ECE 486: Control Systems

Lecture 5B: Block Diagrams

Key Takeaways

- This lecture describes a method to construct block diagrams for linear ODEs with constant coefficients.
- The diagrams are constructed from blocks for:
  - integration,
  - addition/subtraction, and
  - multiplication by a gain
- These diagrams will be used later for:
  - Numerical integration of ODEs using a tool call Simulink
  - Developing state-space models. These provide an alternative to the ODE/TF models that we are using as a starting point.

Integrator Block

The integrator is the basic building block for graphical representations of ODEs.

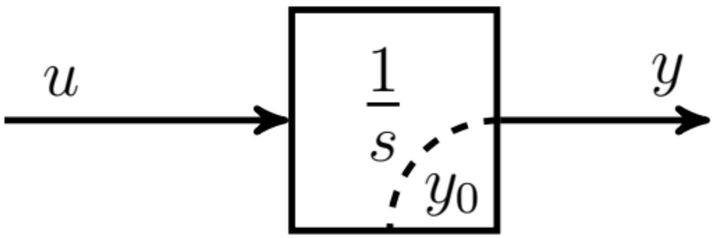
Consider the 1^{st} order system with input *u* and output *y:*

$$\dot{y}(t) = u(t)$$
$$y(0) = y_0$$

$$G(s) = \frac{1}{s}$$

Obtain *y* by integrating *u* from the given initial condition:

$$y(t) = y_0 + \int_0^t u(\alpha)\, d\alpha$$



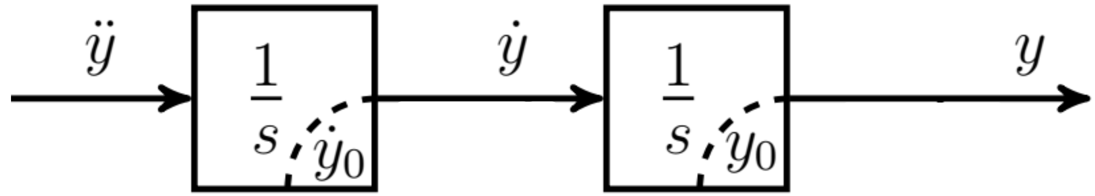
Block Diagram: No Derivatives of Input

Second-order system with no input derivatives:

$$a_2\ddot{y}(t) + a_1\dot{y}(t) + a_0 y(t) = b_0 u(t)$$

$$\text{ICs: } y(0) = y_0;\ \dot{y}(0) = \dot{y}_0$$

First draw two integrators to go from $\ddot{y}(t)$ to $y(t)$:



An n^{th}-order system would require n integrators

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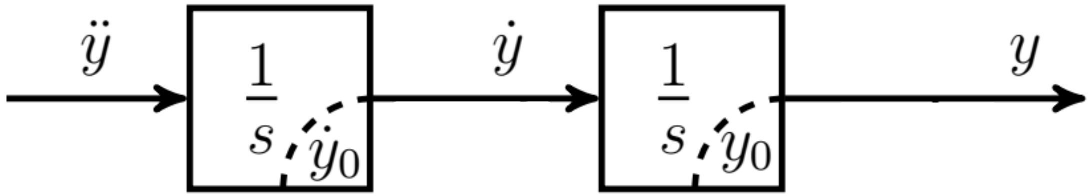
$$\text{ICs: } y(0) = y_0;\ \dot{y}(0) = \dot{y}_0$$

Next, solve for the highest derivative output term:

$$a_2\ddot{y}(t) = b_0 u(t) - a_1\dot{y}(t) - a_0 y(t)$$



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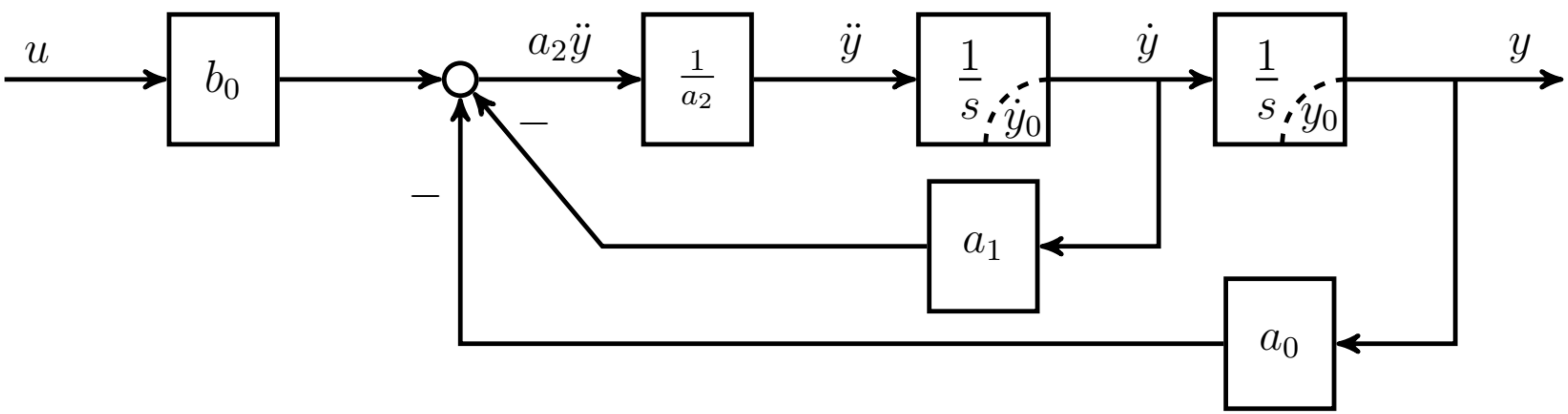

Block Diagram: With Input Derivatives

Second-order system with input derivatives:

$$a_2\ddot{y}(t) + a_1\dot{y}(t) + a_0 y(t) = b_2\ddot{u}(t) + b_1\dot{u}(t) + b_0 u(t)$$
$$\text{ICs: } y(0) = y_0;\ \dot{y}(0) = \dot{y}_0$$

We could modify our previous block diagram to add derivative blocks ($\frac{d}{dt}$) to compute $\dot{u}$ and $\ddot{u}$ from u.

Computational Issue: Differentiating tends to increase numerical errors.

Theoretical Issue: This requires the input signal *u* to be twice differentiable.

Block Diagram: With Input Derivatives

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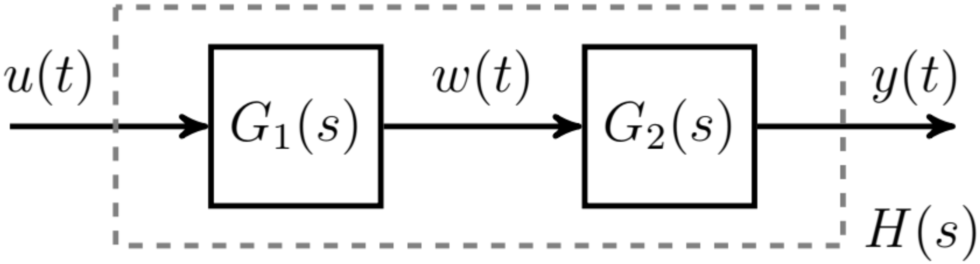
$$\text{ICs: } y(0) = y_0;\ \dot{y}(0) = \dot{y}_0$$

Instead, represent the system as a serial interconnection:

$$H(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0}$$



$$G_{u\to w}(s) = \frac{1}{a_2 s^2 + a_1 s + a_0} \quad \text{and} \quad G_{w\to y}(s) = \frac{b_2 s^2 + b_1 s + b_0}{1}$$

Block Diagram: With Input Derivatives

Second-order system with input derivatives:

$$a_2\ddot{y}(t) + a_1\dot{y}(t) + a_0 y(t) = b_2\ddot{u}(t) + b_1\dot{u}(t) + b_0 u(t)$$

$$\text{ICs: } y(0) = y_0;\ \dot{y}(0) = \dot{y}_0$$

Instead, represent the system as a serial interconnection:

$$\boxed{a_2\ddot{w}(t) + a_1\dot{w}(t) + a_0 w(t) = u(t)}$$ No input derivatives

$$y(t) = b_2\ddot{w}(t) + b_1\dot{w}(t) + b_0 w(t)$$

$$\boxed{G_{u\to w}(s) = \frac{1}{a_2 s^2 + a_1 s + a_0}} \text{ and } G_{w\to y}(s) = \frac{b_2 s^2 + b_1 s + b_0}{1}$$

Block Diagram: With Input Derivatives

Second-order system with input derivatives:

$$a_2\ddot{y}(t) + a_1\dot{y}(t) + a_0 y(t) = b_2\ddot{u}(t) + b_1\dot{u}(t) + b_0 u(t)$$

$$\text{ICs: } y(0) = y_0;\ \dot{y}(0) = \dot{y}_0$$