EECS 486: Control Systems

Lecture 5A: Interconnection of Systems

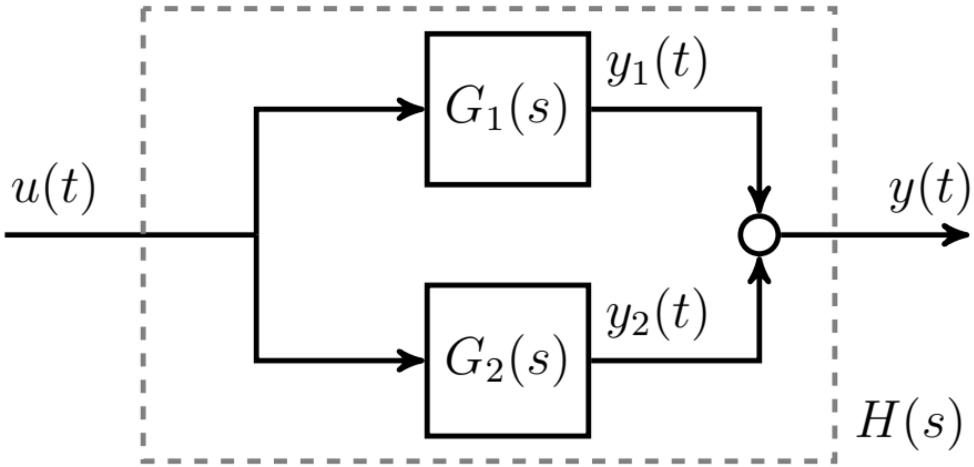
Key Takeaways

Transfer functions can be used to derive models for interconnections of LTI systems.

This lecture covers two specific examples:

- The parallel connection of $G_1(s)$ and $G_2(s)$ is given by $H(s) = G_1(s)+G_2(s)$.
- The serial connection of $G_1(s)$ and $G_2(s)$ is given by $H(s) = G_1(s)\,G_2(s)$.
- The negative feedback interconnection of $G_1(s)$ and $G_2(s)$ is given by $H(s) = \frac{G_1(s)}{1+G_1(s)\,G_2(s)}$.

Parallel Interconnection



$$\begin{aligned} Y(s) &= Y_1(s) + Y_2(s) \\ &= G_1(s)U(s) + G_2(s)U(s) \\ &= [G_1(s) + G_2(s)]\, U(s) \end{aligned}$$

$$\Rightarrow H(s) = G_1(s) + G_2(s)$$

```
>> G1 = tf(3,[1 4]);
```

$G_1(s) = \frac{3}{s+4}$ $\qquad$ $\dot{y}(t) + 4y(t) = 3u(t)$

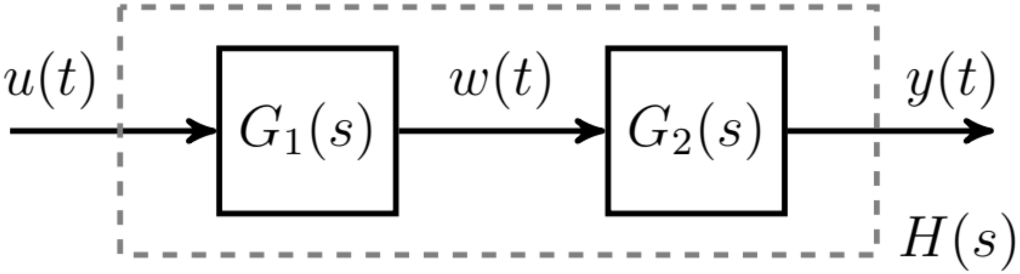
```
>> G2 = tf(5,[9 -6]);
```

$G_2(s) = \frac{5}{9s-6}$ $\qquad$ $9\dot{y}(t) - 6y(t) = 5u(t)$

```
>> H = G1+G2
     32 s + 2
 -----------------
 9 s^2 + 30 s - 24
```

$$9\ddot{y}(t) + 30\dot{y}(t) - 24y(t) = 32\dot{u}(t) + 2u(t)$$

Serial Interconnection



$$Y(s) = G_2(s)W(s)$$
$$= G_2(s)G_1(s)U(s)$$

$$\Rightarrow H(s) = G_2(s) \cdot G_1(s)$$

```
>> G1 = tf(3,[1 4]);
```

$G_1(s) = \frac{3}{s+4}$ $\qquad$ $\dot{y}(t) + 4y(t) = 3u(t)$

```
>> G2 = tf(5,[9 -6]);
```

