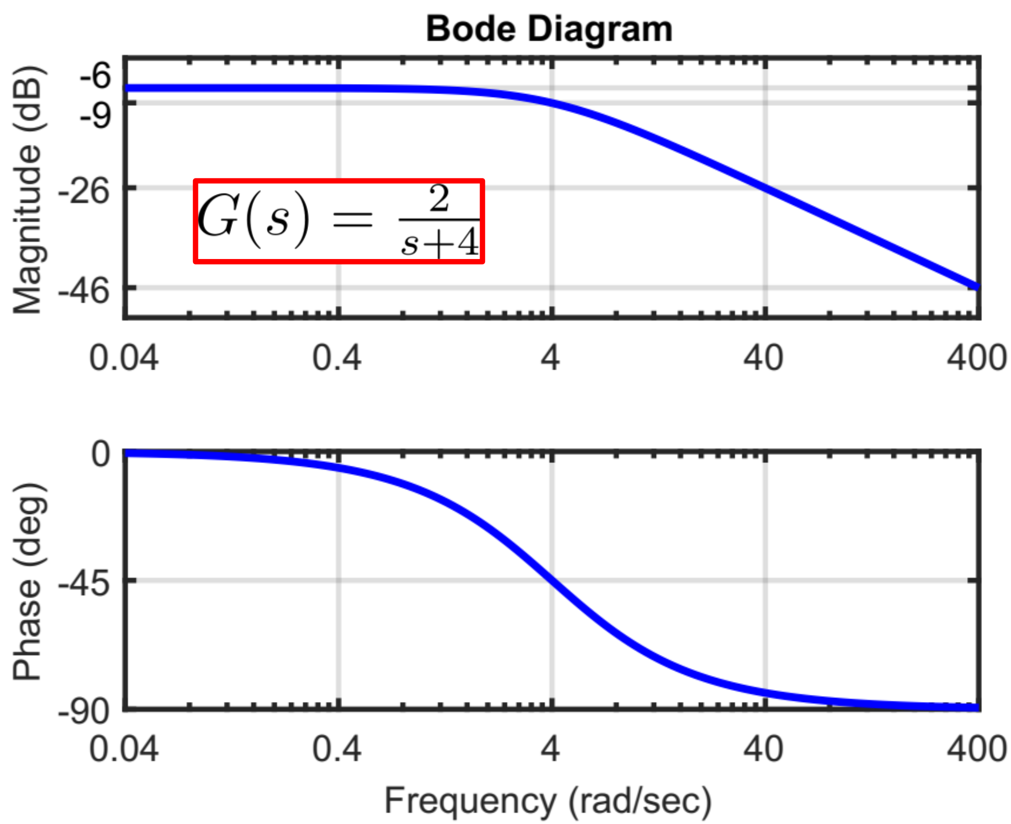
Consider the following first-order system:

$$\dot{y}(t) + a_0 y(t) = b_0 u(t) \qquad G(s) = \frac{b_0}{s+a_0}$$

To start, assume $a_0 > 0$ and $b_0 > 0$.

Bode plots can be generated by Matlab:

```
>> G=tf(2,[1 4]);
>> bode(G);
```



It will be useful to sketch straight-line approximate Bode plots.

Corner Frequency

Consider the following first-order system:

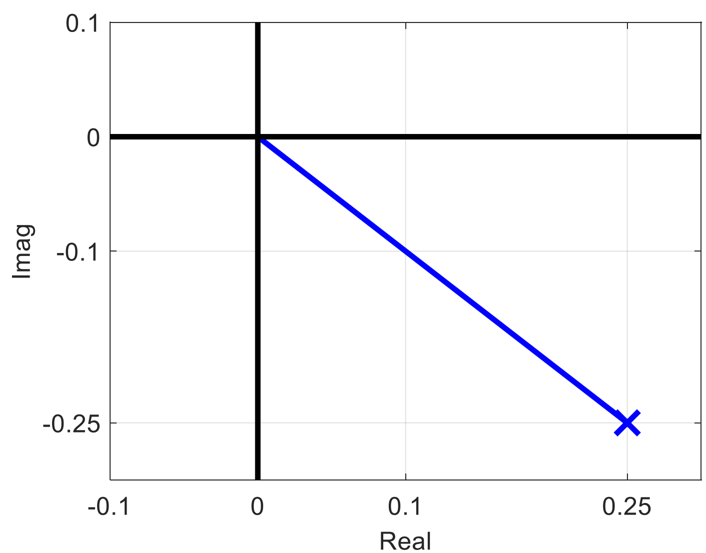
$$\dot{y}(t) + a_0 y(t) = b_0 u(t) \qquad G(s) = \frac{b_0}{s+a_0}$$

