ECE 486 *Control Systems* **Spring 2024**

Exam II Information

The second midterm exam will be held ***in class***

1015 ECE Building *Thu, April 18, 9:30 – 10:50 a.m.*

The exam is closed-book, no calculators allowed (and you will not need one). You can bring one double-sided sheet of notes. The exam will cover all of the material covered in Lectures 13–19, inclusive.
In particular, I expect you to understand perfectly the following concepts:

1. Frequency domain basics: Bode plots, Nyquist plots, how to sketch them, how to relate them for a given transfer function.
2. Bode's gain-phase relationship.
3. Frequency domain design: Crossover frequency; bandwidth; phase and gain margins; PD/lead and PI/lag compensation; choosing lead/lag parameters to satisfy given specs (bandwidth, PM/GM, steady-state tracking errors).
4. The Nyquist Stability Criterion: $N = Z - P$; reading stability ranges given the Nyquist plot and knowledge of open-loop poles and zeros.
5. Reading stability margins (PM and GM) off a Nyquist plot.

The bare minimum of the material you need to know will be attached to the exam and reproduced below. *However,* you are responsible for all of the content outlined above.

Useful Facts

Bode plots A transfer function $G(j\omega)$ is in *Bode form* if it is written as a product of (some or all of) the following three types of factors:

- Type 1 — nth-order zero or pole at the origin, $K_0(j\omega)^n$, $K_0 > 0$, n is an integer
- Type 2 — real zero or pole, $(j\omega\tau + 1)^{\pm 1}$, $\tau > 0$
- Type 3 — complex zero or pole, $\left[\left(\dfrac{j\omega}{\omega_n}\right)^2 + 2\zeta\dfrac{j\omega}{\omega_n} + 1\right]^{\pm 1}$, $\quad \omega_n > 0$, $0 < \zeta < 1$

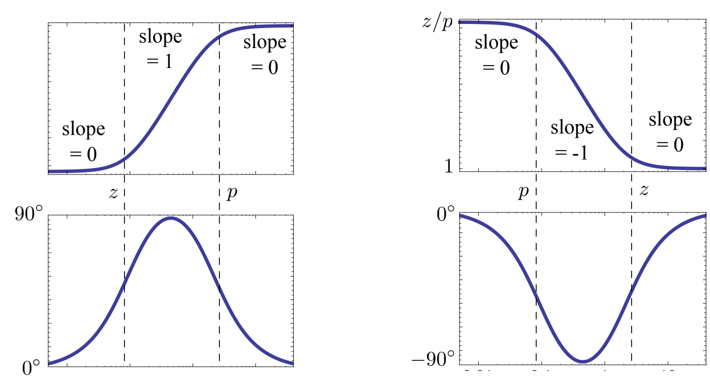
Magnitude and phase relationships:

	low frequency	real zero/pole	complex zero/pole
magnitude slope	n	up/down by 1	up/down by 2
phase	$n \times 90°$	up/down by 90°	up/down by 180°

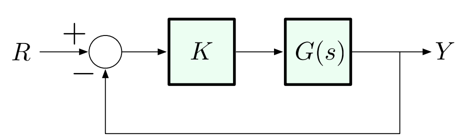
Crossover frequency: $|G(j\omega_c)| = 1$

Bode plots for lead and lag compensators

$$\text{lead: } D(s) = K\frac{s/z+1}{s/p+1}, \qquad z < p \qquad \text{lag: } D(s) = \frac{s+z}{s+p}, \qquad z > p$$



Stability margins — assume K is stabilizing



- **Gain Margin (GM)**: the factor by which K has to be multiplied for the closed-loop system to become unstable
- **Phase Margin (PM)**: the amount by which the phase of $G(j\omega_c)$ differs from $\pm 180°$ (the sign depends on the magnitude slope of the Bode plot of KG)

Nyquist plots For a transfer function $H(s)$, the Nyquist plot is the set of all points

$$\Big(\text{Re } H(j\omega), \text{Im } H(j\omega)\Big), \qquad -\infty < \omega < \infty$$

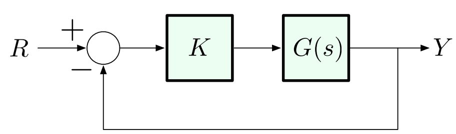
The Argument Principle

$$N = Z - P,$$

where:

- $N = \#(\circlearrowright$ of 0 by the Nyquist plot of $H)$
- $Z = \#(\text{RHP zeros of } H)$
- $P = \#(\text{RHP poles of } H)$

Nyquist Stability Criterion — consider the unity feedback configuration:



Then

$$N = Z - P,$$

where:

- $N = \#(\circlearrowright$ of $-1/K$ by the Nyquist plot of $G)$
- $Z = \#(\text{RHP closed-loop poles})$
- $P = \#(\text{RHP open-loop poles})$

Stability margins from Nyquist plots

$$\text{GM} = \frac{1}{M_{180^\circ}}, \qquad \text{PM} = \varphi$$