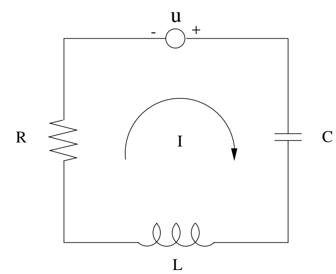
ECE 515, Fall 2019 **Homework 1** **Due Thursday, Sep 12**

Remark: Just a quick reminder that these homeworks are due **online** on the course's Gradescope. We are **not** collecting these during class!

Problems:

1. Consider this electrical circuit with *time-varying* characteristics $R(t)$, $L(t)$ and $C(t)$. Let $q(t)$ denote the charge in capacitor at time t, and $\phi(t)$ be the inductor flux at time t. From physical laws we know:



- Capacitor equation: $q(t) = C(t)V_C(t)$.
- Inductor equation: $\phi(t) = L(t)I(t)$.

Use these laws to derive a dynamical model of this circuit which takes the form $\dot{x} = A(t)x + B(t)u$.

2. Which of the following are vector spaces over $\mathbb{R}$ (with respect to the standard addition and scalar multiplication)? Justify your answers.

a) The set of real-valued $n \times n$ matrices with nonnegative entries, where n is a given positive integer.

b) The set of rational functions of the form $\frac{p(s)}{q(s)}$, where p and q are polynomials in the complex variable s and the degree of q does not exceed a given fixed positive integer k.

c) The space $L^2(\mathbb{R}, \mathbb{R})$ of square-integrable functions, i.e., functions $f : \mathbb{R} \to \mathbb{R}$ with the property that $\int_{-\infty}^{\infty} f^2(t)dt < \infty$. (**Hint:** You may use the Cauchy-Schwarz inequality, and note that this inequality applies to any inner product space.)

3. A single-input, single-output linear time-invariant system is described by the transfer function:

$$G(s) = \frac{s+4}{(s+1)(s+2)(s+3)}$$

a) Obtain a state-space representation in controllable canonical form.

b) Now obtain one in observable canonical form.

c) Use the partial fraction expansion of $G(s)$ to obtain a representation of this model with a diagonal state matrix A.

4. Let A be the linear operator in the plane corresponding to the counter-clockwise rotation around the origin by some given angle θ. Compute the matrix of A relative to the standard basis in $\mathbb{R}^2$.