Problem 1

Let

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 5 & 4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x = x_1$$

- Draw a block diagram representing this system.
- Design a reduced-order Luenberger observer, and draw the block diagram for the system and the observer.

Problem 2

Take any matrix $A \in \mathbb{R}^{m \times n}$. Prove the following statements:

- $\mathcal{R}(A)^{\perp} = \mathcal{N}(A^{\intercal})$
- $\mathcal{N}(A)^{\perp} = \mathcal{R}(A^{\intercal})$

Problem 3

Consider the following dynamics:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$$

In this problem, you will be asked to calculate two Kalman decompositions for this system.

- Calculate $\Sigma_{c\bar{o}} = \mathcal{R}(\mathcal{C}) \cap \mathcal{N}(\mathcal{O})$
- Calculate a Σ_{co} such that $\Sigma_{c\bar{o}} \oplus \Sigma_{co} = \mathcal{R}(\mathcal{C})$
- Calculate a $\Sigma_{\bar{c}\bar{o}}$ such that $\Sigma_{c\bar{o}} \oplus \Sigma_{\bar{c}\bar{o}} = \mathcal{N}(\mathcal{O})$
- Calculate a $\Sigma_{\bar{c}o}$ such that $\Sigma_{c\bar{o}} \oplus \Sigma_{co} \oplus \Sigma_{\bar{c}\bar{o}} \oplus \Sigma_{\bar{c}o} = \mathbb{R}^n$
- As we discussed in class, this need not be unique; calculate a different Σ_{co}, $\Sigma_{\bar{c}\bar{o}}$, and $\Sigma_{\bar{c}o}$ for the same system