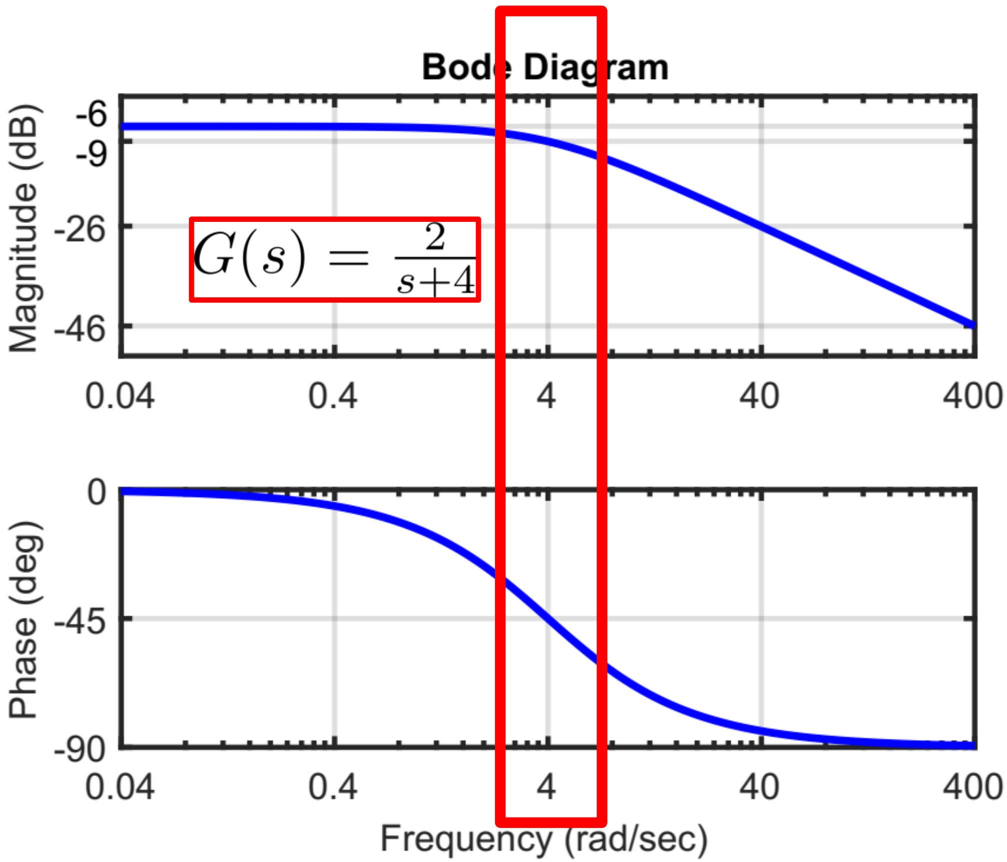
To start, assume $a_0 > 0$ and $b_0 > 0$.

Corner Frequency: $\omega = a_0$

$$G(ja_0) = \frac{b_0}{ja_0 + a_0} = \frac{G(0)}{j+1}$$

$$G(ja_0) = 0.5G(0) - 0.5G(0)j$$





Corner Frequency

Consider the following first-order system:

$$\dot{y}(t) + a_0 y(t) = b_0 u(t) \qquad G(s) = \frac{b_0}{s+a_0}$$

To start, assume $a_0 > 0$ and $b_0 > 0$.

