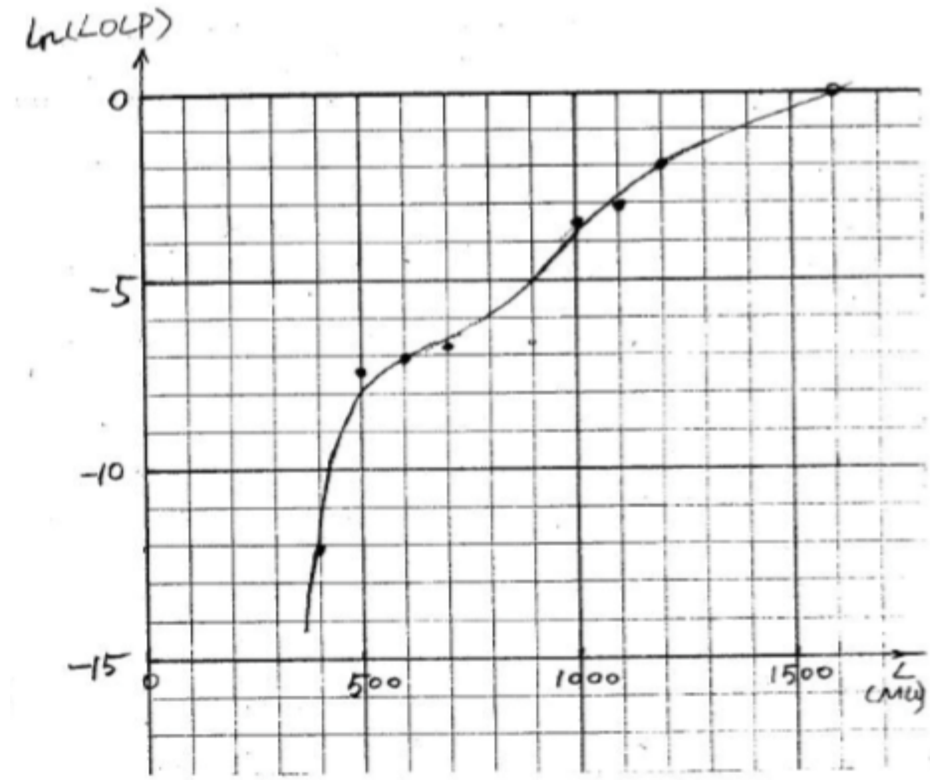
ECE 588
Problem Set 3: Solution

Problem 1

a) **We make use of the results in HW 1 to obtain the following *LOLP* vs. load table**

load (*MW*)	*LOLP*	ln(*LOLP*)
400	0.000006	-12.024
500	0.000600	-7.4190
600	0.000894	-7.0200
700	0.001088	-6.8230
1000	0.030194	-3.5000
1100	0.049400	-3.0080
1200	0.058906	-2.8320
1600	1.000000	0.0000

Below we provide a plot of the ln(*LOLP*) vs. load.



b) **We know from the lecture notes that**

$$H(x) \triangleq P\{\underset{\sim}{R} < x\} = Ke^{\alpha x}$$

Consider the *LOLP* value at the load level of 1,200 *MW*. In order to determine α we make use of two arbitrary points:

$$x = 0: \qquad H(0) = Ke^{\alpha 0} = 0.058906$$

$$x = -100: \quad H(-100) = Ke^{\alpha(-100)} = 0.0494$$

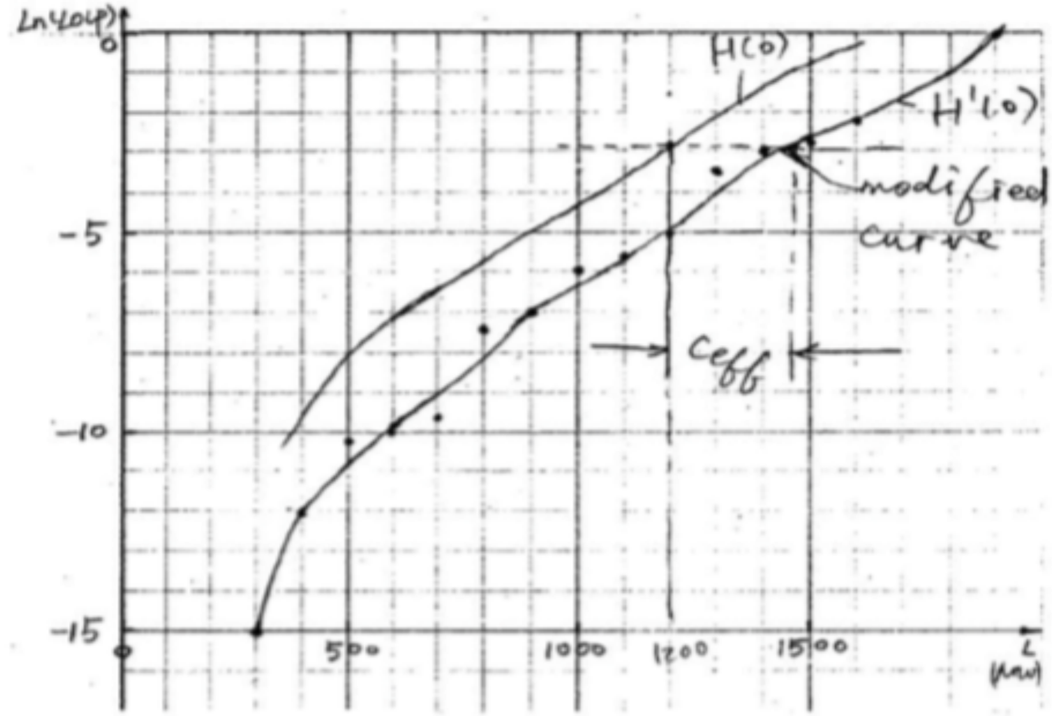
Therefore,

$$\frac{e^{0}}{e^{-100\alpha}} = \frac{0.058906}{0.049400} \Rightarrow \alpha = 0.00176$$

c) **After adding the new 300 *MW* unit, we get a new table of *LOLP* vs. load values.**

load (*MW*)	*LOLP*	ln(*LOLP*)
300	$3*10^{-7}$	-15.02
400	$6*10^{-6}$	-12.02
500	0.0000357	-10.24
600	0.0000504	-9.90
700	0.0000601	-9.72
800	0.0006244	-7.38
900	0.0009037	-7.01
1000	0.0025433	-5.97
1100	0.0035036	-5.65
1200	0.0039789	-5.53
1300	0.0316296	-3.45
1400	0.0498753	-3.00
1500	0.0589060	-2.83
1600	0.1059607	-2.24
1800	1.0000000	0.00

The modified *LOLP* vs. load curve is provided below.



From the lecture notes we have proved that the effective load carrying capability (*ELCC*) of a resource is given by

$$c_{eff} = \frac{1}{\alpha}\left\{-\ell n\left[\left(1-p\right)+pe^{-\alpha c}\right]\right\}$$

At a load level of 1,200 *MW* we have computed $\alpha = 0.00176$. Therefore, for $c = 300$ *MW* and $p = 0.95$ we obtain

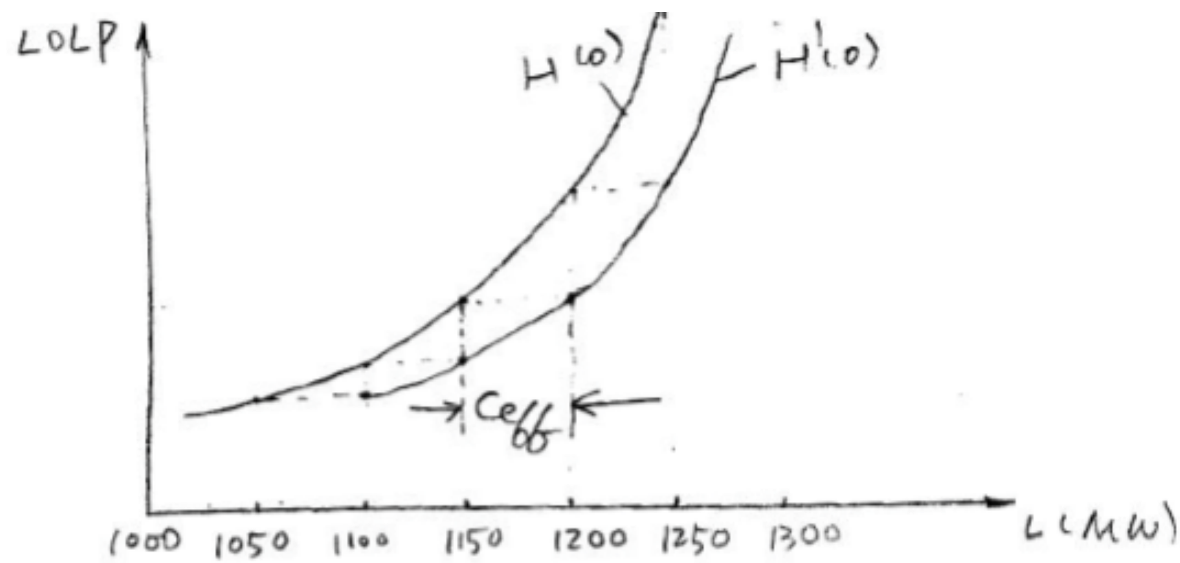
$$c_{eff} = 280.58\ MW$$

d) With the application of the *DSM* program, for load levels greater then 1,100 *MW* we derive the new *LOLP* values as follows:

$$LOLP'(1100) = LOLP(1050) = 0.0494$$
$$LOLP'(1150) = LOLP(1100) = 0.0494$$
$$LOLP'(1200) = LOLP(1150) = 0.058906$$
$$LOLP'(1250) = LOLP(1200) = 0.058906$$
$$LOLP'(1550) = LOLP(1500) = 0.058906$$

When the load values are below 1100 *MW*, the *LOLP* values remain the same. Since we assume that the *DSM* program is 100% available, which implies that $p_{DSM} = 1.0$, then the *ELCC* of the *DSM* program is

$$c_{eff} = 50\ MW$$



Problem 2

The generation and load data for system A are given in lecture 3 (*Markov Models for Reliability Evaluation*) and 4 (*Frequency and Duration Techniques*) respectively. The generation data for system B are given in lecture 5 (*Reliability of Two-Area Interconnections*) and the load data for system B are given below

j	ℓ_j	α_j	$\lambda_{\ell_{j+}}(1/day)$	$\lambda_{\ell_{j-}}(1/day)$	$p_j = \alpha_j e\,(j \neq 0)$
0	**0**	**-**	**2**	**0**	**0.5**
1	**80**	**0.1**	**0**	**2**	**0.05**
2	**96**	**0.2**	**0**	**2**	**0.10**
3	**112**	**0.5**	**0**	**2**	**0.25**
4	**120**	**0.2**	**0**	**2**	**0.10**

The availability table for system B is

k	available capacity x	p_k	$P\left\{\sum_{i=1}^{3} \underset{\sim}{A}_i \leq x\right\}$
0	**0**	**0.000064**	**0.000064**
1	**40**	**0.003072**	**0.003136**
2	**80**	**0.038400**	**0.041536**
3	**120**	**0.0737280**	**0.115309**
4	**160**	**0.8847360**	**1.000000**

We compute the following representative values for the *two-area interconnection* example given in lecture 5. For system A as an isolated system we obtain

$$P\{\underset{\sim}{R}^A = -100\} = P\left\{\sum_{i=1}^{4} \underset{\sim}{A}_i = 0\right\} P\{\underset{\sim}{L} = 100\} + P\left\{\sum_{i=1}^{4} \underset{\sim}{A}_i = 50\right\} P\{\underset{\sim}{L} = 150\}$$
$$= (0.0000026)(0.1 * 0.5) + (0.0002458)(0.2 * 0.5)$$
$$= 0.0000001 + 0.0000246 = 0.0000247$$

$$P\{\underset{\sim}{R}^A \leq -100\} = P\{\underset{\sim}{R}^A = -100\} + P\{\underset{\sim}{R}^A = -120\} + P\{\underset{\sim}{R}^A = -140\} + P\{\underset{\sim}{R}^A = -150\}$$
$$= 0.0000259$$

For the frequency computation we define the set

$$\mathbb{S}(-100) = \{i : \underset{\sim}{R}^A = r_i^A \leq -100\}$$

then the frequency

$$\mathfrak{F}\left\{\mathbb{S}\left(-100\right)\right\} = \mathfrak{F}\left\{\underset{\sim}{R}^{A} \le -100\right\} = \sum_{i\in\mathbb{S}(-100)} p_i * \left(\sum_{j\notin\mathbb{S}(-100)} \lambda_{ji}\right) =$$

$$= p^{A}_{-100}\left(0.079+2\right) + p^{A}_{-120}\left(0.105+2\right)$$

$$+p^{A}_{-140}\left(0.105+2\right) + p^{A}_{-150}\left(2\right) + p^{A}_{-100}\left(0.105+2\right)$$

