ECE 588 Problem Set 5 – Solutions

Problem 1

Consider

$$\left(\omega_j - \frac{1}{\eta_g \eta_t} h_j \rho - \frac{1}{\eta_t} \nu_j\right) g_j = 0 \qquad j = K+1, \dots, J$$

Since $\nu_j > 0$ if unit j is used to charge

$$\omega_j - \{(\rho - \beta) h_j - \xi_j\} \eta_t \eta_c = 0 \qquad j = 1, \dots, K$$

or

$$\omega_j = \{(\rho - \beta) h_j - \xi_j\} \eta_t \eta_c$$

We know that $\beta \geq 0$ and $\xi_j \geq 0$

$$\omega_j \leq \rho h_j \eta_t \eta_c \qquad j = 1, \dots, K$$

Let

$$\lambda_c = \max \left\{ \frac{\omega_j}{h_j} : j = 1, \dots, K \right\}$$

Then,

$$\frac{\omega_j}{h_j} \leq \rho \eta_t \eta_c \quad \forall j = 1, \dots, K \implies \lambda_c \leq \rho \eta_t \eta_c$$

Consider

$$\left[\omega_j - \{(\rho - \beta) h_j - \xi_j\} \eta_t \eta_c\right] \nu_j = 0 \qquad j = 1, \dots, K$$

Since $g_j > 0$ if storage generates

$$\omega_j - \frac{1}{\eta_g \eta_t} h_j \rho - \frac{1}{\eta_t} \nu_j = 0 \qquad j = K+1, \dots, J$$

or

$$\omega_j = \frac{1}{\eta_g \eta_t} h_j \rho + \frac{1}{\eta_t} \nu_j$$

Now $\nu_j \geq 0 \implies$

$$\omega_j \geq \frac{1}{\eta_g \eta_t} h_j \rho$$

Let

$$\lambda_g = \min \left\{ \frac{\omega_j}{h_j} : j = K+1, \dots, J \right\}$$

Then,

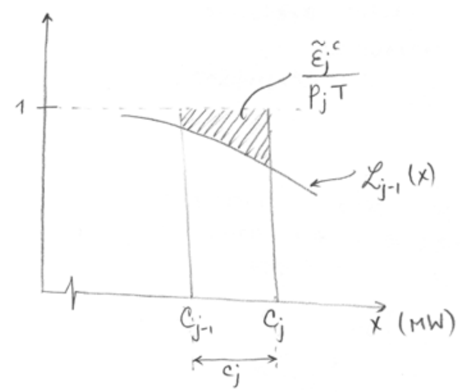
$$\frac{\omega_j}{h_j} \geq \frac{1}{\eta_g \eta_t} \rho \quad \forall j = K+1, \dots, J \implies \lambda_g \geq \frac{1}{\eta_g \eta_t} \rho \quad \text{QED}$$

Problem 2

$\tilde{\varepsilon}_j^c$ = EXPECTED ENERGY THAT CAN BE USED FOR CHARGING AN ENERGY STORAGE PLANT, PROVIDED BY UNIT j

(i) $\tilde{\varepsilon}_j^c = ?$ IF j IS THE FIRST BLOCK OF A 3-STATE UNIT.

GENERALLY, THE ENERGY AVAILABLE TO CHARGE A STORAGE PLANT WITH UNIT j IS DETERMINED ACCORDING TO THE SHADED AREA, AS INDICATED BELOW:



$$\tilde{\varepsilon}_j^c = p_j T \left(c_j - \int_{C_j}^{C_{j-1}} \mathcal{L}_{j-1}(x)\, dx \right)$$

$$= p_j T c_j - \varepsilon_j$$

IF j IS THE FIRST BLOCK OF A 3-STATE UNIT:

$$\tilde{\varepsilon}_j^c = T \left\{ p_j \int_{C_{j-1}}^{C_{j-1}+d_j} (1 - \mathcal{L}_{j-1}(x)\, dx + s_j \int_{C_{j-1}+d_j}^{C_j} (1 - \mathcal{L}_{j-1}(x))\, dx \right\}$$

$$= T \left\{ p_j d_j + s_j (c_j - d_j) \right\} - \varepsilon_j$$

$$= T \left\{ d_j (p_j - s_j) + s_j c_j \right\} - \varepsilon_j$$

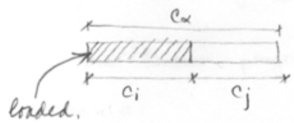
$$= T \left(r_j d_j + s_j c_j \right) - \varepsilon_j \qquad \text{FOR } c_j \geq d_j$$

If $c_j < d_j$

Then it practically becomes a two state unit.

(ii) IF j IS THE SECOND BLOCK OF A 2-STATE UNIT.

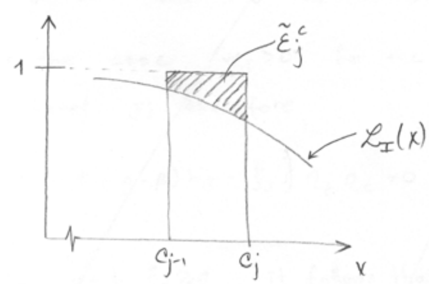
ASSUMING THAT i IS THE FIRST BLOCK TO BE LOADED AND THE TOTAL CAPACITY OF THE 2-STATE UNIT IS C_α:



THE PROBABILITY OF THE UNIT BEING AVAILABLE IS p_α

THE ENERGY AVAILABLE FROM i FOR CHARGING THE STORAGE UNIT IS EQUIVALENT TO THE GENERAL CASE PREVIOUSLY DESCRIBED

WHEN THE SECOND BLOCK IS CHARGED THE AVAILABLE ENERGY FOR CHARGING IS DETERMINED ACCORDING TO THE SHADED AREA:



$$\tilde{\varepsilon}_j^c = p_\alpha T \int_{c_{j-1}}^{c_j} \left(1 - \mathcal{L}_I(x)\right) dx$$

WHERE $\mathcal{L}_I(x) = 1 - F_{\underset{\sim}{I}}(x)$

WHERE $\underset{\sim}{I}$ IS THE EQUIVALENT LOAD RESULTING FROM LOADING ALL UNITS EXCEPT α.

Problem 3

We have the set $\mathcal{S}$ of *SP*'s

$$\mathcal{S} = \{ s_1, s_2, \ldots s_m \}$$

with $\eta_{s_i} \geq \eta_{s_{i+1}} \; \forall i = 1, 2, \ldots (m-1)$.

We adapt the algorithm in order to be able to simulate all of them: