ECE 486: Control Systems

Lecture 8A: Proportional-Integral (PI) Control

Key Takeaways

This lecture describes proportional-integral control. The controller sets the plant input with two terms: (i) proportional to the error and (ii) proportional to the integral of the error.

Key properties of PI control:

1. Integral control is that it achieves zero error in steady state (assuming system is stable and reaches a steady-state).
2. Initial transient is dominated by the proportional term while the steady state is dominated by the integral term.

The two terms in a PI controller can be used to provide better trade-offs in the control design.

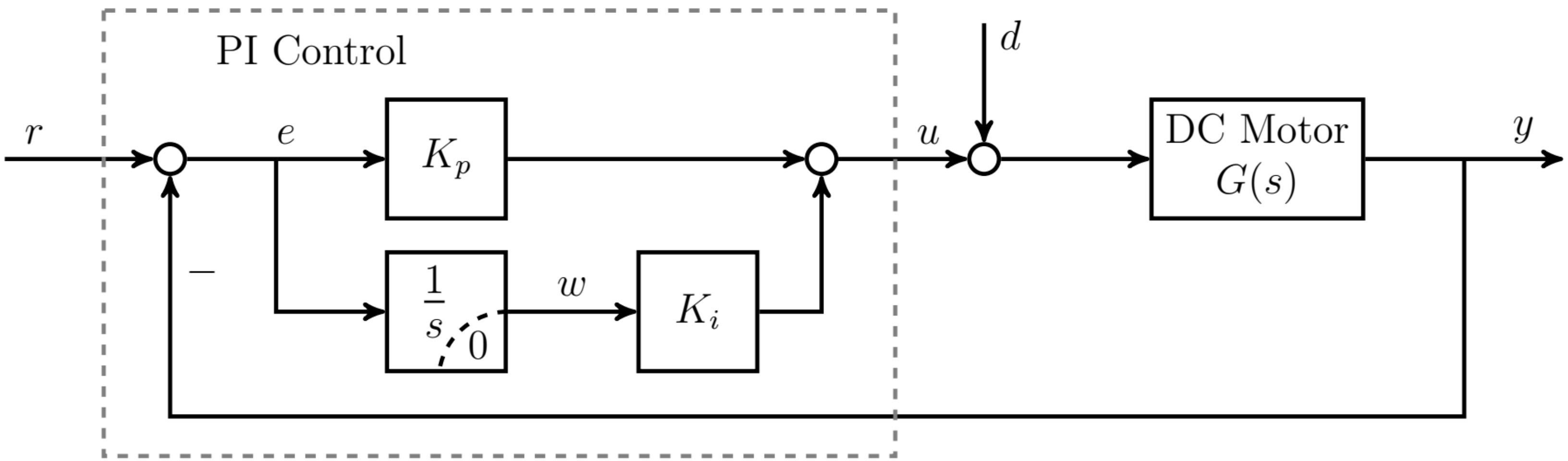
Proportional-Integral (PI) Control

Closed-loop, proportional-integral control for DC motor:

1. User specifies the desired motor speed, *r(t)*
2. Controller computes the tracking error *e(t) = r(t)-y(t)*
3. Controller sets input voltage to:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau$$

where K_p and K_i are gains to be selected.



Effect of P and I Terms

P Control: $u(t) = K_p e(t)$

K_p affects settling time, steady-state error, control input

PI Control: $u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau$

Use two gains to independently modify the transient and steady-state characteristics:

P-Term: Reacts to present (current error) and dominates during initial transient.

