ECE588: Solution Homework 1

- **Problem 1** : Every multi-state unit has the following distribution function:

$$f_{\underline{A}_i}(x) = p_i\delta(x - c_i) + \sum_{j=1}^{k-2} s_i^j \delta(x - d_i^j) + (1 - \sum_{j=1}^{k-2} s_i^j - p_i)\delta(x) \quad (1)$$

Where $0 < d_i^{k-2} < d_i^{k-3} < ...d_i^1 < c_i$.

To find the distribution function of the sum of a set of this random variables, due to statistical independence, we can use the fact that the distribution of $A = A_1 + A_2 + ... + A_n$ is equal to the n-convolution of the distributions,

$$f_{\underline{A}}(x) = f_{\underline{A}_1}(x) * f_{\underline{A}_2}(x) * f_{\underline{A}_3}(x).... * f_{\underline{A}_N}(x) \quad (2)$$

The above equation is the generalization to the case with 2 random variables like lecture notes. Then using this distribution, we can find $P\{\sum_{l=1}^{i} \underline{A}_l \leq x\}$ in the following way,

$$P\{\sum_{l=1}^{i} \underline{A}_l \leq x\} = \int_0^x f_{\underline{A}}(y)dy \quad (3)$$

We can also find an equivalent formula following the steps in lecture notes for the case of two-state system.

Let

$$\underline{Z} = \sum_{j=1}^{i-1} \underline{A}_j \quad (4)$$

and

$$\underline{Y} = \underline{A}_i \quad (5)$$

, then

$$P\{\sum_{l=1}^{i} \underline{A}_l \leq x\} = F_{\underline{Z}+\underline{Y}}(x) = \int_{-\infty}^{\infty} F_{\underline{Z}}(x - m) f_{\underline{Y}}(m)dm \quad (6)$$

Using (1) into (6) we obtain that,

$$P\{\sum_{l=1}^{i} \underline{A}_l \leq x\} = (1 - \sum_{j=1}^{k-2} s_i^j - p_i)P\{\underline{Z} \leq x\} + \sum_{j=1}^{k-2} s_i^j P\{\underline{Z} \leq x - d_i^j\} + p_i P\{\underline{Z} \leq x - c_i\} \quad (7)$$

- **Problem 2** :

i	$c_i(MW)$	p_i
1	400	.99
2	500	.98
3	600	.97

From the above table it is possible to build first a table from operation of units 1 and 2:

- Capacity=0, 1 down and 2 down, $(1-p_1)(1-p_2) = 0.01 \times 0.02 = 0.0002$
- Capacity=400, 1 up and 2 down, $(p_1)(1-p_2) = 0.99 \times 0.02 = 0.0198$
- Capacity=500, 1 down and 2 up, $(1-p_1)(p_2) = 0.01 \times 0.98 = 0.0098$
- Capacity=900, 1 up and 2 up, $(p_1)(p_2) = 0.99 \times 0.98 = 0.9799 = 0.9702$

Now adding unit 3, we obtain:

- Capacity=0, 1 down, 2 down and 3 down, $(1-p_1)(1-p_2)(1-p_3) = 0.0002 \times 0.03 = 0.000006$
- Capacity=400, 1 up, 2 down and 3 down, $(p_1)(1-p_2)(1-p_3) = 0.0198 \times 0.03 = 0.000594$
- Capacity=500, 1 down, 2 up and 3 down, $(1-p_1)(p_2)(1-p_3) = 0.0098 \times 0.03 = 0.000294$
- Capacity=600, 1 down, 2 down and 3 up, $(1-p_1)(1-p_2)(p_3) = 0.0002 \times 0.97 = 0.000194$
- Capacity=900, 1 up, 2 up and 3 down, $(p_1)(p_2)(1-p_3) = 0.9702 \times 0.03 = 0.029106$
- Capacity=1000, 1 up, 2 down and 3 up, $(p_1)(1-p_2)(p_3) = 0.0198 \times 0.97 = 0.019206$
- Capacity=1100, 1 down, 2 up and 3 up, $(1-p_1)(p_2)(p_3) = 0.0098 \times 0.97 = 0.009506$
- Capacity=1500, 1 up, 2 up and 3 up, $(p_1)(p_2)(p_3) = 0.9702 \times 0.97 = 0.941094$

- **Problem 3** :

Day	Sun	Mon	Tue	Wed	Thur	Fri	Sat
Daily peak *MW*	600	1100	1200	1100	1200	1200	800

