ECE 598HH: Advanced Wireless Networks and Sensing Systems

Lecture 4: OFDM

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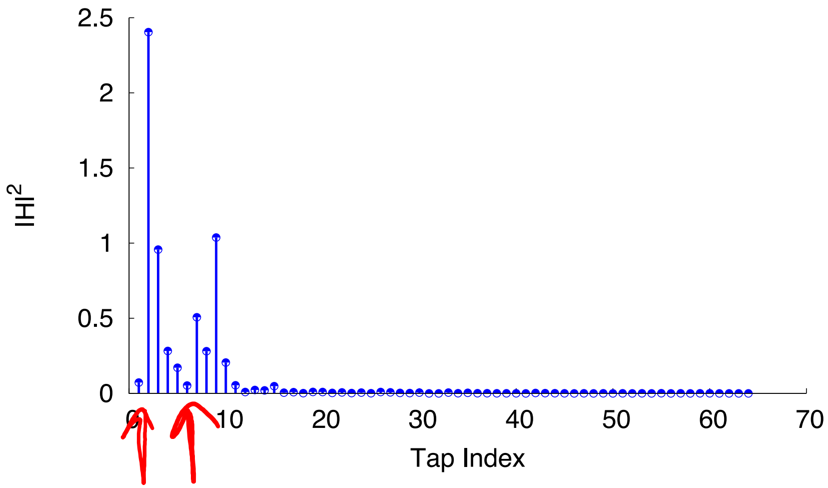
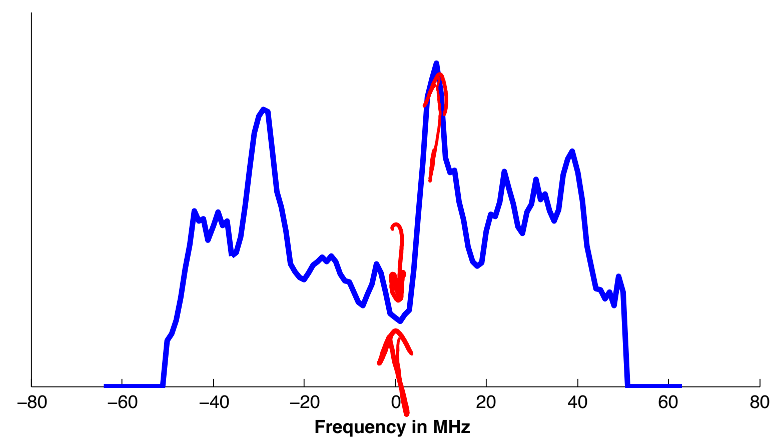
Wireless Channel

$$y(t) = \sum_k h_k x(t - \tau_k) = h(t) * x(t) \Leftrightarrow H(f)X(f)$$





- Multi-tap Channel → ISI (Inter Symbol Interference)
- h is varies with f → Frequency Selective Fading
- $y(t) = \boxed{h(t) * x(t)} \Leftrightarrow \underline{H(f)X(f)}$

$x(t) = I$!

