ECE 486: Control Systems

Lecture 8A: Proportional-Integral (PI) Control

Key Takeaways

This lecture describes proportional-integral control. The controller sets the plant input with two terms: (i) proportional to the error and (ii) proportional to the integral of the error.

Key properties of PI control:

1. Integral control is that it achieves zero error in steady state (assuming system is stable and reaches a steady-state).
2. Initial transient is dominated by the proportional term while the steady state is dominated by the integral term.

The two terms in a PI controller can be used to provide better trade-offs in the control design.

Problem 1

Consider the following plant:

$$2\dot{y}(t) + 6y(t) = 8u(t)$$

with a PI controller in the following form:

$$u(t) = K_p\, e(t) + K_i \int_0^t e(\tau)\, d\tau$$

A) What is the ODE model for the closed-loop from *r* to *y*?

B) Choose (K_p,K_i) so that the closed-loop: i) is stable and ii) has ζ =0.5, iii) settling time $T_s \leq 0.5sec$, and (iv) zero steady-state error due to a unit step reference.

C) How will the response change if K_i is increased further? Would you recommend increasing K_i based on your analysis?

Solution 1A

$$2\dot{y}(t) + 6y(t) = 8u(t)$$
$$u(t) = K_p\, e(t) + K_i \int_0^t e(\tau)\, d\tau$$

A) What is the ODE model for the closed-loop from *r* to *y*?

Solution 1B

$$2\dot{y}(t) + 6y(t) = 8u(t)$$
$$u(t) = K_p\, e(t) + K_i \int_0^t e(\tau)\, d\tau$$

B) Choose (K_p,K_i) so that the closed-loop: i) is stable and ii) has ζ =0.5, iii) settling time $T_s \leq 3sec$, and (iv) zero steady-state error due to a unit step reference.

Solution 1C

$$2\dot{y}(t) + 6y(t) = 8u(t)$$
$$u(t) = K_p\, e(t) + K_i \int_0^t e(\tau)\, d\tau$$

C) How will the response change if K_i is increased further? Would you recommend increasing K_i based on your analysis?

Solution 1-Extra Space

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Lecture 8B: Proportional-Derivative (PD) Control

Key Takeaways

This lecture describes proportional-derivative control. The controller sets the plant input with two terms: (i) proportional to the error and (ii) proportional to the derivative of the error.

Key properties of PD control:

1. Some plants cannot be stabilized by P or PI control. This motivates the use of PD.
2. A basic implementation of PD control will amplify noise.
3. Common implementations use a “smoothed” derivative or a direct measurement the derivative of the output.

Problem 2

Consider the following plant:

$$\ddot{y}(t) - 2\dot{y}(t) + y(t) = u(t)$$

with a PD controller in the following form:

$$u(t) = K_p\left(r(t) - y(t)\right) - K_d\dot{y}(t)$$

A) What is the ODE model for the closed-loop from *r* to *y*?

B) Choose (K_p,K_d) so that the closed-loop: i) is stable and ii) has (ω_n ,ζ)= (2 rad/sec,0.5).

C) What is the steady-state error if *r* is a unit step reference?

D) Would you increase or decrease Kp to reduce the steady-state error? What other impacts does this change have on the response?

Solution 2A

$$\ddot{y}(t) - 2\dot{y}(t) + y(t) = u(t)$$

$$u(t) = K_p\left(r(t) - y(t)\right) - K_d\dot{y}(t)$$

A) What is the ODE model for the closed-loop from *r* to *y*?

Solution 2B

$$\ddot{y}(t) - 2\dot{y}(t) + y(t) = u(t)$$

$$u(t) = K_p\left(r(t) - y(t)\right) - K_d\dot{y}(t)$$

B) Choose (K_p,K_d) so that the closed-loop: i) is stable and ii) has (ω_n ,ζ)= (2 rad/sec,0.5).

Solution 2C

$$\ddot{y}(t) - 2\dot{y}(t) + y(t) = u(t)$$

$$u(t) = K_p\left(r(t) - y(t)\right) - K_d\dot{y}(t)$$

C) What is the steady-state error if *r* is a unit step reference?

Solution 2D

$$\ddot{y}(t) - 2\dot{y}(t) + y(t) = u(t)$$

$$u(t) = K_p\left(r(t) - y(t)\right) - K_d\dot{y}(t)$$

D) Would you increase or decrease Kp to reduce the steady-state error? What other impacts does this change have on the response?

Solution 2-Extra Space