Using the definition of $LOLP_{week}$,

$$LOLP_{week} = \frac{1}{7}\sum_{i=1}^{7} LOLP_{d_i} \tag{8}$$

So we need to calculate $LOLP(600)$, $LOLP(800)$, $LOLP(1100)$, $LOLP(1200)$. To do this we use information of problem 2 and definition of $LOLP$,

$$LOLP(l) = P\{\sum_i \underline{A} < l\} \tag{9}$$

The random variable $\sum_i \underline{A}$ has in this case the distribution,

$$\begin{aligned} f(x) &= 0.000006\delta(x) + 0.000594\delta(x-400) + 0.000294\delta(x-500) + 0.000194\delta(x-600) \\ &+ 0.029106\delta(x-900) + 0.019206\delta(x-1000) + 0.009506\delta(x-1100) + 0.941094\delta(x-1500) \end{aligned} \tag{10}$$

So using $LOLP(l) = \int_{-\infty}^{l} f(x)dx$ we obtain,

$$\begin{aligned} LOLP(600) &= 0.000006 + 0.000594 + 0.000294 = 0.000894 \\ LOLP(800) &= 0.000894 + 0.000194 = 0.001088 \\ LOLP(1100) &= 0.001088 + 0.029106 + 0.019206 = 0.0494 \\ LOLP(1200) &= 0.0494 + 0.009506 = 0.058906 \end{aligned} \tag{11}$$

Finally $LOLP_{week}$ is,

$$LOLP_{week} = \frac{1}{7}\{0.000894+2\times0.0494+3\times0.058906+0.001088 = \frac{0.2775}{7}\}[\frac{days}{days}] = 0.2775[\frac{days}{week}] \quad (12)$$

Weekly $LOLP$ is expressed in $days/week$ that means that the probability of loss of load is 0.2775 days by week or 1 day in 3.6036 weeks.

- **Problem 4** : Without loss loss of generality we can suppose that the period is 1 day (any other period can be scaled).In this case $LOLE^d$ will be:

$$LOLE^d = LOLP(l_{day})\frac{days}{day} \quad (13)$$

In the same period of time, the $LOLE^h$ will be:

$$LOLE^h = \sum_{i=1}^{24} LOLP(l_i)\frac{hours}{24hours} \quad (14)$$

We can restrict the maximum value of (14) if we use the maximum value of $LOLP(l_i)$ in the period, that correspond to the peak hourly load :

$$LOLE^h = \sum_{i=1}^{24} LOLP(l_i)\frac{hours}{24hours} \leq \sum_{i=1}^{24} LOLP(l_{peak})\frac{hours}{24hours} = 24LOLP(l_{peak})\frac{hours}{24hours} \quad (15)$$

But $LOLP(l_{peak})$ in this case is equal to $LOLP(l_{day})$ if we use the assumption that the daily load is computed using the peak hourly load and by (13) the value is the same that $LOLE^d$. Using this fact in (15), we arrive to:

$$LOLE^h \leq 24LOLE^d \quad (16)$$

- **Problem 5** :

We use $T = 1$ with the different peak-loads, $l = 600, 800, 1100, 1200$.

$$\begin{aligned} \mathcal{U}(600) &= (600-0)P\{0\} + (600-400)P\{400\} + (600-500)P\{500\} \\ &= 600\times0.000006 + 200\times0.000594 + 100\times0.000294 = 0.1518MWh \end{aligned} \quad (17)$$

$$\begin{aligned} \mathcal{U}(800) &= (800-0)P\{0\} + (800-400)P\{400\} + (800-500)P\{500\} \\ &= 800\times0.000006 + 400\times0.000594 + 300\times0.000294 = 0.3306MWh \end{aligned} \quad (18)$$

$$\begin{aligned} \mathcal{U}(1100) &= (1100-0)P\{0\} + (1100-400)P\{400\} + (1100-500)P\{500\} + (1100-600)P\{600\} \\ &+ (1100-900)P\{900\} + (1100-1000)P\{1000\} \\ &= 1100\times0.000006 + 700\times0.000594 + 600\times0.000294 + 500\times0.000194 \\ &+ 200\times0.029106 + 100\times0.019206 = 8.4376MWh \end{aligned} \quad (19)$$

$$\begin{aligned} \mathcal{U}(1200) &= (1200-0)P\{0\} + (1200-400)P\{400\} + (1200-500)P\{500\} + (1200-600)P\{600\} \\ &+ (1200-900)P\{900\} + (1200-1000)P\{1000\} + (1200-1100)P\{1100\} \\ &= 1200\times0.000006 + 800\times0.000594 + 700\times0.000294 + 600\times0.000194 \\ &+ 300\times0.029106 + 200\times0.019206 + 100\times0.009506 = 14.3282MWh \end{aligned} \quad (20)$$