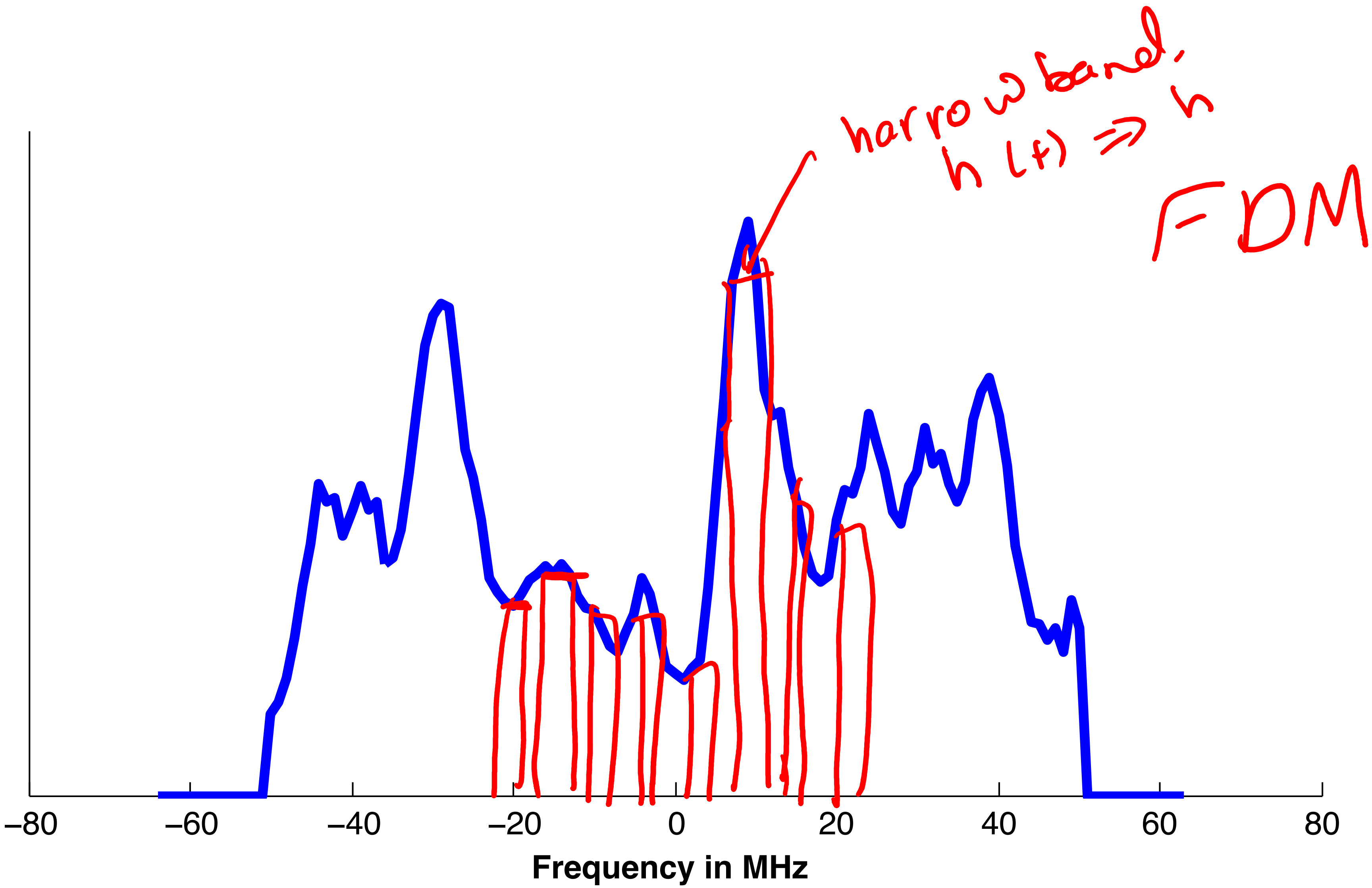
$y(t) = h\, x(t) = I h$

- Solution:

OFDM: Orthogonal Frequency Division Multiplexing

- Idea: transmit symbols in frequency not time.

