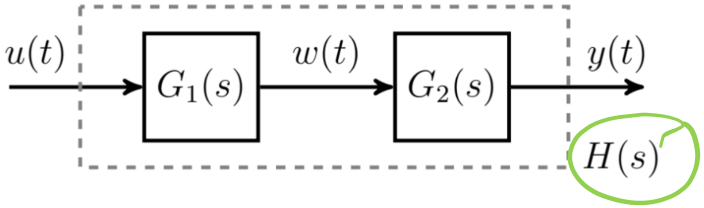
ECE 486: Control Systems

Lecture 5A: Interconnection of Systems

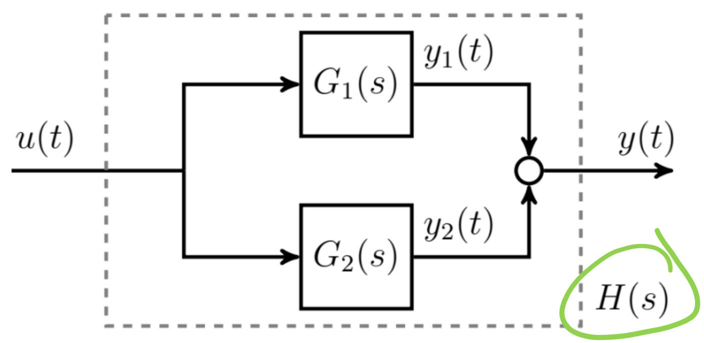
Problem 1

A) Suppose $G_1(s) = \frac{3}{s+2}$ and $G_2(s) = \frac{5}{s+7}$. What is the ODE for serial connection *H(s)=G₂(s) G₁(s)*?

B) Suppose $G_1(s) = \frac{5}{s+7}$ and $G_2(s) = \frac{3}{s+2}$. What is the ODE for serial connection $H(s)=G_2(s)\,G_1(s)$?



C) Suppose $G_1(s) = \frac{3}{s+2}$ and $G_2(s) = \frac{5}{s+7}$. What is the ODE for parallel connection $H(s)=G_1(s) + G_2(s)$?

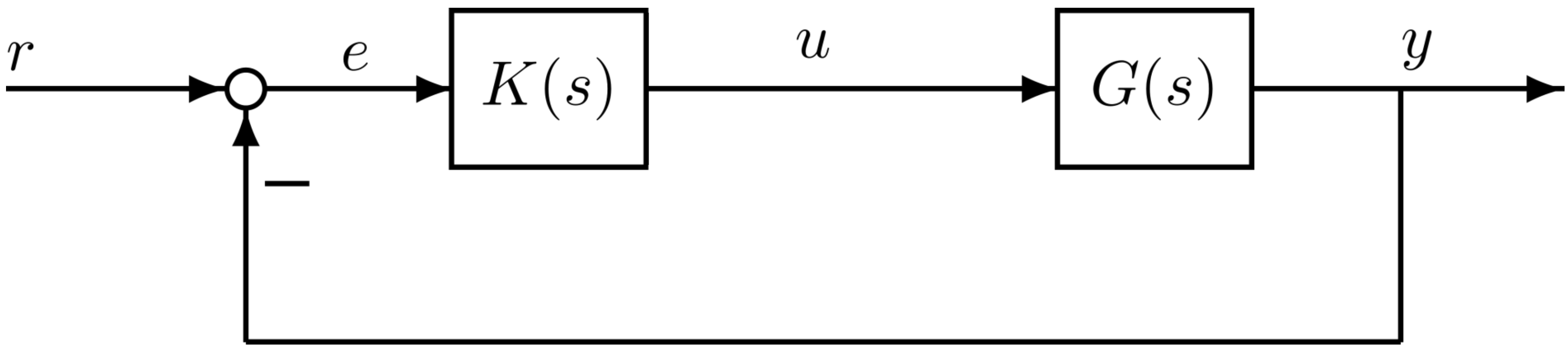


Problem 1

D) Consider the feedback system below with:

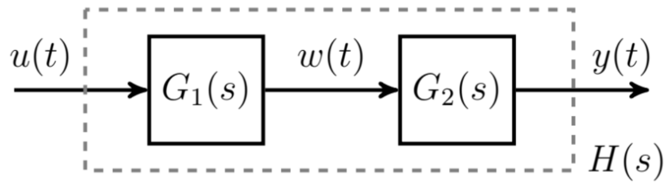
$$\dot{y}(t) + 5y(t) = 5u(t) \text{ and } u(t) = 2e(t) + 4\int_0^t e(\tau)\,d\tau$$

Obtain a model of the closed-loop from *r* to *y* with transfer functions, and compare your answers in Matlab using the function `feedback`.



Solution 1A

A) Suppose $G_1(s) = \frac{3}{s+2}$ and $G_2(s) = \frac{5}{s+7}$. What is the ODE for serial connection *H(s)=G₂(s) G₁(s)*?



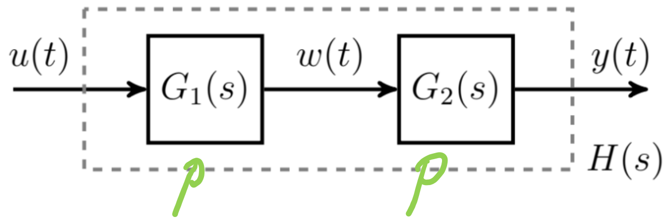
$$H = G_2 G_1 = \left(\frac{3}{s+2}\right)\left(\frac{5}{s+7}\right) = \frac{15}{(s+2)(s+7)}$$

$$= \frac{15}{s^2+9s+14}$$

$$\ddot{y} + 9\dot{y} + 14y = 15u$$

Solution 1B

B) Suppose $G_1(s) = \frac{5}{s+7}$ and $G_2(s) = \frac{3}{s+2}$. What is the ODE for serial connection *H(s)=G₂(s) G₁(s)*?

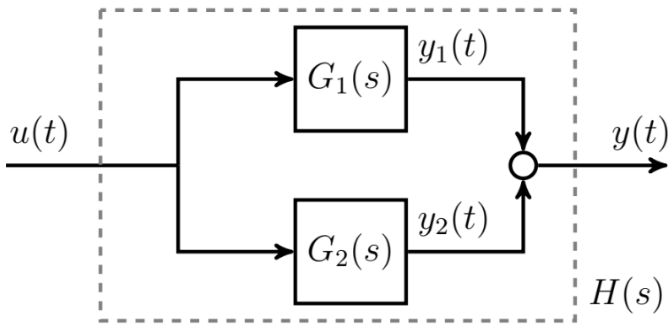


Same $\qquad G_2 G_1 = G_1 G_2$

[1 input and 1 output]

Solution 1C

C) Suppose $G_1(s) = \frac{3}{s+2}$ and $G_2(s) = \frac{5}{s+7}$. What is the ODE for parallel connection *H(s)=G₁(s) + G₂(s)*?



$$H = G_1 + G_2 = \frac{3}{s+2} + \frac{5}{s+7}$$

$$= \frac{3(s+7) + 5(s+2)}{(s+2)(s+7)} = \frac{8s + \overset{31}{\cancel{24}}}{s^2 + 9s + 14}$$

$$\ddot{y} + 9\dot{y} + 14y = 8\dot{u} + 24u$$

Solution 1D

Consider the feedback system below with:

$$\dot{y}(t) + 5y(t) = 5u(t) \text{ and } u(t) = 2e(t) + 4\int_0^t e(\tau)\,d\tau$$

