ECE 486: Control Systems

Lecture 4A: Time Domain Performance

Key Takeaways

This lecture defines important performance characteristics for a system in terms of its step response.

The performance characteristics include:

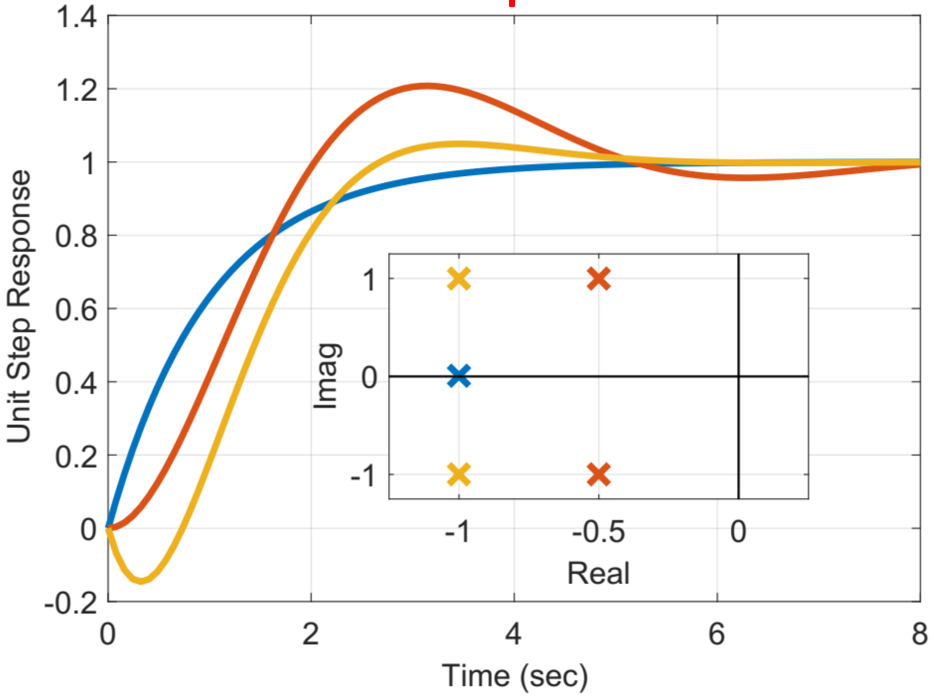
- Stability
- Final Value
- Settling Time
- Overshoot
- Rise Time
- Undershoot

Step Response

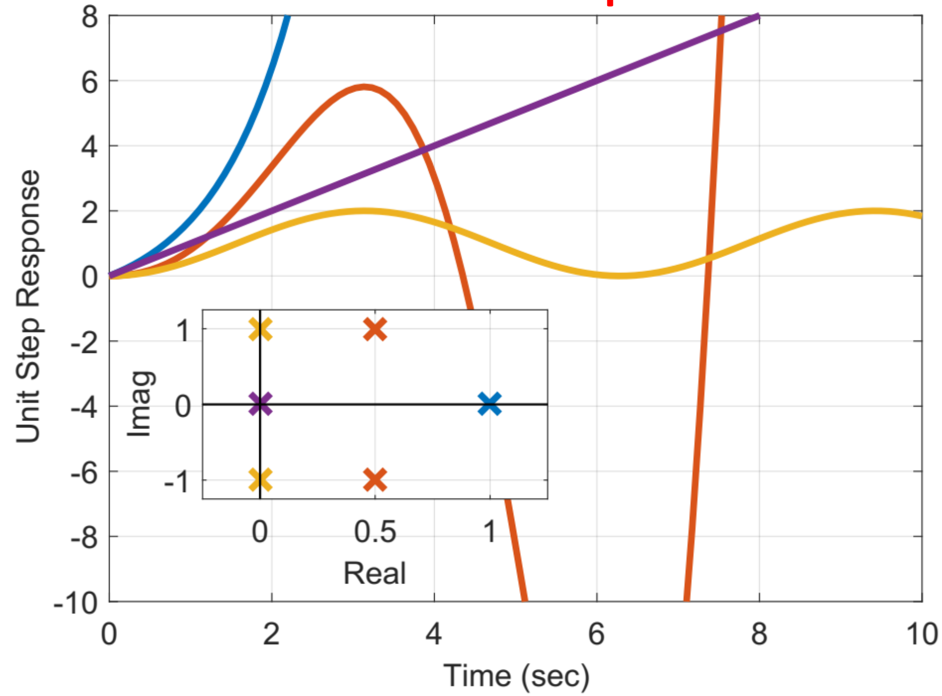
Consider the response of an LTI system with zero initial conditions and $u(t) = \bar{u}$ for $t \geq 0$ where $\bar{u}$ is a constant.

