

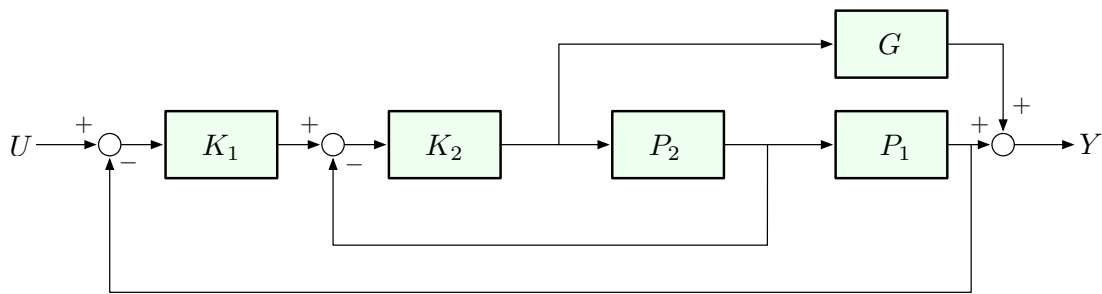
Reading Assignment:

FPE, Sections 3.3-3.6.

Problems:

(unless otherwise noted, you can use a calculator/computer to arrive at numerical answers)

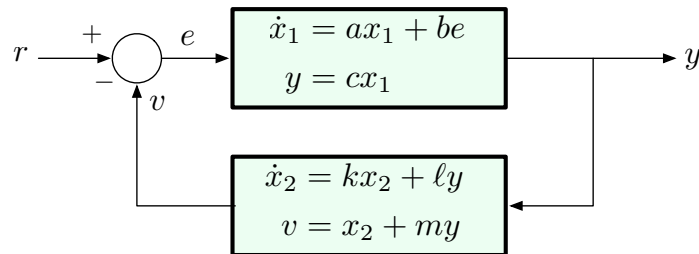
1. Consider the system given by the block diagram below:



Compute the transfer function from the input U to the output Y .

(Your answer should be an expression involving the transfer functions K_1, K_2, P_1, P_2, G .)

2. Consider two systems in a negative feedback configuration shown below:



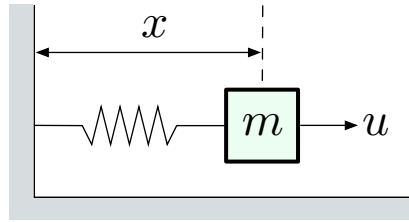
All state variables, all inputs, and all outputs are scalars.

- (i) Find the transfer function from the input R to the output Y .

(Your answer should be a ratio of two polynomials in s with coefficients expressed in terms of the system parameters a, b, c, k, ℓ, m .)

- (ii) Write down the conditions that must be satisfied by the system parameters a, b, c, k, ℓ, m for this transfer function to be stable (i.e., for all poles to have negative real parts).

3. Consider the mass-spring system shown in the diagram below:



As shown in class, this system has the following state-space model with $x_1 = x$ and $x_2 = \dot{x}$:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\rho}{m} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix} u, \quad y = (1 \quad 0) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},$$

where k is the spring constant and ρ is the friction coefficient.

- (i) Determine the transfer function of this system.
- (ii) Now suppose that the output equation is replaced by

$$y = (c_1 \quad c_2) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},$$

where c_1 and c_2 are arbitrary constants. Find the values of c_1 and c_2 to guarantee that the transfer function of the resulting system has the form of the prototype 2nd-order response discussed in class. Write down the expressions for the parameters ω_n and ζ in terms of k , ρ , and m .

- (iii) Determine the steady-state response of the system from part (ii) to the sinusoidal external force $u(t) = \cos(\omega t)$.
- (iv) Sketch the plots of the magnitude and the phase shift of the steady-state response from part (iii) as functions of the input frequency ω . What happens when the input frequency ω matches the system's natural frequency ω_n ? This is the phenomenon known as *resonance*.

4. Consider the transfer function

$$H(s) = \frac{16}{s^2 + 4s + 16}$$

- (i) Suppose that you are given the following time-domain specs: $t_r \leq 0.6$, $t_s \leq 1.6$ (where $t_s = t_s^{5\%}$). Plot the admissible pole locations in the s -plane corresponding to these two specs. Does the given system satisfy these specs?
- (ii) Suppose that in addition to the specs from (i), we have the following spec on the overshoot: $M_p \leq 1/e^2$. Plot the admissible pole locations in the s -plane corresponding to all three specs. Does the given system satisfy the new spec?

- (iii) Now suppose that you are given the two specs from (i) *plus* the following spec on the peak time: $t_p \leq 1$ (*instead of* the overshoot spec). Plot the admissible pole locations in the s -plane corresponding to these three specs. (Pole locations for peak time were not discussed in class, so you need to derive this.) Does the given system satisfy the new spec?