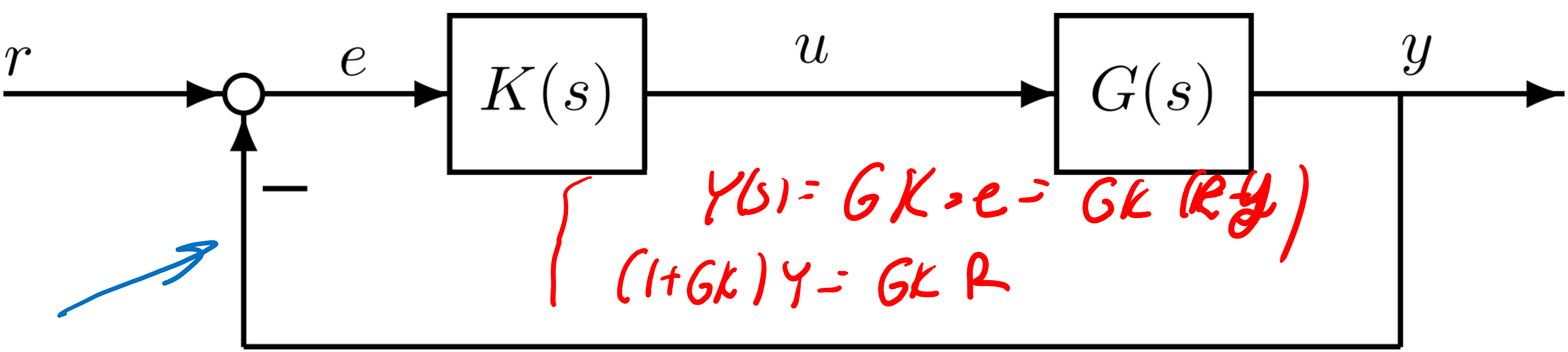
B) Obtain a model of the closed-loop from *r* to *y* with transfer functions.

$$T_{r\to y}(s) = \frac{G(s)K(s)}{1+G(s)K(s)} = \frac{\left(\frac{5}{s+5}\right)\left(\frac{2s+4}{s}\right)}{1+\left(\frac{5}{s+5}\right)\left(\frac{2s+4}{s}\right)} = \frac{\left(\frac{10s+20}{s^2+5s}\right)}{\left(\frac{s^2+15s+20}{s^2+5s}\right)}$$

$$= \frac{10s+20}{s^2+15s+20} \cdot \boxed{\frac{s^2+5s}{s^2+5s}} = \frac{10s+20}{s^2+15s+20}$$

$s = 0, -5$

$\frac{2s+4}{s}$ $\quad$ $\frac{5}{s+5}$



$Y(s) = GK \cdot e = GK(R-Y)$

$(1+GK)Y = GK\,R$

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Lecture 5B: Block Diagrams

Problem 2

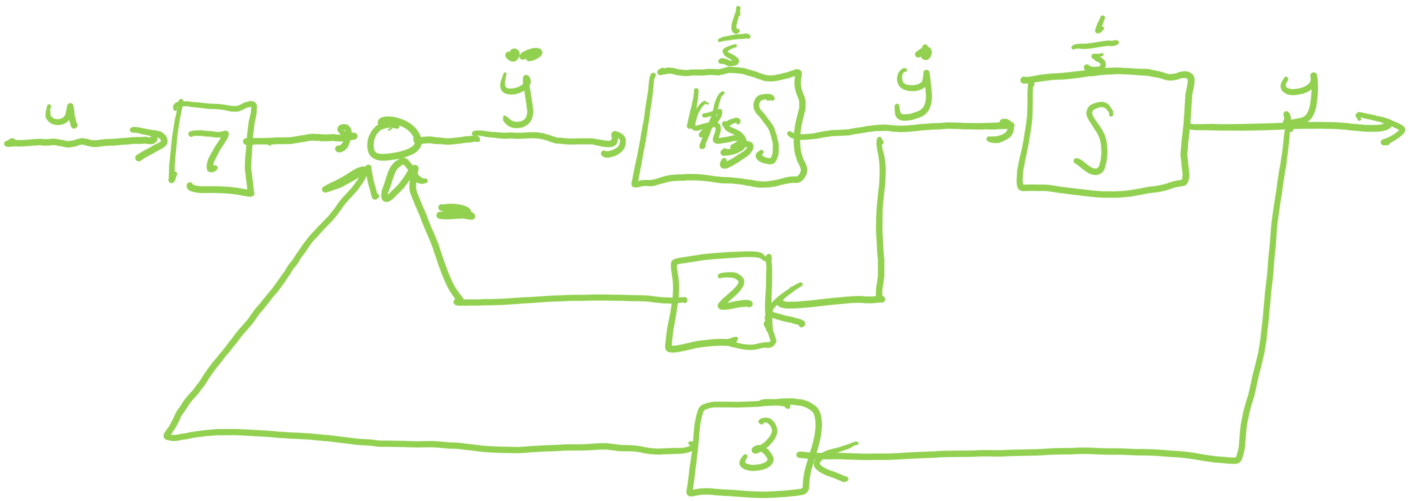
A) Draw a block diagram for $G_1(s) = \frac{7}{s^2+2s-3}$ using integrator, summation, and gain blocks.

