ECE/CS 541: Computer System Analysis

Homework #0 (**Due 09/04/18**)

Student Name: Student's name *Netid: netid*

The goal of this assignment is to get you familiar with Python, its plotting environment, and to gain some first hand experience with random variables and their distributions. You can find some helpful code snippets on the course webpage.

Please submit a single PDF file of your plots and answers to Compass by Tuesday 09/04 at 11 :59 pm.

Exercise 1.

In this exercise, we are interested in exponentially distributed random variables. Use the `scipy.stats` module along with `matplotlib` to :

— Create three random variables $X_1 \sim \exp(\lambda_1 = 1)$, $X_2 \sim \exp(\lambda_2 = 2)$, $X_3 \sim \exp(\lambda_3 = 3)$

— On the same graph, plot the *pdf* of all three random variables.

— On the same graph, plot the *cdf* of all three random variables.

— What do you notice about the distribution functions as λ increases ?

Exercise 2.

Let $X_1, X_2, \ldots, X_n$ be n independent and identically distributed (often referred to as i.i.d.) random variables. Let σ_i be the standard deviation of the random variable X_i. Since all X_i's are i.i.d., we know that $\sigma_i = \sigma_j \ \forall i, j$.

Define the random variable S_n as

$$S_n = \frac{1}{\sqrt{n} \times \sigma} \sum_{i=1}^{n} X_i$$

Part 1.

Assume that $X_i \sim \exp(1) \ \forall i$. We are interested in values of $n \in \{10, 100, 1000, 5000, 10000\}$. For each n,

1. Write a program that generates 500 samples from S_n. In other words, generate 500 samples from each X_i and compute their element-wise sum.
2. Plot a (normalized) histogram of the distribution of the samples. This will serve as a good approximation of S_n's pdf.

Part 2.

Repeat part 1 with $X_i \sim \text{Gamma}(1.99) \ \forall i$.

Using your plots, answer the following question

What can you say about the distribution of S_n as $n \to \infty$? Is this behavior dependent on the distribution of the X_i's ?

Exercise 3.

Let X_1 and X_2 be two independent, exponentially distributed random variables, with $\lambda_1 = 1$ and $\lambda_2 = 2$, respectively. Define the random variable Z such that the cdf of Z is given by Equation (1).

$$F_Z(z) = 1 - (1 - F_{X_1}(z))(1 - F_{X_2}(z)), \quad \text{for } z \geq 0 \tag{1}$$

1. Create a Python class called `ece541_rv` that inherits `scipy.stats.rv_continuous` and define its cdf as per Equation (1).
2. Print out the mean and the variance of Z.
3. Plot the pdf and the cdf of Z.
4. What can you say about the distribution of Z ? Can you say anything about its parameter(s) ? It might help to plot X_1, X_2, and Z on the same graph. You might also find it useful to expand $F_Z(z)$.
5. **Bonus Question :** Try to derive the relationship between Z and X_1 and X_2.
   Hint : Replace $F_X(x)$ by its definition in terms of probabilities and try to reverse engineer the relationship. Solving this can help you prepare for Homework #1.