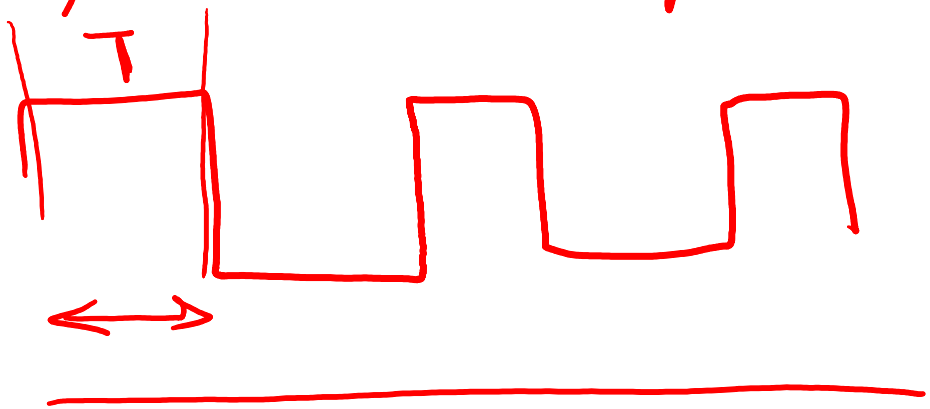
Transmit In Frequency Domain

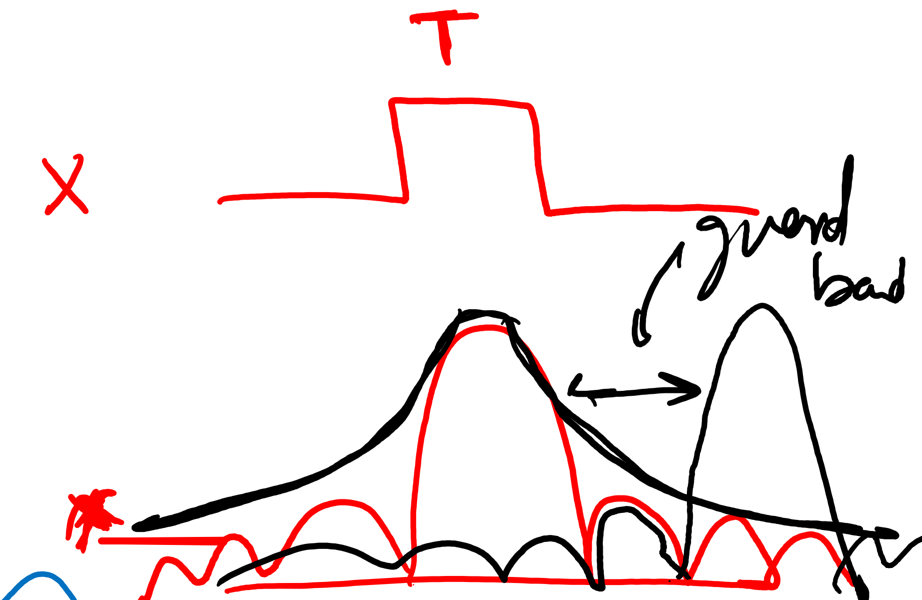


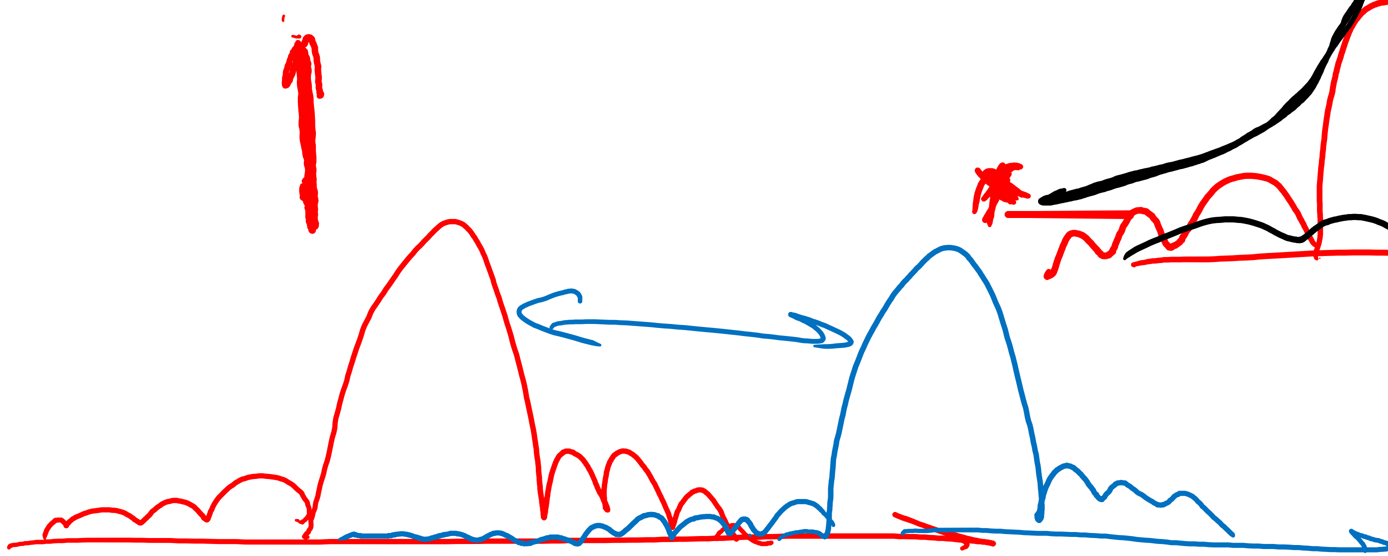
Orthogonality

$$\text{sinc}(x) = \frac{\sin(x)}{x}$$