The solution is: $y(t) = \bar{y} + c_1 e^{s_1 t} + c_2 e^{s_2 t} + \cdots + c_n e^{s_n t}$

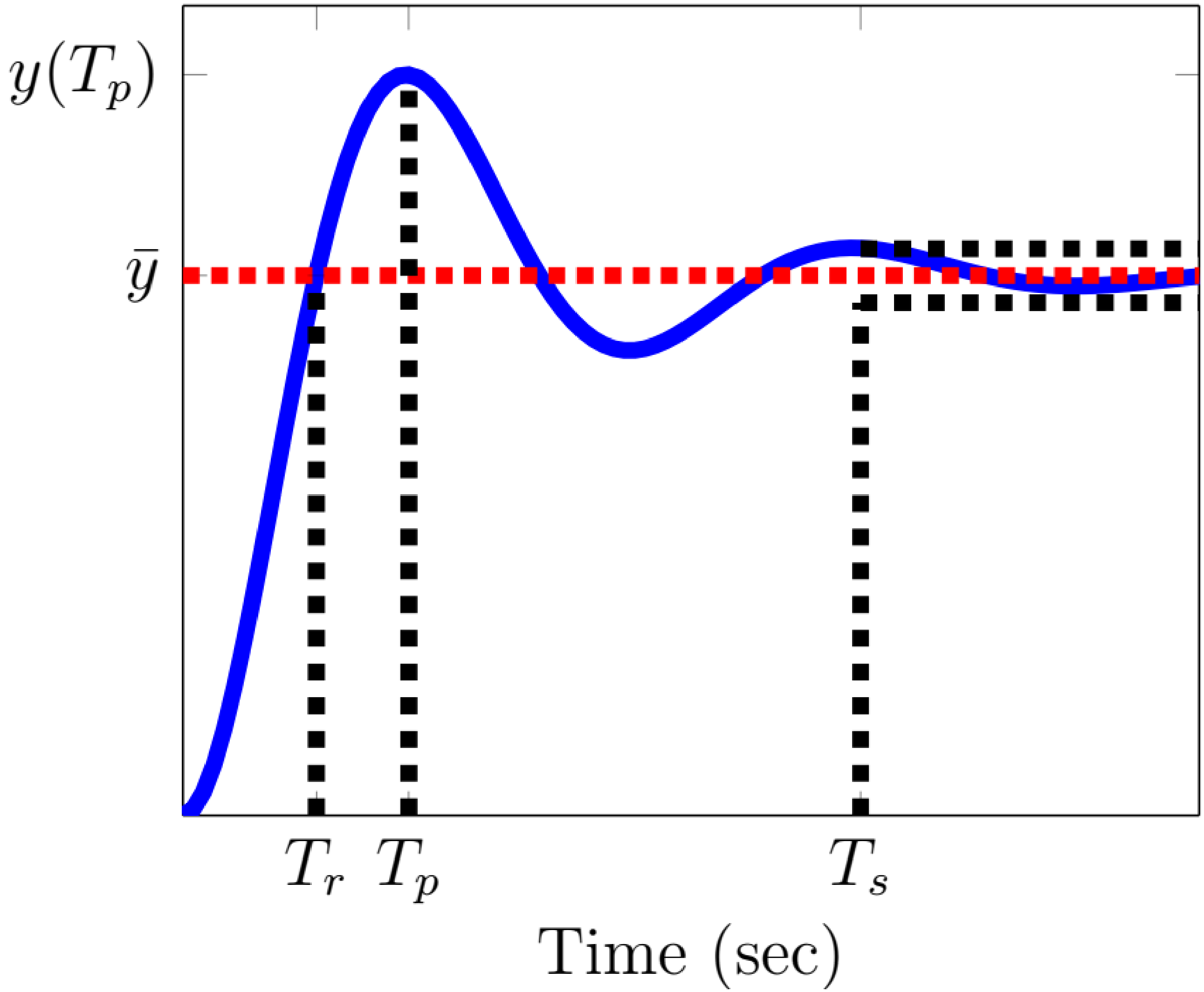
Stable Responses



Unstable Responses



Key Properties of Stable Step Responses



Final (Steady-State) Value

The solution is:

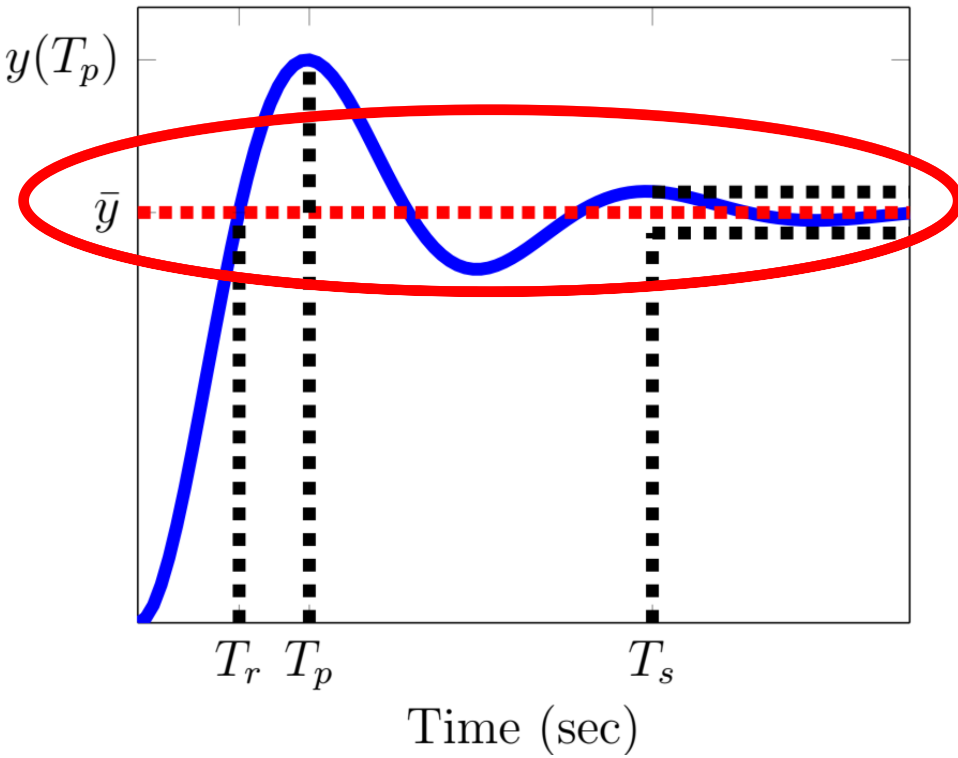
$$y(t) = \bar{y} + c_1 e^{s_1 t} + c_2 e^{s_2 t} + \cdots + c_n e^{s_n t}$$

If system is stable then

$$y(t) \to \bar{y} \text{ as } t \to \infty$$

$\bar{y}$ is the final value or

steady-state value.



Final (Steady-State) Value

Suppose the ODE is:

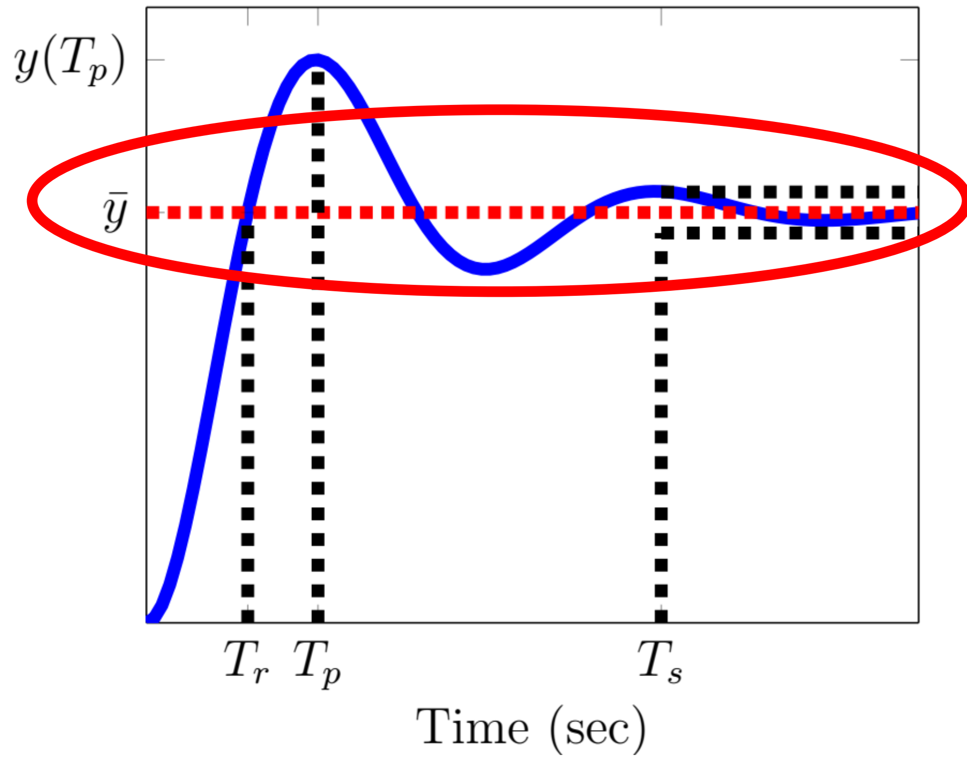
$$a_2\ddot{y}(t) + a_1\dot{y}(t) + a_0 y(t) = b_1\dot{u}(t) + b_0 u(t)$$

If $u(t) = \bar{u}$ and y(t) $\rightarrow \bar{y}$ then all derivatives go to zero:

$$a_0\bar{y} = b_0\bar{u}$$
$$\Rightarrow \bar{y} = \frac{b_0}{a_0}\bar{u}$$

Recall that $G(0) = \frac{b_0}{a_0}$ is the DC (steady-state) gain of the system. Thus:

$$\bar{y} = G(0)\bar{u}$$

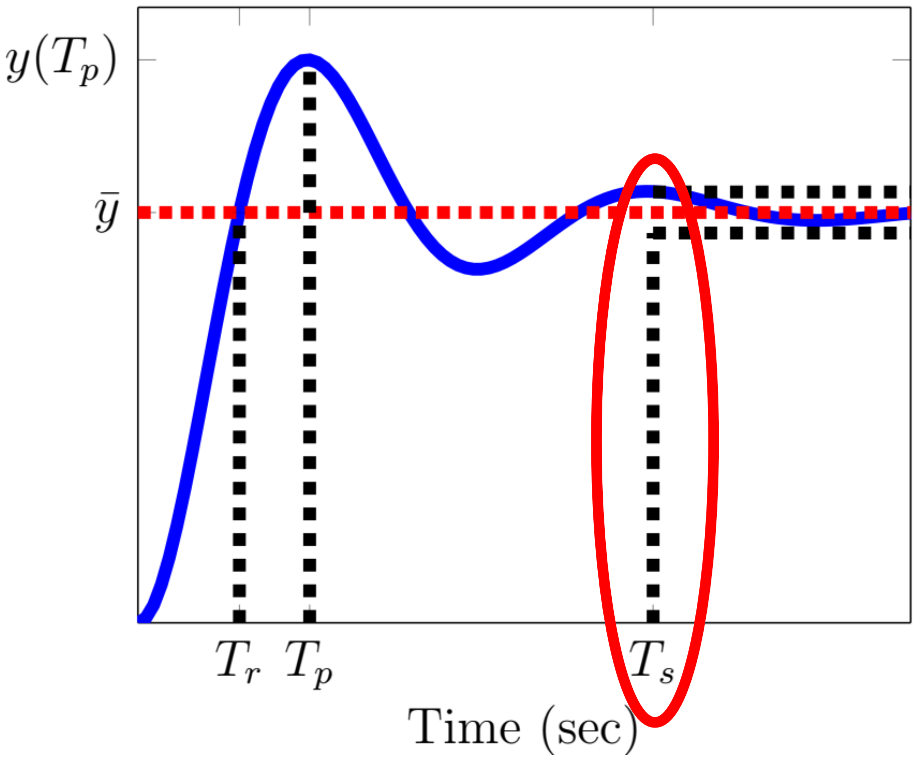


Settling Time

The settling time T_s is the time for the output to converge within ±5% of the final value, $[0.95\bar{y}, 1.05\bar{y}]$.