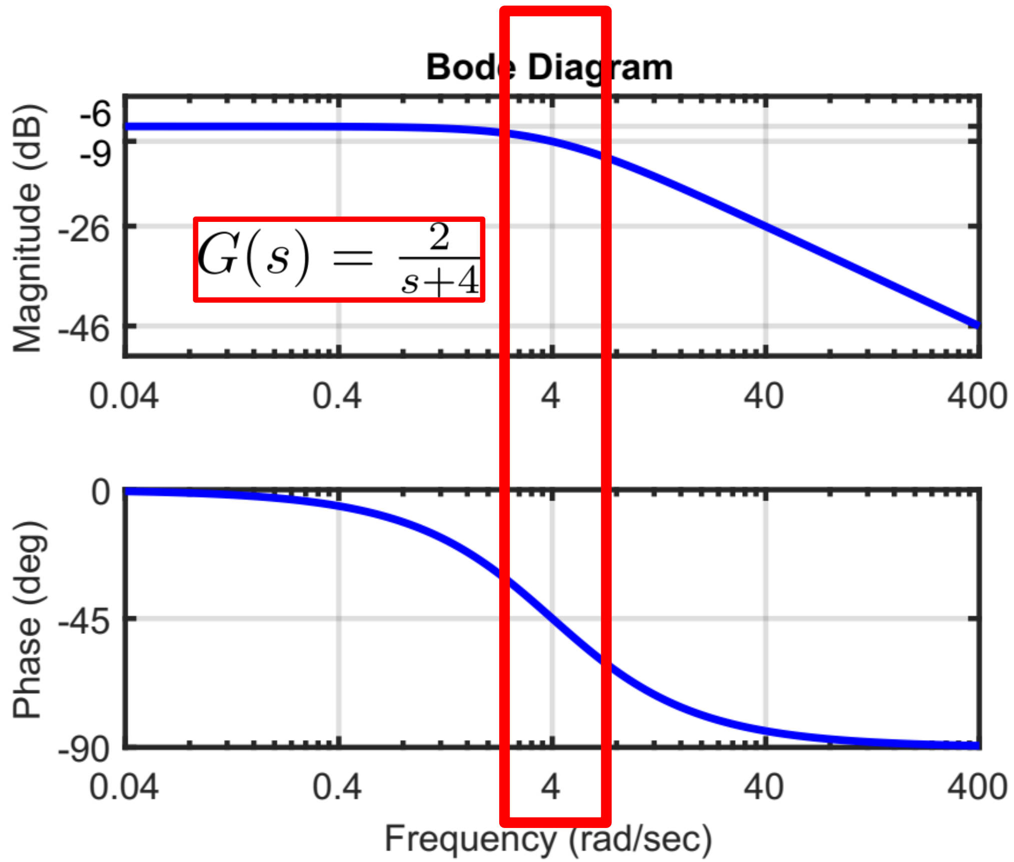
Corner Frequency: $\omega = a_0$

$$G(ja_0) = \frac{b_0}{ja_0 + a_0} = \frac{G(0)}{j+1}$$

$$G(ja_0) = 0.5G(0) - 0.5G(0)j$$

$$\angle G(ja_0) = -45^o$$

$$|G(ja_0)| = \frac{1}{\sqrt{2}}|G(0)|$$



Time constant is $\tau = \frac{1}{a_0}$.

Larger corner frequency

⇔ Faster Response

Low-Frequency Approximation

Consider the following first-order system:

$$\dot{y}(t) + a_0 y(t) = b_0 u(t) \qquad G(s) = \frac{b_0}{s+a_0}$$

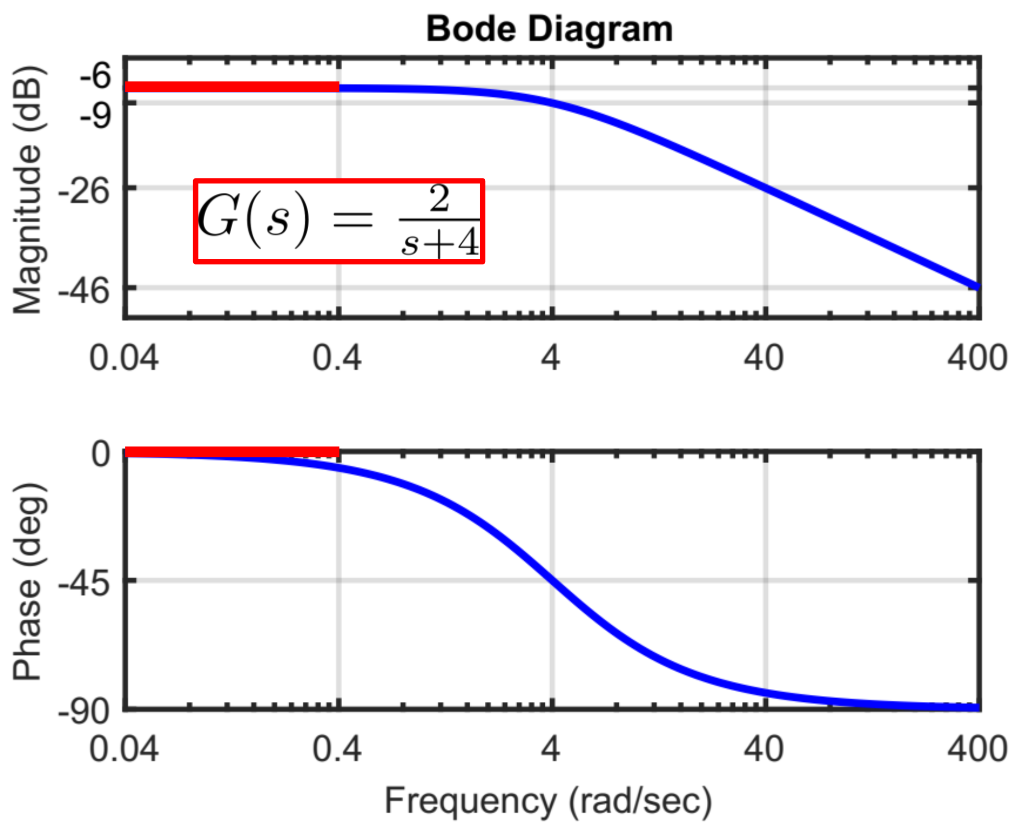
To start, assume $a_0 > 0$ and $b_0 > 0$.

Low Frequency: $\omega \leq \frac{a_0}{10}$

$$G(j\omega) \approx \frac{b_0}{a_0}$$

$$\angle G(j\omega) = 0^o$$

$$|G(j\omega)| = G(0)$$



High-Frequency Approximation

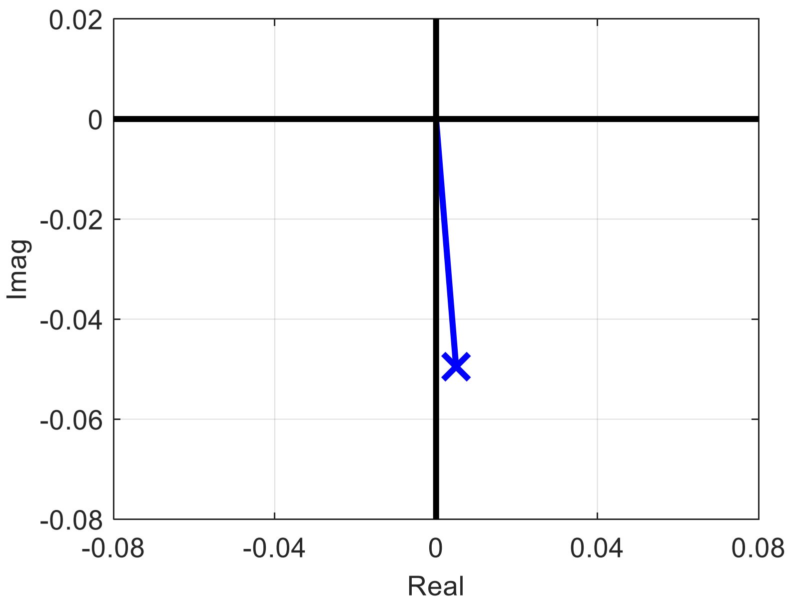
Consider the following first-order system:

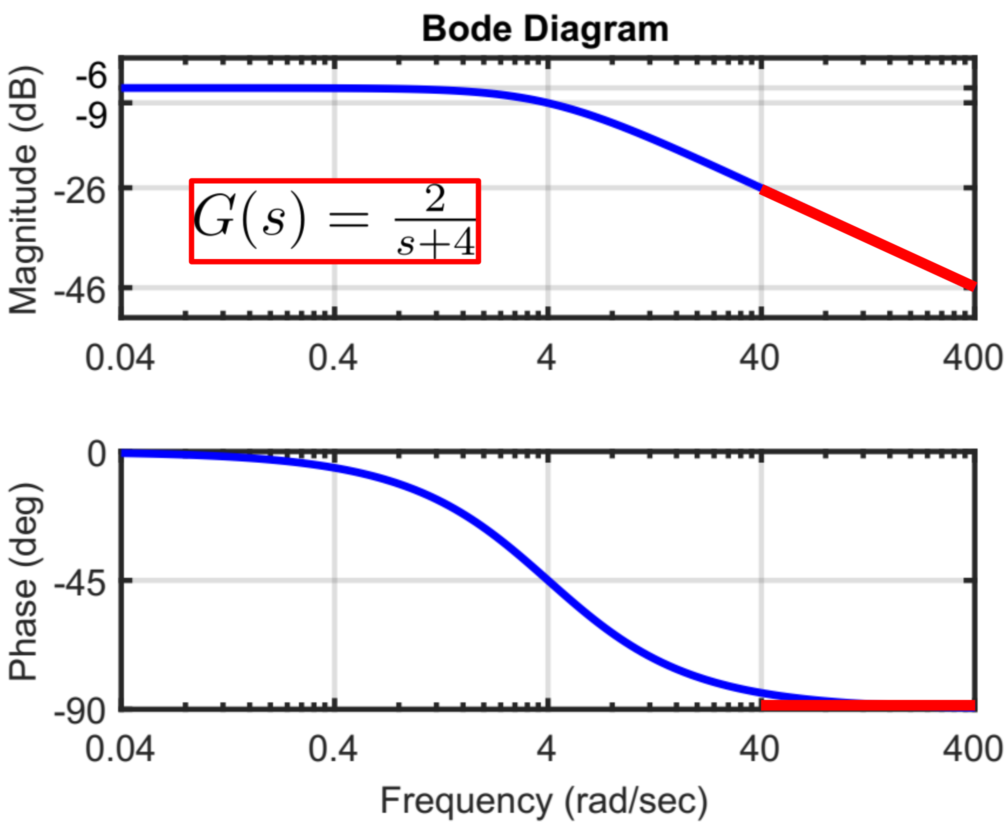
$$\dot{y}(t) + a_0 y(t) = b_0 u(t) \qquad G(s) = \frac{b_0}{s+a_0}$$

To start, assume $a_0 > 0$ and $b_0 > 0$.

High Frequency: $\omega \geq 10a_0$

$$G(j\omega) \approx \frac{b_0}{j\omega} = -\frac{b_0}{\omega} j$$





High-Frequency Approximation

Consider the following first-order system:

$$\dot{y}(t) + a_0 y(t) = b_0 u(t) \qquad G(s) = \frac{b_0}{s+a_0}$$

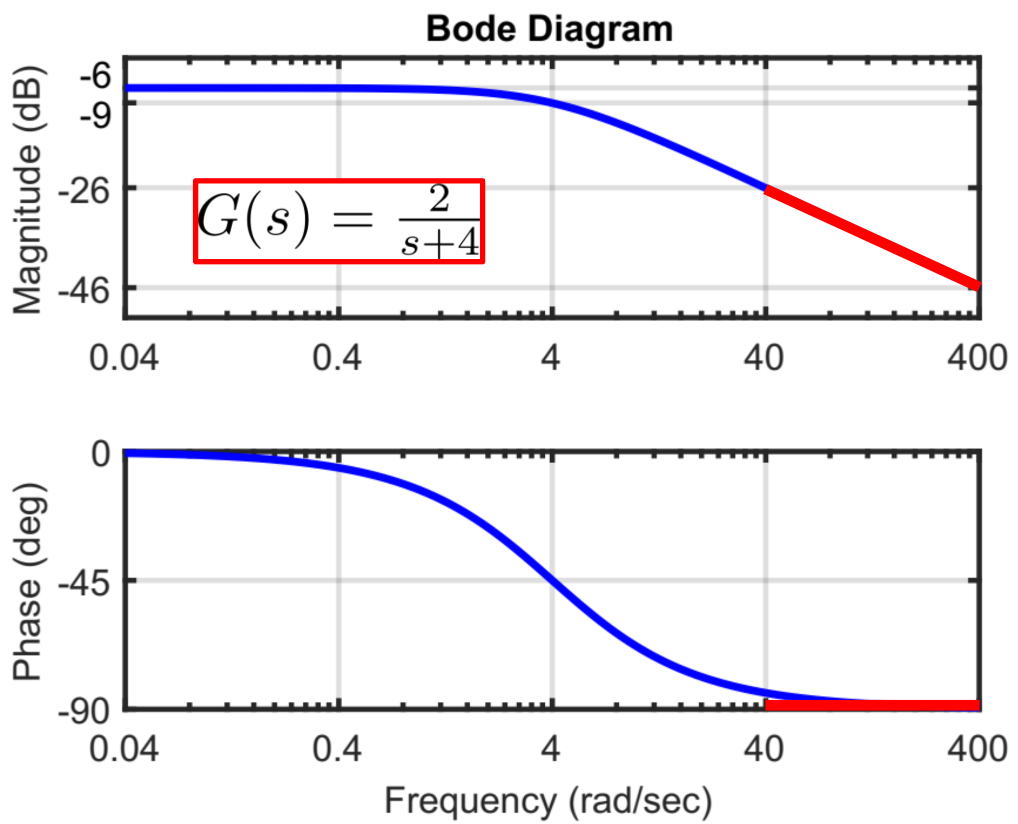
To start, assume $a_0 > 0$ and $b_0 > 0$.