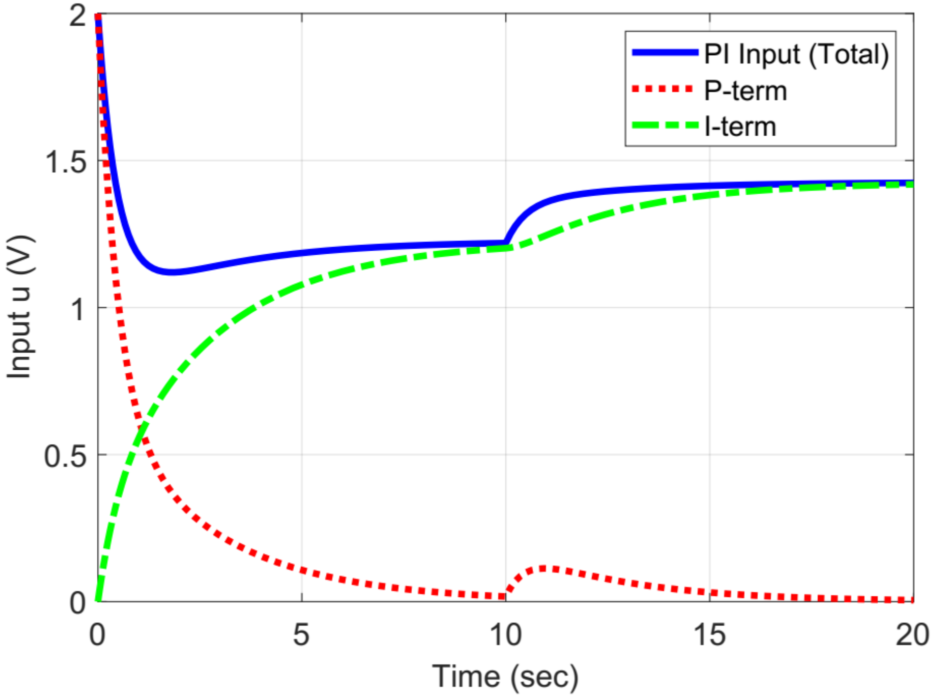
I-Term: Reacts to past (integral of error) and dominates during steady-state.

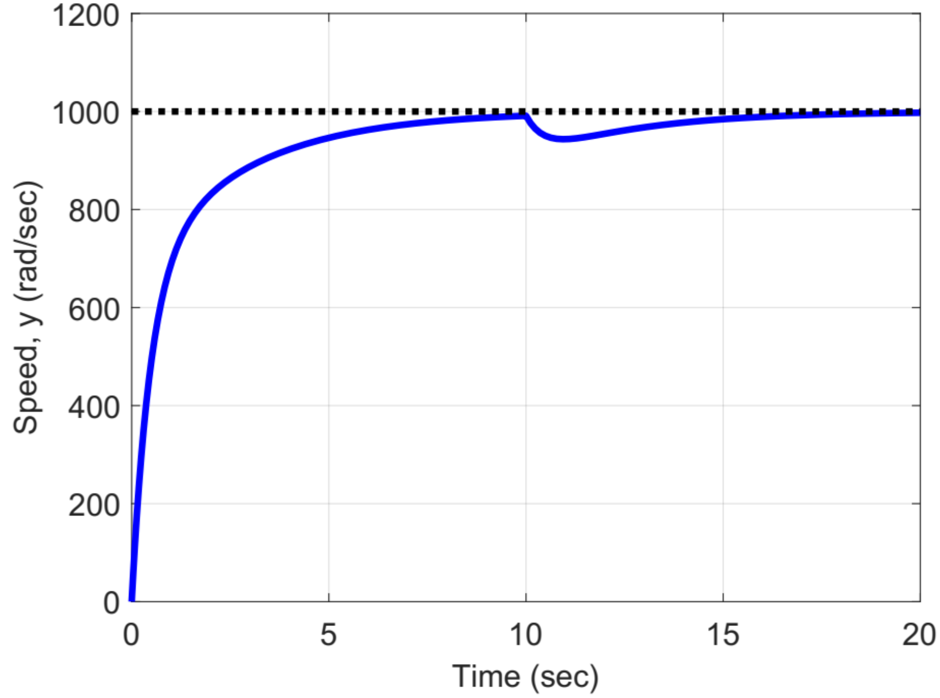
Effect of P and I Terms

PI Control: $u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau$

Simulation with $K_p = 0.002$, $K_i = 0.001$, and

- $r(t) = 1000\frac{rad}{sec}$
- $d(t) = -0.2V$ for $t \geq 10\text{sec}$





Model for Closed-Loop Control

Recall the first-order model for the motor:

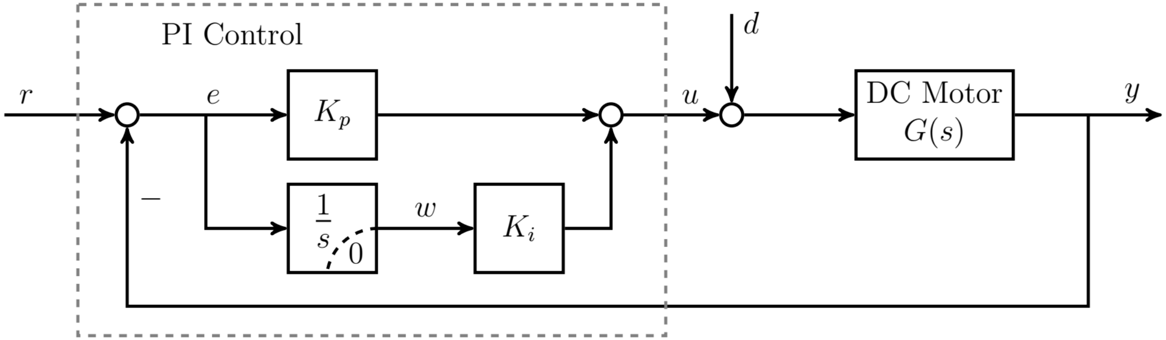
$$\dot{y}(t) + a_0\, y(t) = b_0\, u(t) + b_0\, d(t)$$
$$\Rightarrow \ddot{y}(t) + a_0\, \dot{y}(t) = b_0\, \dot{u}(t) + b_0\, \dot{d}(t)$$

PI Control

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau \quad \Rightarrow \quad \dot{u}(t) = K_p \dot{e}(t) + K_i e(t)$$

Substitute $\dot{u}$ in motor model and combine *y* terms:

$$\ddot{y}(t) + (a_0 + b_0 K_p)\, \dot{y}(t) + (b_0 K_i)\, y(t) = (b_0 K_p)\, \dot{r}(t) + (b_0 K_i)\, r(t) + b_0 \dot{d}(t)$$



Closed-Loop Response

The dynamics of the closed-loop system are:

$$\ddot{y}(t) + (a_0 + b_0 K_p)\, \dot{y}(t) + (b_0 K_i)\, y(t) = (b_0 K_p)\, \dot{r}(t) + (b_0 K_i)\, r(t) + b_0 \dot{d}(t)$$

$$T_{r \to y}(s) = \frac{(b_0 K_p)s + (b_0 K_i)}{s^2 + (a_0 + b_0 K_p)s + (b_0 K_i)}$$

- Closed-loop is stable if and only if $a_0+b_0K_p>0$, $b_0K_i>0$.
- We can place the two closed-loop poles anywhere by proper choice of (K_p, K_i). [Always true if plant is 1st order.]
- If the system reaches a steady-state then:

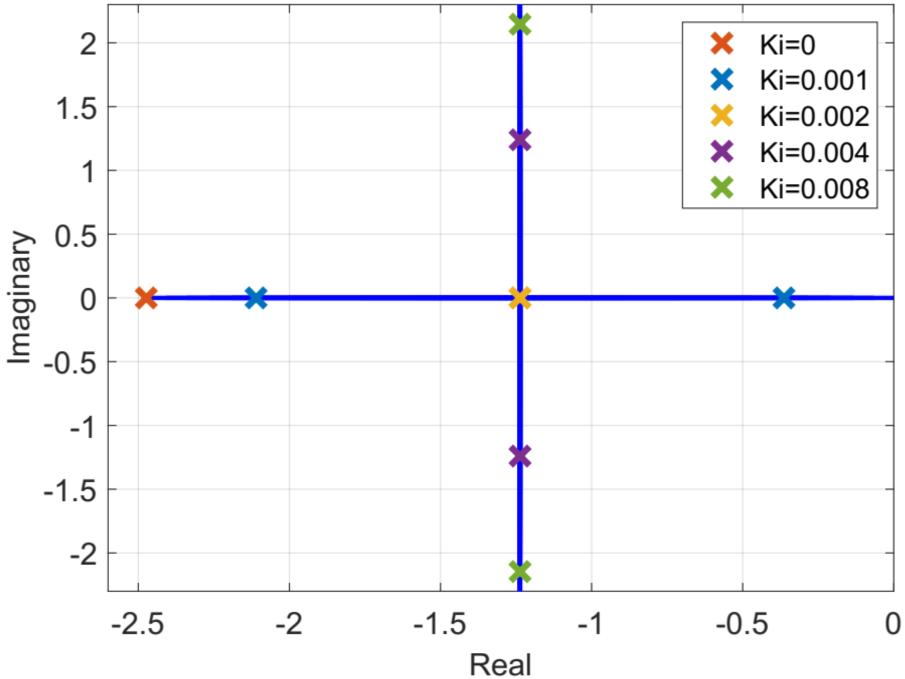
$$(b_0 K_i)\, \bar{y} = (b_0 K_i)\, \bar{r} \quad \Rightarrow \quad \bar{y} = \bar{r}$$

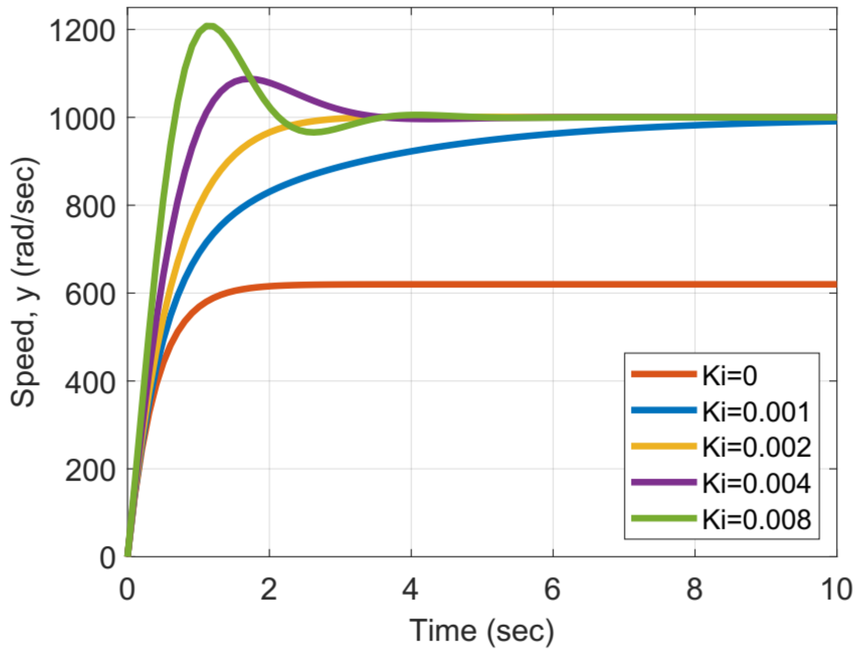
Key property of integral control: If the system converges to a steady state then there is no error.

Effect of Integral Gain

- Small K_i
  - Closed-loop is overdamped with two real poles.
  - Increasing K_i (with K_p fixed) causes the dominant (slow) pole to move into the LHP. This speeds up the response.