$G_2(s) = \frac{5}{9s-6}$ $\qquad$ $9\dot{y}(t) - 6y(t) = 5u(t)$

```
>> H = G2*G1
     15
-----------------
9 s^2 + 30 s - 24
```

$$9\ddot{y}(t) + 30\dot{y}(t) - 24y(t) = 15u(t)$$

Negative Feedback Interconnection

Consider the negative feedback connection with $G_1(s)$ in the forward path and $G_2(s)$ in the feedback path.

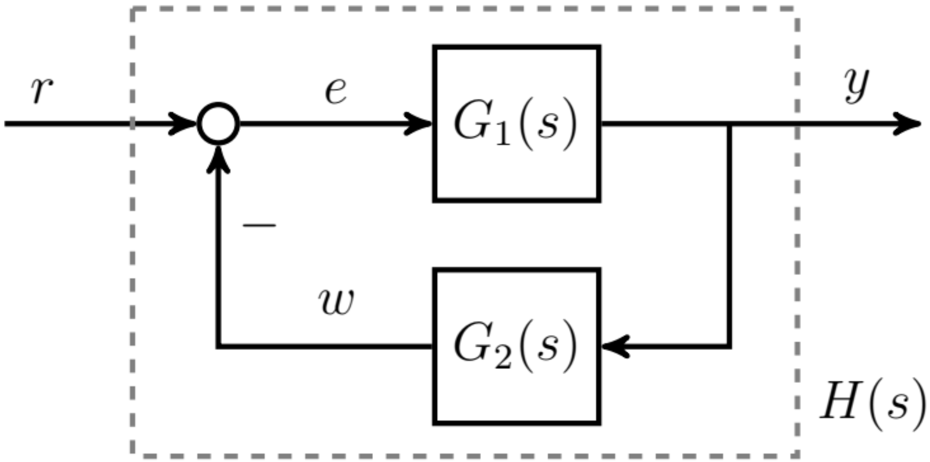
The relations are:

$$Y(s) = G_1(s)\left(R(s) - W(s)\right)$$
$$W(s) = G_2(s)Y(s)$$

Eliminating $W(s)$ and solving for $Y(s)$ yields:

$$Y(s) = \frac{G_1(s)}{1 + G_1(s)G_2(s)}R(s)$$

$$\Rightarrow H(s) = \frac{G_1(s)}{1+G_1(s)G_2(s)}$$



For positive feedback:

$$H(s) = \frac{G_1(s)}{1-G_1(s)G_2(s)}$$

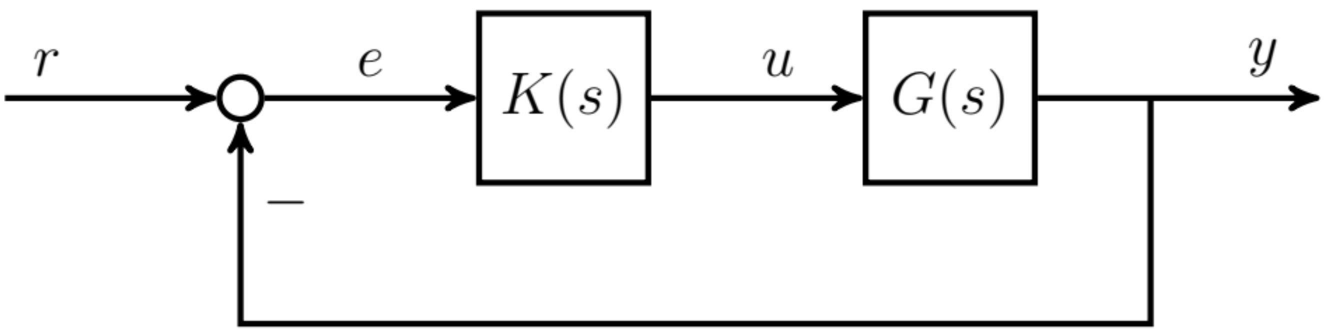
Unity Feedback Interconnection

The system from *r* to *y* has:

- Forward Path: $G_1(s) = G(s)K(s)$
- Feedback Path: $G_2(s) = 1$

The transfer function from *r* to *y* is:

$$T_{r\to y}(s) = \frac{G_1(s)}{1+G_1(s)G_2(s)} = \frac{G(s)K(s)}{1+G(s)K(s)}$$



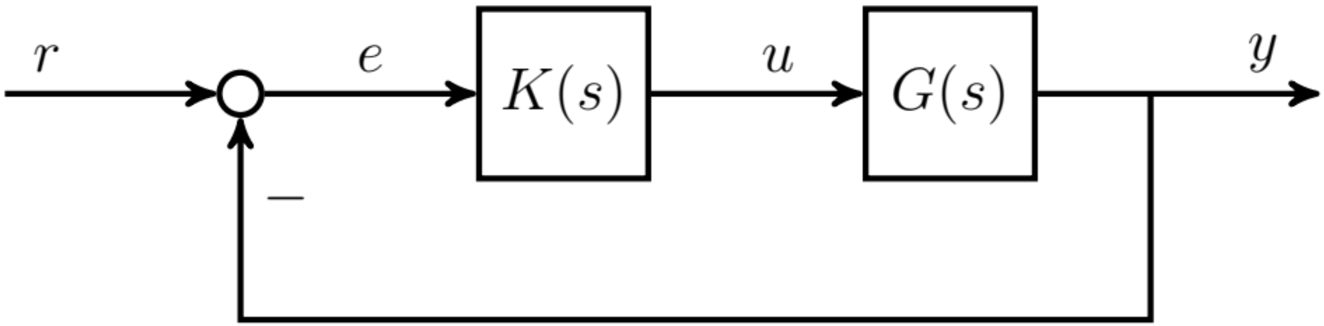
Example

The system from *r* to *e* has:

- Forward Path: $G_1(s) = 1$
- Feedback Path: $G_2(s) = G(s)K(s)$

The transfer function from *r* to *e* is:

$$T_{r\rightarrow e}(s) = \frac{G_1(s)}{1+G_1(s)G_2(s)} = \frac{1}{1+G(s)K(s)}$$



Non-minimal Realizations

Some care is required when interpreting: $H(s) = \frac{G_1(s)}{1+G_1(s)G_2(s)}$

Consider the negative feedback interconnection of:

$$G_1(s) = \frac{3}{s+4} \qquad \dot{y}(t) + 4y(t) = 3\left(r(t) - w(t)\right)$$

$$G_2(s) = \frac{5}{9s-6} \qquad 9\dot{w} - 6w(t) = 5y(t)$$

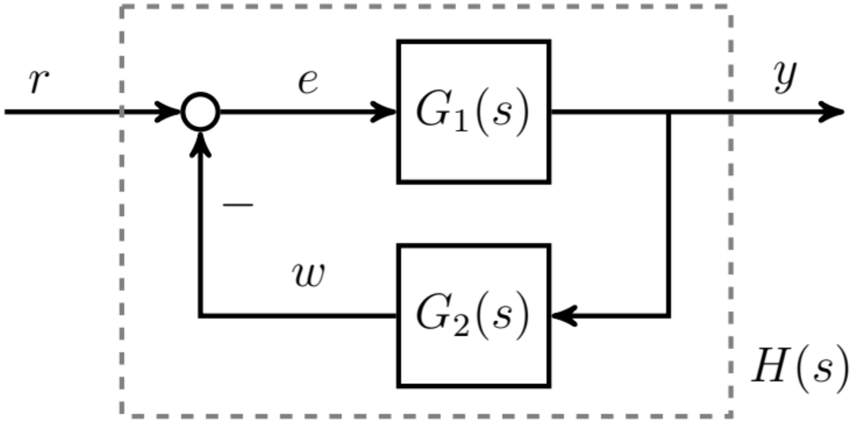
A direct use of the negative-feedback formula gives:

$$H(s) = \frac{\frac{3}{s+4}}{1+\frac{3}{s+4}\cdot\frac{5}{9s-6}}$$

$$= \frac{3}{(s+4)(9s-6)+3\cdot 5} \cdot \frac{\frac{1}{s+4}}{\frac{1}{(s+4)(9s-6)}}$$

$$= \frac{3(9s-6)}{(s+4)(9s-6)+3\cdot 5} \cdot \frac{s+4}{s+4}$$

This has a (fictitious) pole/zero cancellation at *s=-4*.