High Frequency: $\omega \geq 10a_0$

$$G(j\omega) \approx \frac{b_0}{j\omega} = -\frac{b_0}{\omega}j$$

$$\angle G(j\omega) \approx -90^o$$

$$|G(j\omega)| \approx \frac{b_0}{\omega}$$



Magnitude rolls-off at -20dB per decade (similar to 1/s).

Middle-Frequency Approximation

Consider the following first-order system:

$$\dot{y}(t) + a_0 y(t) = b_0 u(t) \qquad\qquad G(s) = \frac{b_0}{s+a_0}$$

To start, assume $a_0 > 0$ and $b_0 > 0$.

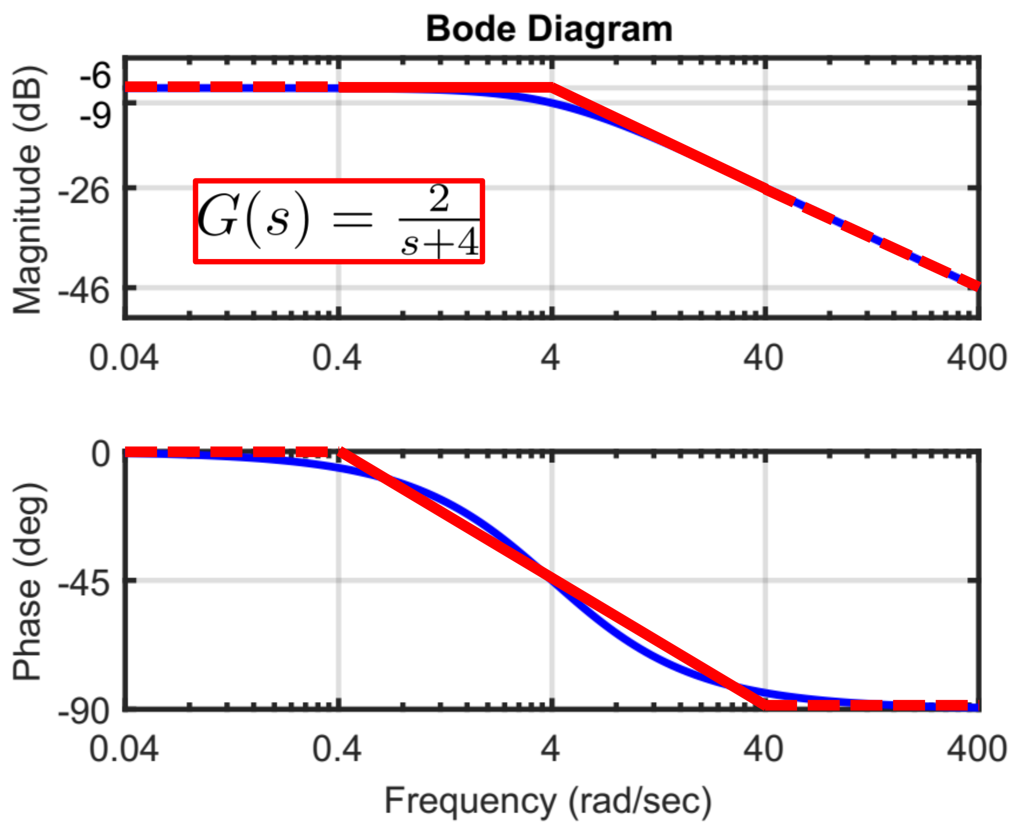
Middle Frequency:

$$\frac{a_0}{10} \leq \omega \leq 10a_0$$

Straight line approximation to connect low/high freqs.

