Please read the following information carefully and start preparing for the exam.

Time. The final exam will be held on *Tuesday, Dec 12, at 9:00am*, in our regular classroom. Please note that this start time is *1 hour later* than what's indicated in the university's official final exam schedule. There will be no conflict exam offered at any other time. This is designed to be a 2-hour exam, so you should be able to finish around 11:00am. Students requiring special accommodations should contact me at this time to discuss specific arrangements.

Topics covered:

- All topics listed in midterm 1 and midterm 2 information sheets (available on the class website)
- Nyquist plots and Nyquist stability criterion (proof via argument principle not tested on the exam)
- State-space models and their transfer functions, coordinate transformations, canonical forms
- Controllability, pole placement by full-state feedback
- Observability, observer design
- Combining full-state feedback and observer: dynamic output feedback

The emphasis will be on the material after midterm 2, as well as on connections between these more recent topics and earlier topics.

What to bring. The exam is closed-book, closed-notes. You may bring three (double-sided) sheets of notes with any necessary formulas. (Hint: bring the two sheets from the two midterms plus one new one.) A calculator will not be necessary or helpful.

Tips for preparing. As usual, make sure to follow up on all lecture material, readings, and homework problems and solutions, but keep in mind that the exam questions will be more conceptual. You may also find useful the slides by Prof. Max Raginsky and the state-space notes by Prof. Ali Belabbas posted on the class website. The last homework to be posted on Nov 30 will consist partially of exam material. (You will not need to submit this homework, and its solutions will be posted on the class website.) On the next pages are two final exams from past semesters. Solutions to the first practice exam will be discussed in a pre-recorded lecture which will be posted on the Mediaspace channel next week; solutions to the second practice exam will be posted on the class website.

Office hours. I will hold extra office hours before the exam, details to be announced. Please also take advantage of homework TAs' office hours to clear up any homework-related questions.

YOUR NAME: ______________________________

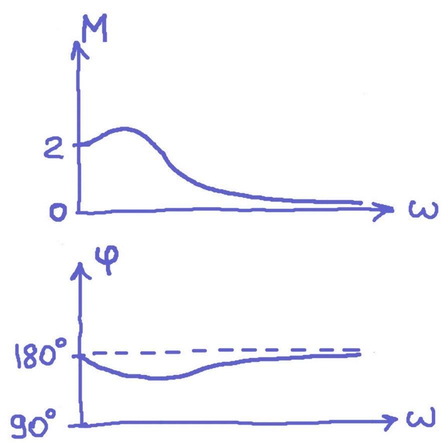
Instructions: Please write your answers clearly and concisely, and try to fit them in the space provided (use the back of the page if necessary). There are scratch pages at the end, you don't need to submit them.

1. Consider the plant with transfer function

$$G(s) = \frac{s^3 - 2s + 1}{s^4 + 2s^3 + s^2 + 4s + 1}$$

What is the steady-state response of this system to the unit step input?

2. Suppose that the Bode plots for a given plant's transfer function were obtained experimentally and look as shown in the figure.



a) Suppose we know that the plant is open-loop stable (i.e., all open-loop poles are in the left-half plane). Determine the range of $K > 0$ for which the standard negative unity feedback with constant gain K stabilizes the plant.

b) How does your answer change if the plant has one right-half-plane open-loop pole?

3. Consider the system

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$

where $x \in \mathbb{R}^2$ is the state and $u \in \mathbb{R}^1$ is the control input.

a) Characterize all possible closed-loop poles (eigenvalues) that can be obtained by applying state feedback. In other words, specify which pairs of points in the complex plane can be assigned as eigenvalues of the closed-loop system.

b) Suppose now that you are allowed to change one element of the matrix A. Changing which element would have the most significant effect (with respect to the same question as in part a))? Explain.

4. Consider the plant with transfer function

$$G(s) = \frac{s-1}{s-2}$$

in standard feedback configuration with a controller of the form

$$K\frac{(s-z_1)\cdots(s-z_m)}{(s-p_1)\cdots(s-p_n)}$$

where z_i are controller zeros, p_i are controller poles, and $K > 0$ is a gain. Show that if the controller is stable (has all its poles p_i in the left-half plane), then the closed-loop system can never be stable. On the other hand, show that with an unstable controller (having at least some right-half plane poles) closed-loop stability is possible.

5. Consider the system

$$\dot{x} = Ax + f(y, u)$$
$$y = Cx$$

where x is the state, u is the input, y is the measured output, A and C are matrices, and f is a known nonlinear function. Design an observer for this system. (By an observer we mean a system with inputs u and y whose state $\hat{x}$ converges to x.) As in the linear case, for this observer to work, an appropriate assumption on the matrices A and C is needed; state this assumption and explain why it is needed. Be sure to justify that your observer works.

PRACTICE FINAL EXAM #2 (solutions will be posted)

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1. Let the plant be given by the equations

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -2x_1 + u$$
$$y = x_1 + x_2$$

a) Draw a block diagram of this system. The blocks you can use are integrators, gains, and summing junctions. Make sure to label all signals.

b) Derive the plant transfer function.

c) Let the controller be given by the transfer function

$$\frac{5}{s+3}$$

Write down two different state-space realizations of this controller, both of them stable.

d) Consider now the closed-loop system obtained by interconnecting the plant from part a) and the controller given by one of your realizations (choose any one) from part c). By "interconnecting" we mean that the plant's output is the controller's input, the controller's output is minus the plant's input (negative feedback configuration), and the state of the overall closed-loop system consists of the plant's state and the controller's state. Write down the differential equations describing this closed-loop system.

e) Is it true that the state of the closed-loop system decays to 0 from arbitrary initial condition as time goes to ∞?

2. Suppose that a plant with transfer function $G(s)$ is put in standard negative feedback configuration with gain $K = 1$, and suppose we know that this results in a stable closed-loop system (in other words, we know that the gain $K = 1$ is stabilizing). Further, suppose we know that the Nyquist plot of $G(s)$ *never enters inside* the disk in the complex plane with radius 1 centered at the point -1.

a) Show that the gain margin is $+\infty$, in the sense that any gain $K > 1$ is also stabilizing.

b) Show that the phase margin is at least $60°$.

3. Consider the system

$$\dot{x} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$

a) Is it possible to place closed-loop eigenvalues at arbitrary locations by full-state feedback? If yes, explain how. If not, explain what closed-loop eigenvalues are achievable and why others are not.

b) *For extra credit:* Let the initial condition be $x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, and for each $t \geq 0$ let R_t be the set of all states $x(t)$ that can be reached from this initial condition after time t by applying arbitrary controls. Describe what these sets R_t look like. Your answer may involve a picture, but it must be supported by mathematical explanations. (Hint: you can use the fact that for a *controllable* linear system, any initial state can be steered to any other state in arbitrary time by applying some control.)