Slightly different definitions are occasionally used, e.g. 1% or 2% settling times.

This is one measure for the speed of response.



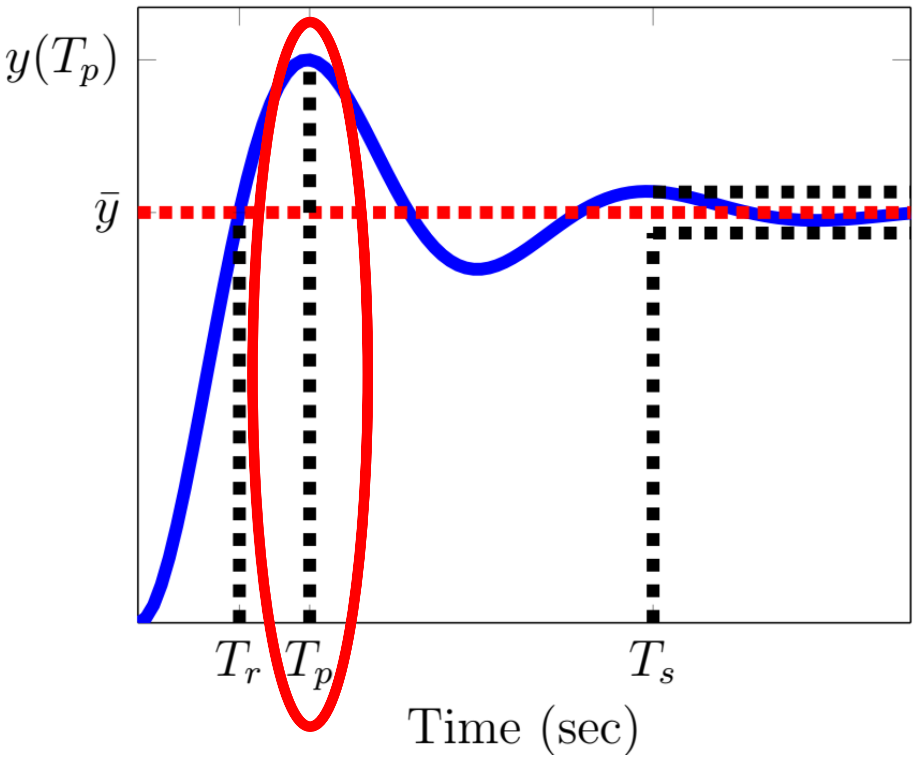
Peak Overshoot

Certain responses overshoot (exceed) the final value and oscillate before converging.

The peak value $y(T_p)$ occurs at the peak time T_p.

The peak overshoot is defined as:

$$M_p = \frac{y(T_p) - \bar{y}}{\bar{y}}$$



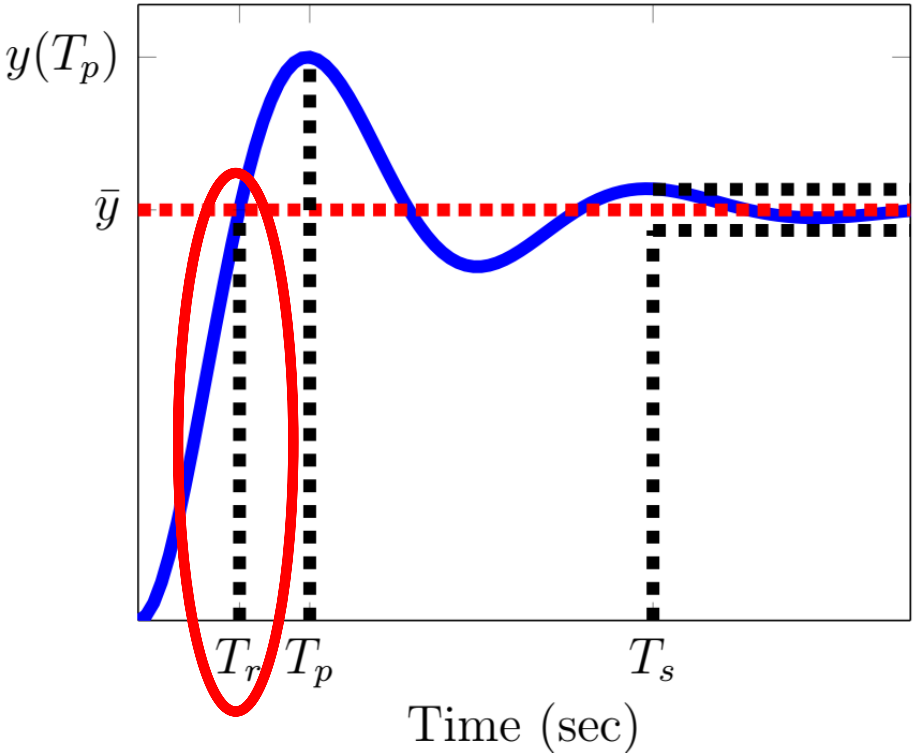
This is a unitless quantity and is sometimes reported as a percent (=100% x M_p).

Rise Time

The rise time T_r is the time required to first reach the steady-state value: $y(T_r) = \bar{y}$

Sometimes, the steady-state value is never reached.
Slightly different definitions are sometimes used.

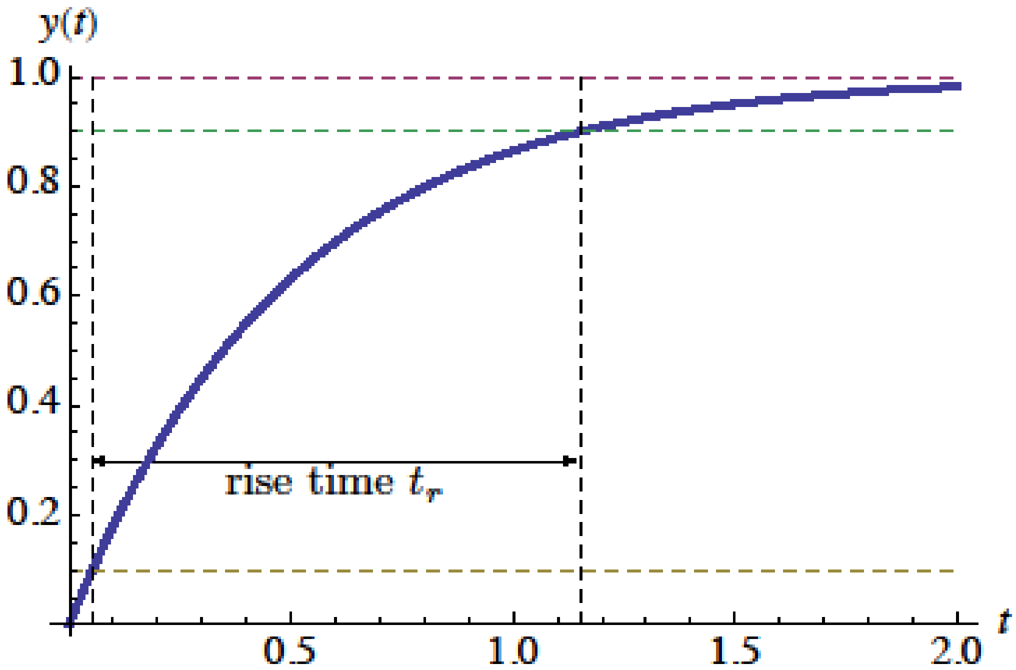
This is a measure for the initial speed of response.



Rise Time

Rise time can also be defined as the time it takes to get from 10% of steady-state value to 90%. (This is used in HWs and labs.)

Still a measure for the initial speed of response.



Undershoot

- The yellow step response initially moves negative before reversing direction toward the final value.
- This is called undershoot.
- Undershoot is related to the zeros in the transfer function.

This is not common but does appear in some systems. It creates fundamental challenges in feedback design.