symbol $\Rightarrow$ Time period T





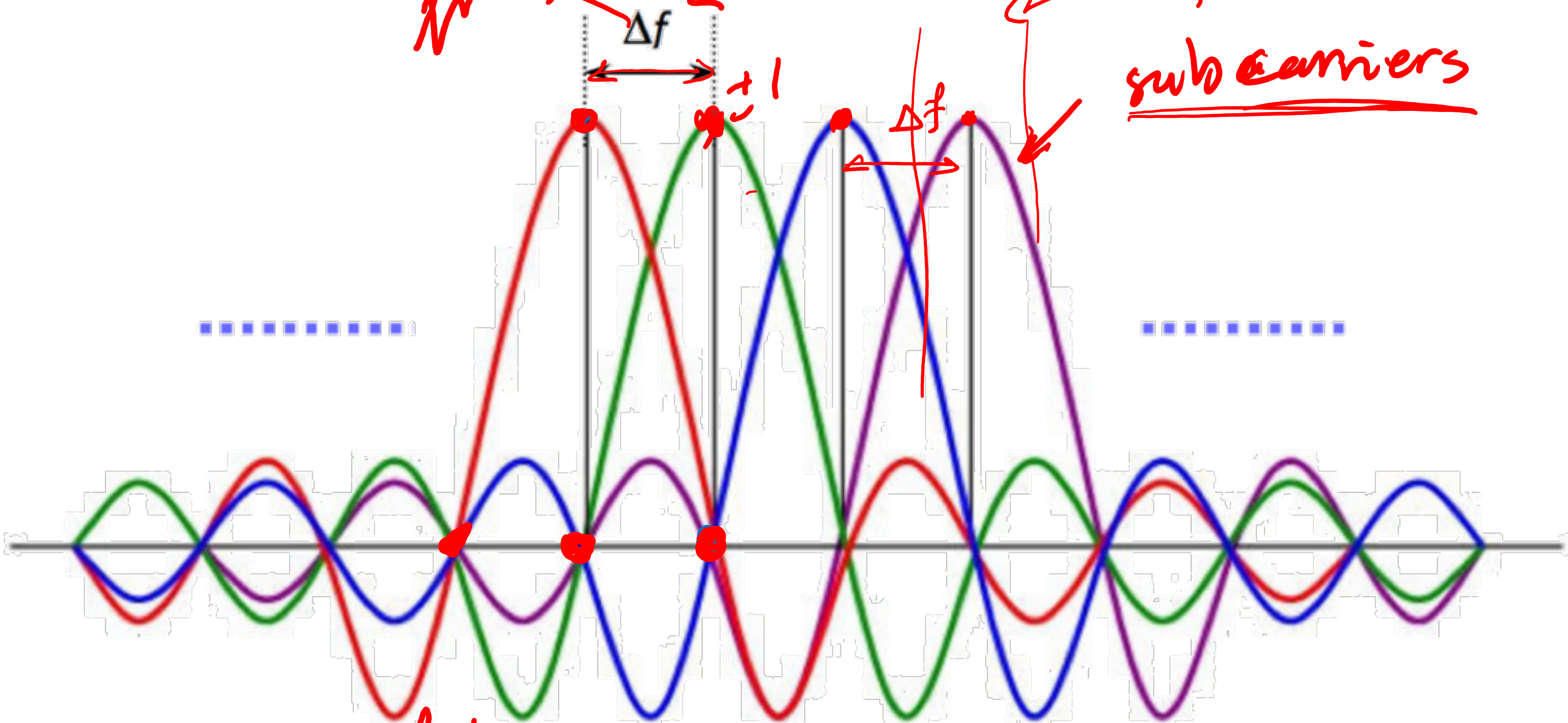


Orthogonality

$\frac{B}{N} \leftarrow \Delta f \ll$ Guard Band

bin

subcarriers



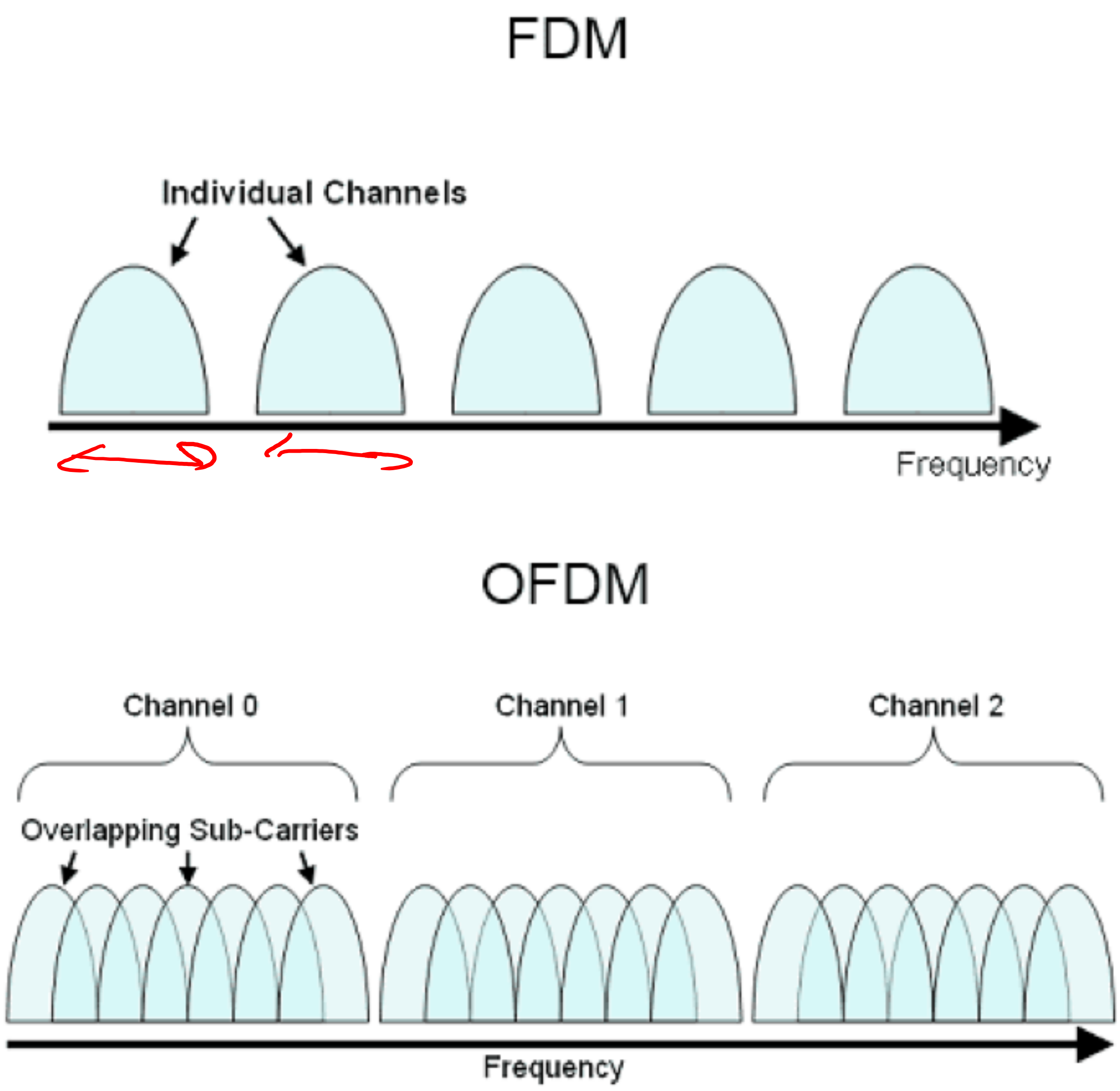
$+1$

Δf

$x(t) \times e^{j2\pi f_c t} \longrightarrow$

2.4 GHz : carrier frequency

Orthogonality



Discrete Fourier Transform

$$x(t) \Rightarrow X(f) = \frac{1}{N}\sum_{t=1}^{N} x(t)\, \underbrace{e^{-j2\pi f t / N}}$$