$$= 0.0000246 * 2.079 + 0.0000003 * 2.105 +$$

$$+ 0.0000007 * 2.105 + 0.0000003 * 2$$

$$+ 0.0000001 * 2.105 = 0.0000541$$

For system B as an isolated system we obtain

$$P\left\{\underset{\sim}{R}^{B} = 120\right\} = P\left\{\sum_{i=1}^{3} \underset{\sim}{A}^{B}_{i} = 120\right\} P\left\{\underset{\sim}{L} = 0\right\} = 0.0737280 * 0.5 = 0.036864$$

$$P\left\{\underset{\sim}{R}^{B} \le 120\right\} = \sum_{r_i^B \le 120} P\left\{\underset{\sim}{R}^{B} \le r_i^B\right\} = 0.557632$$

$$\mathfrak{F}\left\{\underset{\sim}{R}^{B} \le 120\right\} = p^{B}_{120}\left(0.026301\right) + p^{B}_{80}\left(2\right) + p^{B}_{64}\left(2\right) + p^{B}_{48}\left(2\right) +$$

$$+p^{B}_{40}\left(2\right) = 0.8857$$

For the interconnected systems A and B with *unconstrained* tie-line capacity we can derive the probabilities of any of the failure states as follows

$P\left\{\underset{\sim}{R}^{A} = r_i^A \text{ and } \underset{\sim}{R}^{B} = r_i^B\right\} = P\left\{\underset{\sim}{R}^{A} = r_i^A\right\} P\left\{\underset{\sim}{R}^{B} = r_i^B\right\}$ **due to statistical independence of the two areas. Hence,**

$$P\left\{\underset{\sim}{R}^{A} = -20 \text{ and } \underset{\sim}{R}^{B} = 160\right\} = P\left\{\underset{\sim}{R}^{A} = -20\right\} P\left\{\underset{\sim}{R}^{B} = 160\right\}$$

$$= 0.0008847 * 0.442368 = 0.000391$$

Problem 3

i)

$$P\left\{\underset{\sim}{R}^{A} < 0\right\} = P\left\{\underset{\sim}{R}^{A^0} + \underset{\sim}{A}^{AB} < 0\right\} =$$
$$P\left\{\underset{\sim}{R}^{A^0} + \underset{\sim}{A}^{AB} < 0 \middle| \underset{\sim}{A}_t = c_t\right\} p_t + P\left\{\underset{\sim}{R}^{A^0} < 0 \middle| \underset{\sim}{A}_t = 0\right\}(1 - p_t)$$

We know that

$$P\left\{\underset{\sim}{R}^{A^0} < 0\right\} = 0.0041052$$

and,

$$P\left\{\underset{\sim}{R}^{A^0} + \underset{\sim}{A}^{AB} < 0\right\} = \sum_{i,j} p_{ij}$$

with states $i, j \in \left\{\underset{\sim}{R}^{A^0} + 50 < 0 \middle| \underset{\sim}{R}^{B} \geq 50\right\} \cup \left\{\underset{\sim}{R}^{A^0} + \underset{\sim}{R}^{B} < 0 \middle| 0 < \underset{\sim}{R}^{B} < 50\right\} \cup \left\{\underset{\sim}{R}^{A^0} < 0 \middle| \underset{\sim}{R}^{B} \leq 0\right\}$

From the table in slide 29, lecture 5 we obtain by adding all the values below the red staircase line plus the *<u>values above the staircase for</u>* $r_j^A \leq -70$ (that give negative reserves when we add the 50 MW assistance from system B). Hence,

$$P\left\{\underset{\sim}{R}^{A^0} + \underset{\sim}{A}^{AB} < 0\right\} = 0.6031 \times 10^{-3}$$

Consequently,

$$P\left\{\underset{\sim}{R}^{A} < 0\right\} = 0.999\left(0.6031 \times 10^{-3}\right) + 0.001\left(0.0041052\right) = 0.000607$$

For $c_t = 100\ MW$ **we can derive in a similar way**

$$P\left\{\underset{\sim}{R}^{A^0} + \underset{\sim}{A}^{AB} < 0\right\} = \sum_{i,j} p_{ij} = 0.000548$$

with states $i, j \in \left\{\underset{\sim}{R}^{A^0} + 100 < 0 \middle| \underset{\sim}{R}^{B} \geq 100\right\} \cup \left\{\underset{\sim}{R}^{A^0} + \underset{\sim}{R}^{B} < 0 \middle| 0 < \underset{\sim}{R}^{B} < 100\right\} \cup \left\{\underset{\sim}{R}^{A^0} < 0 \middle| \underset{\sim}{R}^{B} \leq 0\right\}$