Magnitude: Lines meet at corner frequency.

Phase: Line passes through -45° at corner frequency.



General First-Order System

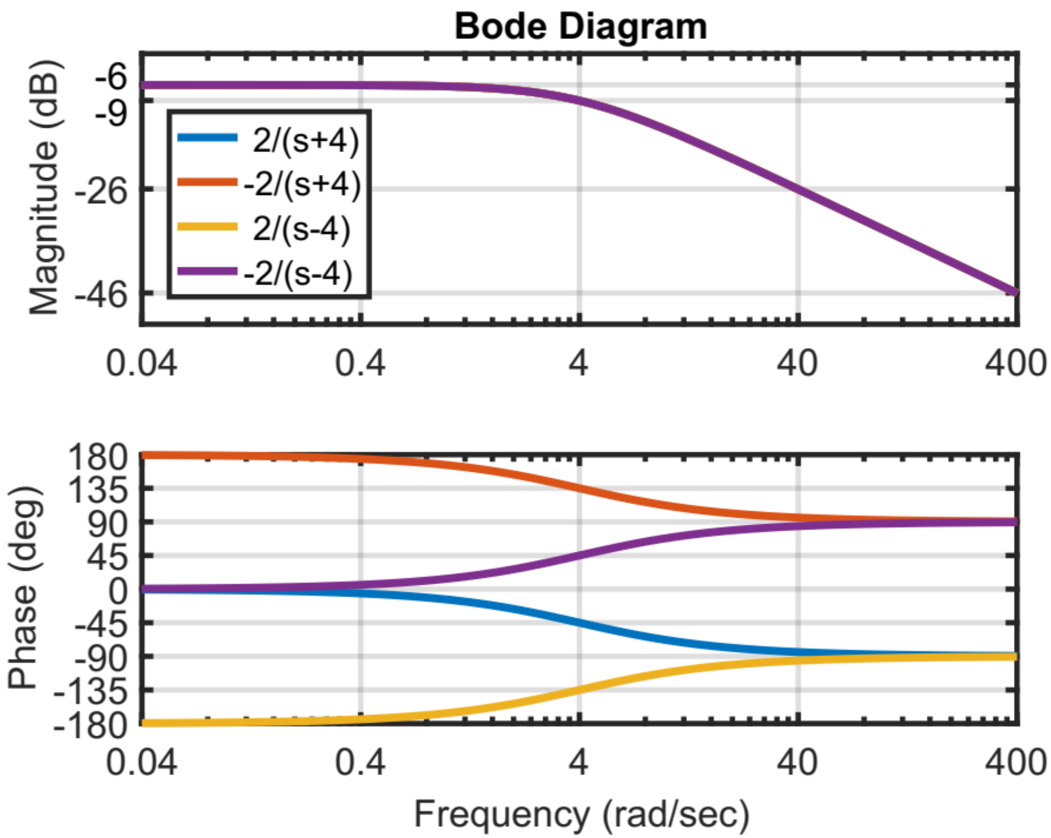
Consider the following first-order system:

$$\dot{y}(t) + a_0 y(t) = b_0 u(t) \qquad G(s) = \frac{b_0}{s+a_0}$$

Allow a_0 and b_0 to take any sign.

Bode Plots:

- Use same procedure for straight-line approximation.
- Magnitude is unchanged.
- Phase changes by $\pm 90^o$ but details depend on signs of (a_0, b_0).
- Bode plots can be drawn for unstable systems.



First-Order Zero

Consider the following first-order system:

$$a_0 y(t) = \dot{u}(t) + b_0 u(t) \qquad G(s) = \frac{s+b_0}{a_0}$$

Allow a_0 and b_0 to take any sign.

Bode Plots:

- Use same procedure for straight-line approximation.
- Corner frequency at the zero $\omega = |a_0|$
- Magnitude rises at +20dB per decade at high frequencies.