$$X(0) = \frac{1}{N}\sum x(t)$$

IDFT

$$x(t) = \sum_{f=1}^{N} X(f)\, e^{j2\pi f t / N}$$

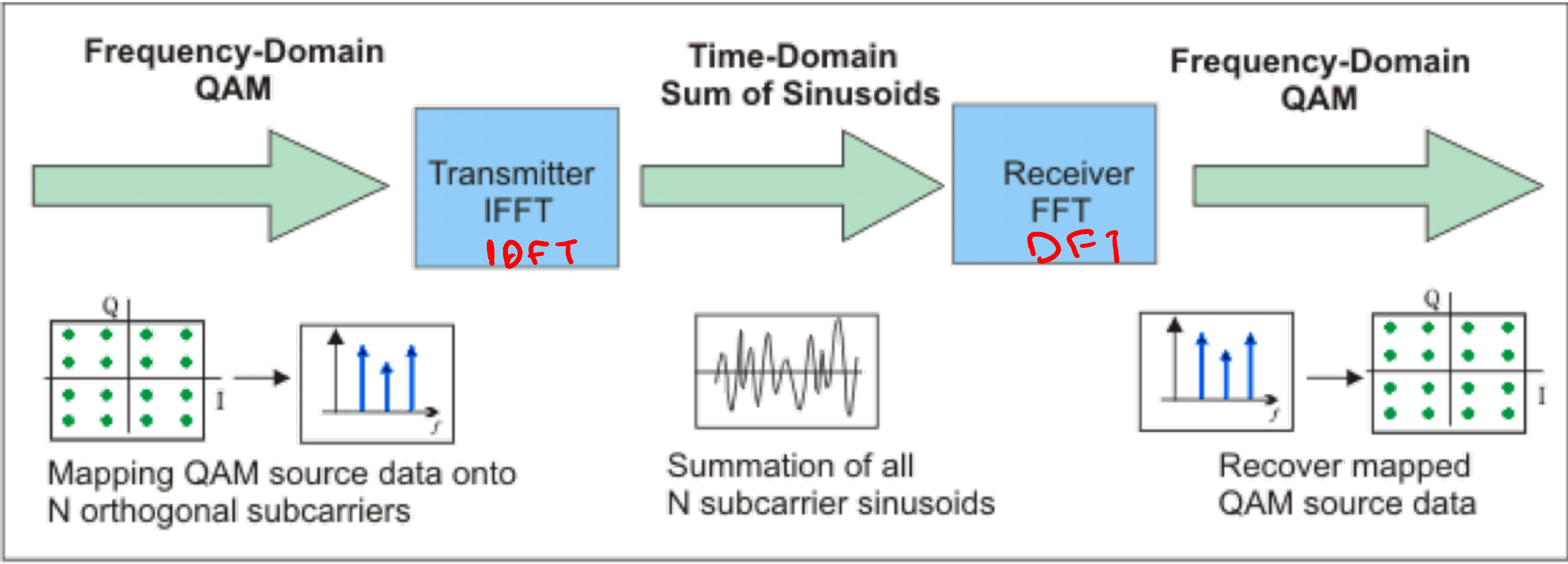
Bits ⇒ modulate I) ⇒ $X(f)$ ⇒ $x(t) = \sum_{f=1}^{N} X(f)\, e^{j2\pi f t / N}$