Therefore,

$$P\left\{\underset{\sim}{R}^{A} < 0\right\} = 0.999\left(0.000548\right) + 0.001\left(0.0041502\right) = 0.000552$$

ii) For the frequency evaluation we proceed as follows

$$\mathfrak{F}\left\{\underset{\sim}{R}^{A}<0\right\}=\mathfrak{F}\left\{\underset{\sim}{R}^{A}<0 \middle| A_{t}=c_{t}\right\}p_{t}+\mathfrak{F}\left\{\underset{\sim}{R}^{A^{0}}<0 \middle| A_{t}=0\right\}(1-p_{t})+\lambda_{t}p_{t}\left(\sum_{i,j}p_{ij}\right)$$
$$\textit{with } i,j\in\left\{\underset{\sim}{R}^{A^{0}}+\underset{\sim}{A}^{AB}\geq 0\right\}\cap\left\{\underset{\sim}{R}^{A^{0}}<0\right\}$$

For $c_t = 50\ MW$:

$$\mathfrak{F}\left\{\underset{\sim}{R}^{A^{0}}<0\right\}=\mathfrak{F}\left\{\underset{\sim}{R}^{A^{0}}\leq -20MW\right\}=0.0084239\ (\textit{from table in slide } 27)$$
$$\sum_{i,j}p_{ij}=0.0035573\ \textit{for } i,j\in\left\{\underset{\sim}{R}^{A}+\underset{\sim}{A}^{AB}\geq 0\right\}\cap\left\{\underset{\sim}{R}^{A^{0}}<0\right\}$$

For the evaluation of the term $\mathfrak{F}\left\{\underset{\sim}{R}^{A}<0 \middle| A_{t}=50\right\}$ we compute the following tables

$\underset{\sim}{A}_j^{AB}(MW)$ x	$p_{\underset{\sim}{A}_j^{AB}}$	$\mathfrak{F}\left\{\underset{\sim}{R}^{A^{0}}<-A_j^{AB}\right\}$
50	0.631143	0.0002330
48	0.221184	0.0020727
40	0.093696	0.0020727
24	0.007373	0.0066093
8	0.018432	0.0084239
0	0.028172	0.0084239

The total frequency contribution of this case is given by

$$\sum_{j}p_{\underset{\sim}{A}_j^{AB}}\ \mathfrak{F}\left\{\underset{\sim}{R}^{A^{0}}<-A_j^{AB}\right\}=0.001241$$

r_i^A	p_i^A	$\mathfrak{F}\left\{\underset{\sim}{A}^{AB}<-r_i^A\right\}$
200 to 0	-	0
-20	0.0008847	0.0942699
-40	0.0022119	0.1091936
-50	0.0008970	0.7319554
-70 to -150	0.0001121	1

The total contribution of the second case is

$$\sum_i p_i^A \,\mathcal{F}\left\{\underset{\sim}{A}^{AB} < -r_i^A\right\} = 0.001094$$

$$\mathcal{F}\left\{\underset{\sim}{R}^A < 0 \middle| A_t = 50\right\} = 0.001241 + 0.001094 = 0.002335$$

Hence,

$$\mathcal{F}\left\{\underset{\sim}{R}^A < 0\right\} = 0.002335(0.999) + 0.0084239(0.001) + (0.00274)(0.999)0.0035573 = 0.002351$$

For c_t =100 *MW* the first and last term of the frequency computation remain the same as in the previous case. For the second term, we follow the same procedure

$\underset{\sim}{A}_j^{AB}$	$p_{\underset{\sim}{A}_j^{AB}}$	$\mathcal{F}\left\{\underset{\sim}{R}^{A^0} < -A_j^{AB}\right\}$
100	**0.479232**	**0.0000021**
80	**0.063437**	**0.0001819**
64	**0.088474**	**0.0002330**
48	**0.221184**	**0.0020727**
40	**0.093696**	**0.0020727**
24	**0.007373**	**0.0066903**
8	**0.018432**	**0.0084239**
0	**0.028172**	**0.0084239**