B) Draw a block diagram for $G_1(s) = \frac{5s+6}{s^2+2s-3}$ using integrator, summation, and gain blocks.

Solution 2A

A) Draw a block diagram for $G_1(s) = \frac{7}{s^2+2s-3}$ using integrator, summation, and gain blocks.

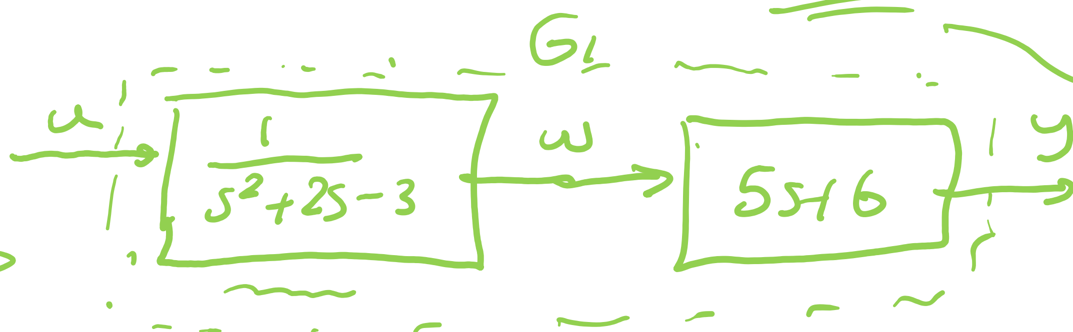
$\ddot{y} + 2\dot{y} - 3y = 7u \rightarrow \ddot{y} = 7u - 2\dot{y} + 3y$



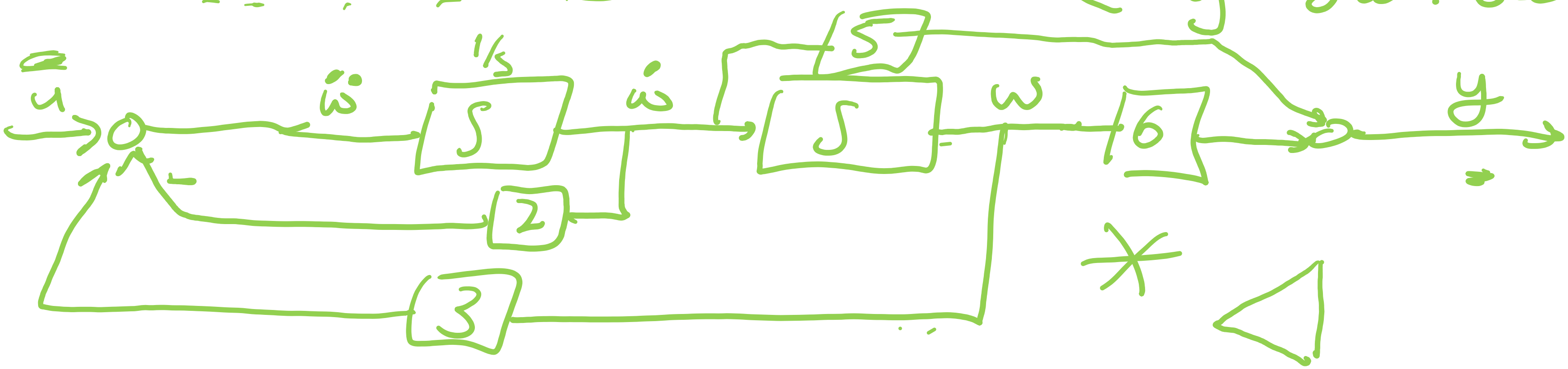
Solution 2B

B) Draw a block diagram for $G_1(s) = \frac{5s+6}{s^2+2s-3}$ using integrator, summation, and gain blocks.