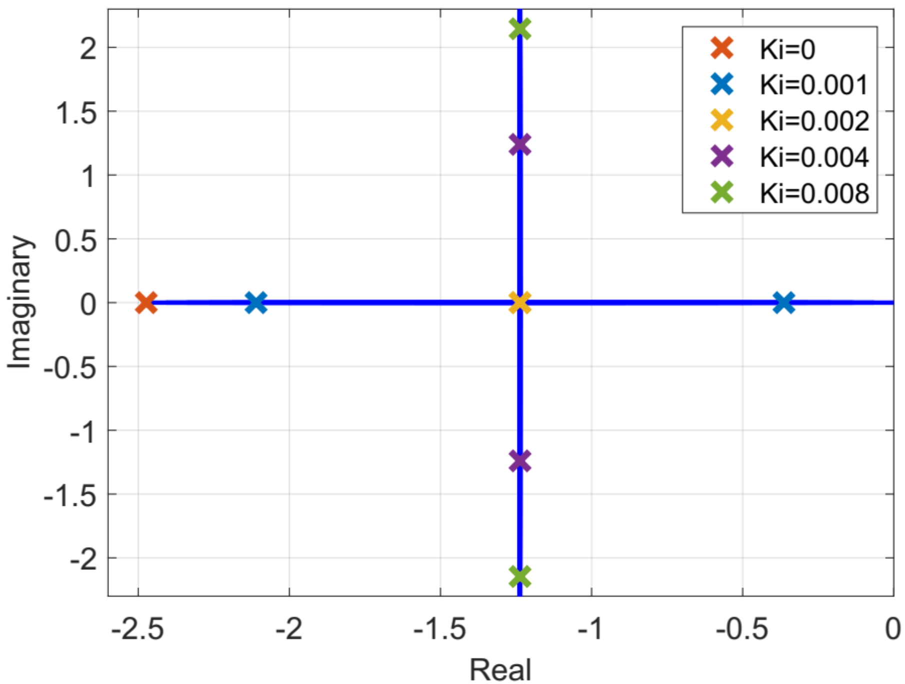
Root locus with K_p = 0.002, varying K_i

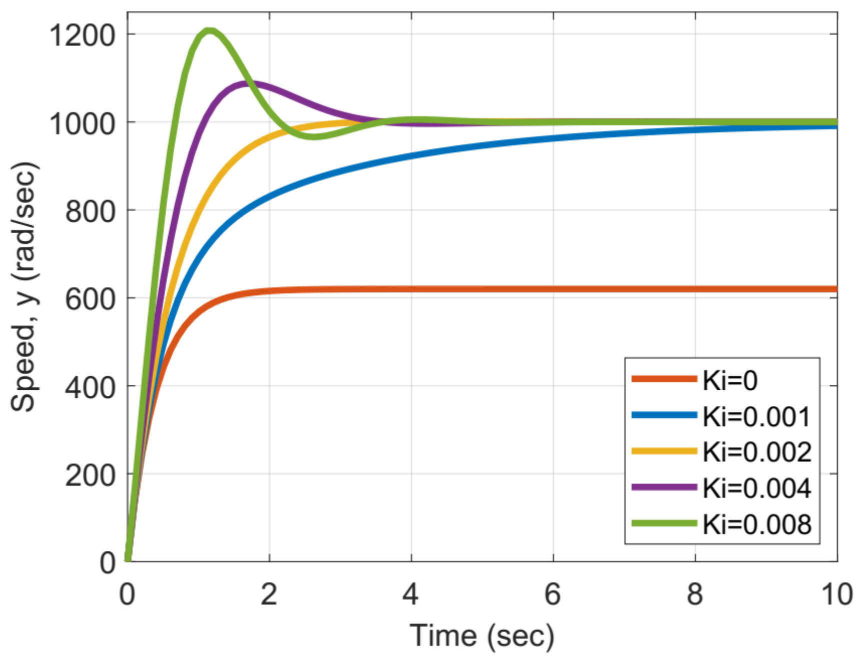




Effect of Integral Gain

- Small K_i
- Large K_i
  - Closed-loop is underdamped with two complex poles.
  - Increasing K_i (with K_p fixed) causes the poles to become more underdamped. This increases overshoot and oscillations.