Therefore the total contribution is computed by the summation

$$\sum_j p_{\underset{\sim}{A}_j^{AB}} \,\mathcal{F}\left\{\underset{\sim}{R}^{A^0} < -A_j^{AB}\right\} = 0.001127$$

r_i^A	p_i^A	$\mathcal{F}\left\{\underset{\sim}{A}^{AB} < -r_i^A\right\}$
200 to 0	**-**	**0**
-20	**0.0008847**	**0.0942699**
-40	**0.0022119**	**0.1091936**
-50	**0.0008970**	**0.7319554**
-70	**0.0000246**	**0.9086114**
-90	**0.0000615**	**0.9590185**
-100	**0.0000247**	**0.9590185**
-120 to -150	**0.0000010**	**1**

$$\sum_i p_i^A \mathfrak{F}\left\{\underset{\sim}{A}^{AB} < -r_i^A\right\} = 0.001088$$

$$\mathfrak{F}\left\{\underset{\sim}{R}^A < 0 \middle| A_t = 100\right\} = 0.001127 + 0.001088 = 0.00215$$

Therefore,

$$\mathfrak{F}\left\{\underset{\sim}{R}^A < 0\right\} = 0.00215(0.999) + 0.0084239(0.001) + (0.00274)(0.999)0.0035573 = 0.002166$$

Problem 4

For the evaluation of the probability and frequency that system B fails, we follow exactly the same procedure as with system A. Therefore,

$$P\left\{\underset{\sim}{R}^B < 0\right\} = P\left\{\underset{\sim}{R}^{B^0} + \underset{\sim}{A}^{BA} < 0 \middle| \underset{\sim}{A}_t = c_t\right\} p_t + P\left\{\underset{\sim}{R}^{B^0} < 0 \middle| \underset{\sim}{A}_t = 0\right\}(1 - p_t)$$

We can evaluate each term in $P\left\{\underset{\sim}{R}^B < 0\right\}$ **as follows**

$$P\left\{\underset{\sim}{R}^{B^0} < 0\right\} = \sum_{i,j} p_{ij} \qquad \forall i, j \in \left\{r_j^B < 0\right\}$$

$$P\left\{\underset{\sim}{R}^{B^0} + \underset{\sim}{A}^{BA} < 0 \middle| \underset{\sim}{A}_t = c_t\right\} = \sum_{i,j} p_{ij} \qquad \forall i, j \in \left\{r_i^A \geq c_t\right\} \cap \left\{r_i^A + r_j^B \geq 0\right\}$$

For the frequency evaluation, we know that system B fails

- **if the tie line is operating, system B fails due to the fact that system B transitions into a state with** $\underset{\sim}{R}^B < 0$ **as a result of unit failures or a load increase in system B and system A provides insufficient assistance to restore area B into a positive margin state. For state *j* of system B, we can express the total frequency contribution of this situation as**

$$p_t \sum_i p_i^A \mathfrak{F}\left\{\underset{\sim}{R}^{B^0} + r_i^A < 0\right\} \quad \forall i \in \left\{0 \leq r_i^A = \underset{\sim}{R}^A \leq c_t\right\}$$

- **If system B receives assistance from system A, but system A transitions to a state of lower reserves (outages of its units or increase in its load) corresponding to a state with** $\underset{\sim}{R}^{B^0} < 0$, **we express the frequency contribution of this case as**

$$p_t \sum_j p_j^B \mathscr{F}\left\{ \underset{\sim}{R}^A + r_j^B < 0 \right\} \quad \forall j \in \left\{ r_j^B < 0 \right\}$$

- **If system B receives assistance from system A and the tie line fails, we can write the frequency contribution of this situation as**

$$\lambda_t p_t \left(\sum_{i,j} p_{ij} \right) \qquad \forall i, j \in \left\{ \underset{\sim}{R}^{B^0} < 0 \right\} \cap \left\{ \underset{\sim}{R}^{B^0} + \underset{\sim}{A}^{BA} \geq 0 \right\} \;\; with \;\; 0 \leq \underset{\sim}{A}^{BA} \leq c_t$$

1. **If the tie line is not operating, system B fails if it transitions to a state with $\underset{\sim}{R}^{B^0} < 0$. We can write its frequency contribution as**

$$\left(1 - p_t\right) \mathscr{F}\left\{ \underset{\sim}{R}^{B^0} < 0 \right\}$$

The total failure frequency is expressed as the sum of the frequencies of all the previous cases. Therefore,

$$\mathscr{F}\left\{ \underset{\sim}{R}^B < 0 \right\} = p_t \left(\sum_i p_i^A \mathscr{F}\left\{ \underset{\sim}{R}^{B^0} + r_i^A < 0 \right\} + \sum_j p_j^B \mathscr{F}\left\{ \underset{\sim}{R}^A + r_j^B < 0 \right\} + \lambda_t \sum_{i,j} p_{ij} \right) + \left(1 - p_t\right) \mathscr{F}\left\{ \underset{\sim}{R}^{B^0} < 0 \right\}$$