ECE 486: Control Systems

Lecture 9B: PID Tuning for Second-Order Systems

Key Takeaways

This lecture describes a method to tune PID controllers using pole placement.

For second-order systems, the approach is to:

- Use PID control and
- Select the gains to place the three closed-loop poles at desired locations.
- A PI controller (without the D-term) should be used if the plant has sufficient damping.

The choice of natural frequency (time constant) is critical.

Design Approach: Pole Placement

1. Approximate the plant dynamics by a first or second-order ODE using the dominant pole approximation.
2. If the dynamics are first-order: Use a PI controller to place the two poles at a desired location.
2. If dynamics are second-order:
   - Use a PID controller to place the three poles.
   - Avoid use of derivative control if plant is well-damped. This will restrict the choice of the three poles.

A reasonable starting point is to place all poles at $s = -\omega_n$.

The choice of natural frequency (time constant) is critical.

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2. If dynamics are second-order:
   - Use a PID controller to place the three poles.
   - Avoid use of derivative control if plant is well-damped. This will restrict the choice of the three poles.
3. Further tuning is often required. Use root locus to tune one gain at a time.
4. Implementation:
   - D-control: Use smoothed derivative or rate feedback
   - I-control: Use anti-windup (to be discussed later)

PID Tuning For Second-Order Systems

Example plant model:

$$\ddot{y}(t) + a_1 \dot{y}(t) + a_0\, y(t) = b_0\, u(t) + b_0\, d(t)$$

where $a_1 = -2$, $a_0 = 17$ and $b_0 = 17$

Formal design requirements can be stated. Roughly a faster closed-loop response will:

- lead to better reference tracking and disturbance rejection,
- but it will also increase the actuator effort and degrade the noise rejection.

Important: Second-order ODE is typically an approximate model.

Formal tools to assess the impact of model uncertainty later.

If the closed-loop is too fast then the unmodeled dynamics will degrade performance and may even cause instability.

Closed-Loop Model

Dynamics of the plant:

$$\ddot{y}(t) + a_1 \dot{y}(t) + a_0\, y(t) = b_0\, u(t) + b_0\, d(t)$$

where $a_1 = -2$, $a_0 = 17$ and $b_0 = 17$

PID controller in rate feedback form:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau - K_d \dot{y}(t)$$

Sub for *u* into plant dynamics and collect terms.

Closed-loop dynamics are:

$$y^{[3]}(t) + (a_1 + b_0 K_d)\ddot{y}(t) + (a_0 + b_0 K_p)\dot{y}(t) + (b_0 K_i) y(t)$$
$$= b_0 K_p\, \dot{r}(t) + b_0 K_i\, r(t) + b_0 \dot{d}(t)$$

The closed-loop characteristic equation is:

$$0 = s^3 + (a_1 + b_0 K_d) s^2 + (a_0 + b_0 K_p) s + (b_0 K_i)$$

PID Tuning

Closed-loop characteristic equation:

$$0 = s^3 + (a_1 + b_0 K_d)s^2 + (a_0 + b_0 K_p)s + (b_0 K_i)$$

Pole Placement:

- Select the desired poles to satisfy for some (ζ, ω_n, p):

$$0 = (s^2 + 2\zeta\omega_n s + \omega_n^2) \cdot (s + p)$$

  Choose $\zeta = 1$ and $p = \omega_n$ as a starting point.

- The desired characteristic equation is:

$$0 = s^3 + (p + 2\zeta\omega_n)s^2 + (2\zeta\omega_n p + \omega_n^2)s + \omega_n^2 p$$

- Match coefficients to the closed-loop characteristic equation:

$$\begin{aligned} a_1 + b_0 K_d &= p + 2\zeta\omega_n \\ a_0 + b_0 K_p &= 2\zeta\omega_n p + \omega_n^2 \\ b_0 K_i &= \omega_n^2 p \end{aligned}$$

Solve these equations for the three gains.

Comparison of Two PID Controllers

K_1 is designed for faster response than K_2.

Design	ζ	$\omega_n, \frac{rad}{sec}$	p	Poles, $s_{1,2}$ and s_3	M_p	τ_{settle}, sec	K_p	K_i	K_d
$K_1(s)$	1	6	6	-6, -6, -6	0	0.5	5.35	12.7	1.18
$K_2(s)$	1	3.5	3.5	-3.5, -3.5, -3.5	0	0.86	1.16	2.52	0.735

Comparison of Two PI Controllers

K_1 is designed for faster response than *K_2*.

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Step responses with $r(t) = 4$, $d(t) = 2$ for $t \geq 1.5$, and sensor noise for $t \geq 4$.