ECE 515/ME 540 (Control System Theory and Design) – Homework 6

Due: Thursday, Oct. 31 at 2pm

Problem 1. Consider the linear, time-varying system:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$
$$y(t) = C(t)u(t)$$

Recall the definition of the observability Grammian:

$$H(t_1, t_0) = \int_{t_0}^{t_1} \phi^{\intercal}(\tau, t_0) C^{\intercal}(\tau) C(\tau) \phi(\tau, t_0) d\tau$$

Consider the function from $\mathbb{R}$ to $\mathbb{R}^{n \times n}$:

$$X : t_0 \mapsto H(t_1, t_0)$$

(Two other ways to write this are: $X(t_0) = H(t_1, t_0)$, or $X(\cdot) = H(t_1, \cdot)$.)

Show that the function X satisfies the linear matrix differential equation:

$$\dot{X}(t) = -A^{\intercal}(t)X(t) - X(t)A(t) - C^{\intercal}(t)C(t) \qquad\qquad X(t_1) = 0_{n\times n}$$

Here, the initial condition $0_{n\times n}$ is the zero matrix in $\mathbb{R}^{n\times n}$.

Problem 2. Consider a linear time-varying system with dynamics:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$
$$y(t) = C(t)x(t)$$

Let's call this system R.

As we've covered before, the dual system is given by the dynamics:

$$\dot{\tilde{x}}(t) = -A^{\intercal}(t)\tilde{x}(t) - C^{\intercal}(t)\tilde{u}(t)$$
$$\tilde{y}(t) = B^{\intercal}(t)\tilde{x}(t)$$

Let's call this dual system $\tilde{R}$.

Consider any state x_0 that is controllable to zero on $[t_0, t_1]$ for R, and any state $\tilde{x}_0$ that is unobservable on $[t_0, t_1]$ for $\tilde{R}$. Show that x_0 and $\tilde{x}_0$ are orthogonal, i.e. $\langle x_0, \tilde{x}_0 \rangle = 0$.

Problem 3. Consider:

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Is the system controllable? If not, put it in Kalman controllability canonical form.
- Is the system observable? If not, put it in Kalman observability canonical form.

Problem 4. Take the transfer function:

$$H(s) = \frac{s+3}{(s+1)(s+2)}$$

- Put this system into controllable canonical form.
- Using static linear state feedback ($u = -Kx$), find a K that places the poles at $-5 \pm 2j$.

Problem 5. Consider the following dynamical system, inspired by a linearization of the pendubot from Chapter 1 in the reader.

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 5 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ -5 & 6 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \\ 0 \\ -3 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x$$

Design a reduced-order Luenberger observer for this system. You may freely use MATLAB or any other computer assistance to do so (and are encouraged to do so!), but still show your work.