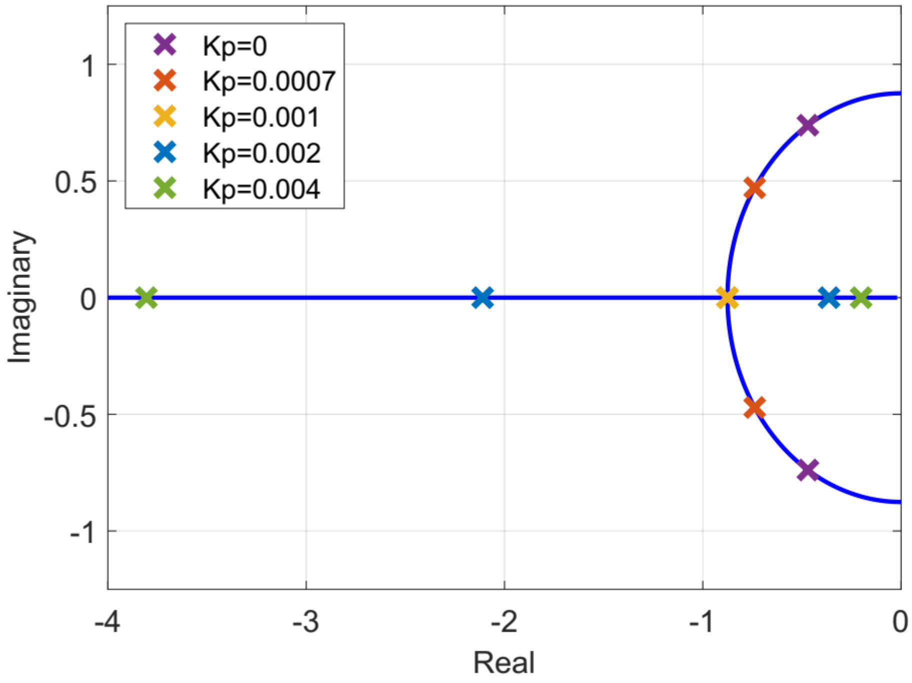


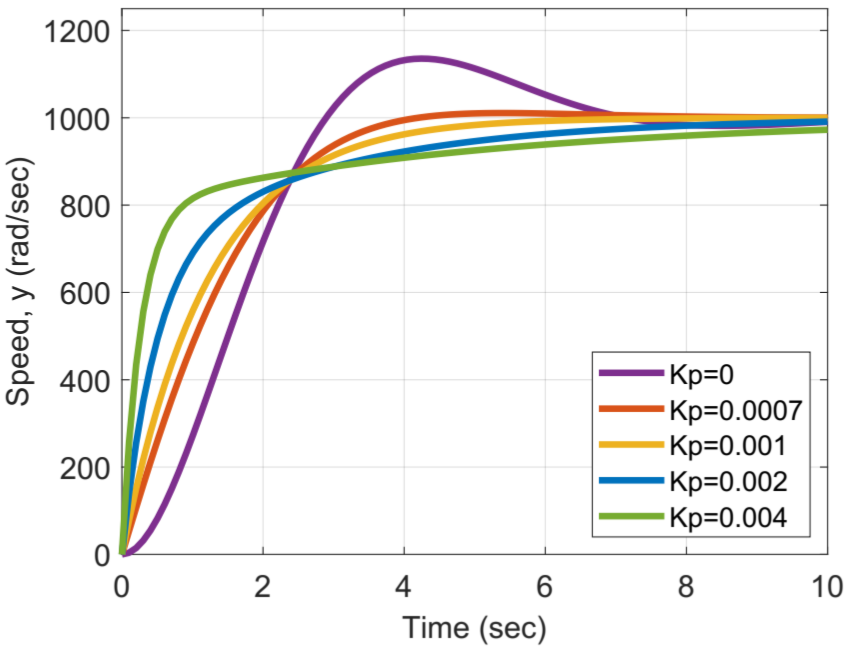


Effect of Proportional Gain

- Small K_p
  - Closed-loop is underdamped with two complex poles.
  - Increasing K_p (with K_i fixed) will increase the damping ratio. This reduces overshoot and oscillations.

Root locus with K_i = 0.001, varying K_p





Effect of Proportional Gain

- Small K_p
- Large K_p
  - Closed-loop is overdamped with two real poles.
  - Increasing K_p (with K_i fixed) causes causes the dominant (slow) pole to move to the imaginary axis. This slows down the response.