Non-minimal Realizations

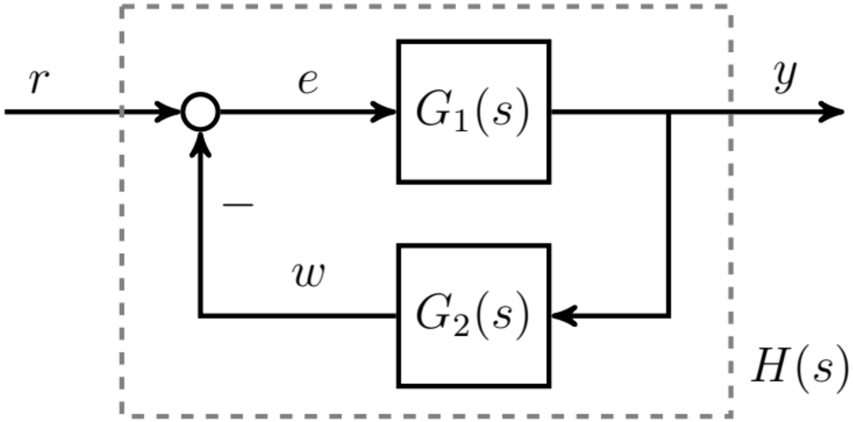
Consider the negative feedback interconnection of:

$$G_1(s) = \frac{3}{s+4} \qquad \dot{y}(t) + 4y(t) = 3\,(r(t) - w(t))$$

$$G_2(s) = \frac{5}{9s-6} \qquad 9\dot{w} - 6w(t) = 5y(t)$$

Matlab code:

```
>> G1 = tf(3,[1 4]); G2 = tf(5,[9 -6]);
>> H = G1/(1+G1*G2)
H =
     27 s^2 + 90 s - 72
---------------------------
9 s^3 + 66 s^2 + 111 s - 36
>> pole(H).'
-4.0000   -3.6103    0.2770
>> zero(H).'
-4.0000    0.6667
```



Minimal Realization with `feedback`

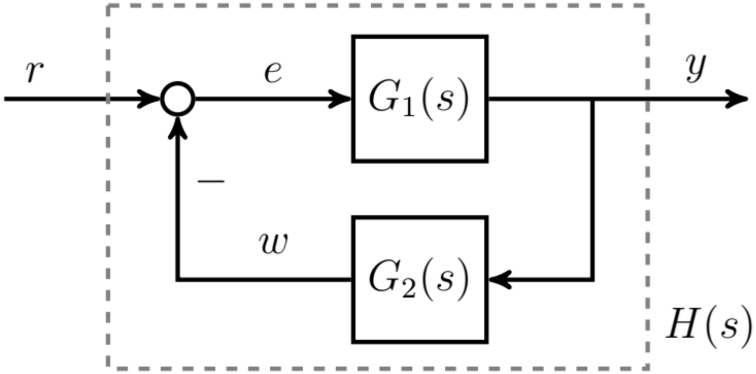
Consider the negative feedback interconnection of:

$$G_1(s) = \frac{3}{s+4} \qquad \dot{y}(t) + 4y(t) = 3\,(r(t) - w(t))$$

$$G_2(s) = \frac{5}{9s-6} \qquad 9\dot{w} - 6w(t) = 5y(t)$$

The `feedback` command computes a minimal realization:

```
>> G1 = tf(3,[1 4]); G2 = tf(5,[9 -6]);
>> H = feedback(G1,G2)
H =
   27 s - 18
----------------
9 s^2 + 30 s - 9
```



Negative feedback interconnections should not be computed using `G1/(1+G1*G2)`. Instead, use the syntax `H=feedback(G1,G2)`.