$$\Rightarrow \ddot{y} + 2\dot{y} - 3y = 5\dot{u} + 6$$



$$\begin{cases} \ddot{w} + 2\dot{w} - 3w = u \\ \ddot{w} = u - 2\dot{w} + 3w \\ y = 5\dot{w} + 6w \end{cases}$$



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Lecture 5C: State-Space Models

Solution 3

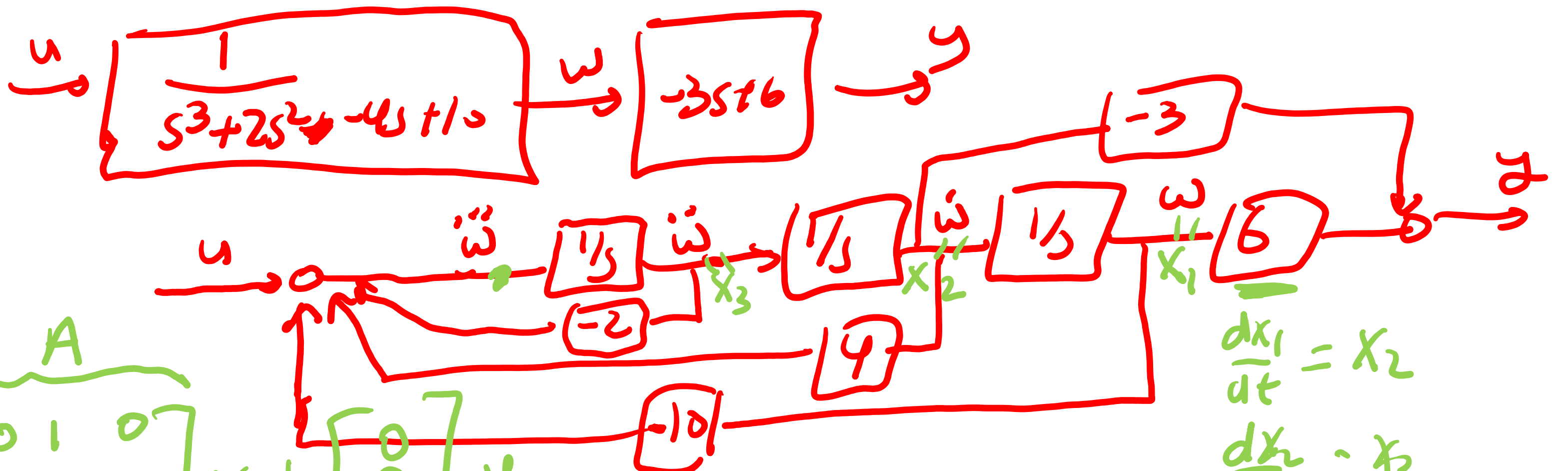
Find a state-space representation for: (S60)

$$y^{[3]}(t) + 2\ddot{y}(t) - 4\dot{y}(t) + 10y(t) = -3\dot{u}(t) + 6u(t)$$

$$\dddot{w} + 2\ddot{w} - 4\dot{w} + 10w = u$$

$$y = -3\dot{w} + 6w$$

$$G(s) = \frac{-3s+6}{s^3+2s^2-4s+10}$$



$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\frac{d}{dt}x = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & 4 & -2 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{B} u$$

$$y = \underbrace{\begin{bmatrix} 6 & -3 & 0 \end{bmatrix}}_{C} x$$

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = x_3$$

$$\frac{dx_3}{dt} = u - 2x_3 + 4x_2 - 10x_1$$