- Band width B ⇒ $\Delta f = \underbrace{\frac{B}{N}}_{\text{width of bin}}$ bandwidth

$$\Delta f = \frac{1}{T} \Rightarrow B \text{ sample/sec}$$

$$\text{Time} = \frac{N}{B} = T$$

Orthogonal Frequency Divison Multiplexing



Simplified OFDM System Block Diagram

DFT $\Rightarrow O(N^2)$

FFT $\Rightarrow O(N \log N)$

OFDM Symbol

bits 01011 $\rightarrow$ Modulation

$\underbrace{+1 \ -1 \ \ldots \ +1}_{N}$

$X(f)$

$\rightarrow$ IDFT $\rightarrow$ $\underbrace{x(1) \ldots N(N)}_{N}$ $\rightarrow$ TX

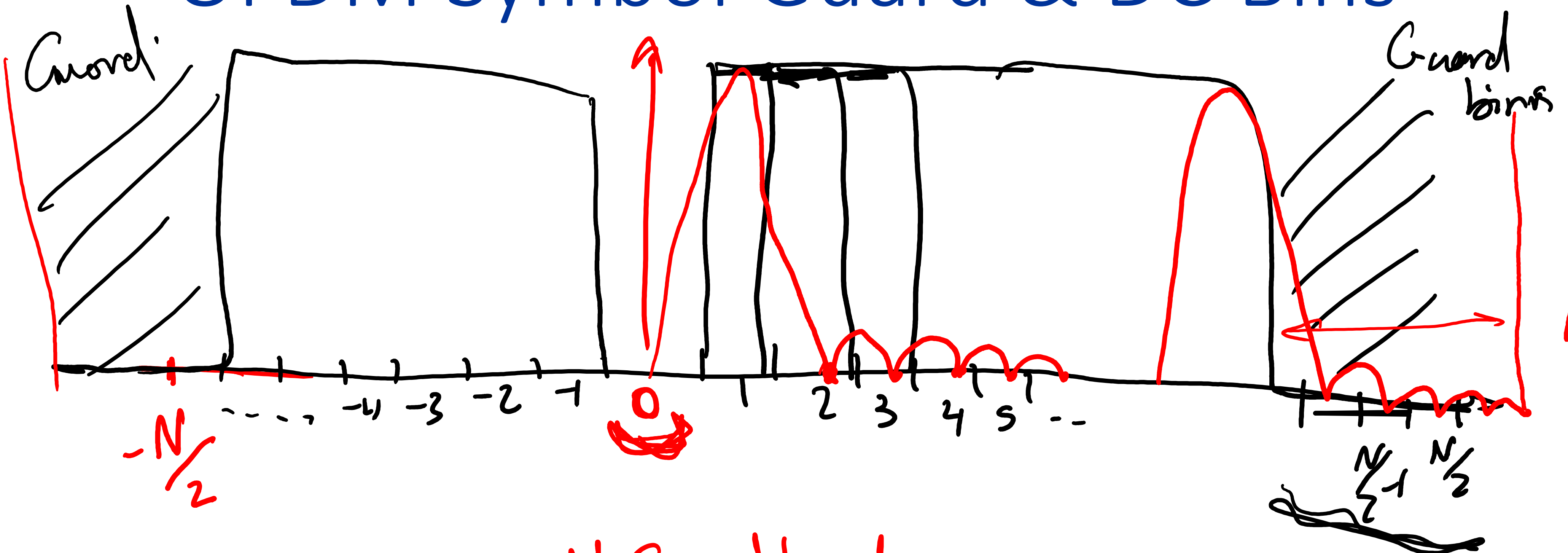
OFDM symbol

$H(f) \cdot X(f)$

RX $h(t) * x(t)$ — DFT — $+1 \ -1 \ +1 \ \ldots \ +1$ $\Rightarrow$ $\frac{1}{H(f)}$ $\Rightarrow$ decode

$H(f_1) \ H(f_2) \ \ldots \ H(f_N)$

OFDM Symbol Guard & DC Bins



N bins ⇒ N Guard bands

OFDM ⇒ 2 Guard bands.

DFT : $0 \rightarrow N-1$

$-\frac{N}{2}-1 \rightarrow N/2$

0 bin not used!

DC of circuit corrupts data bits sent

⇒ Do not use 0 bin

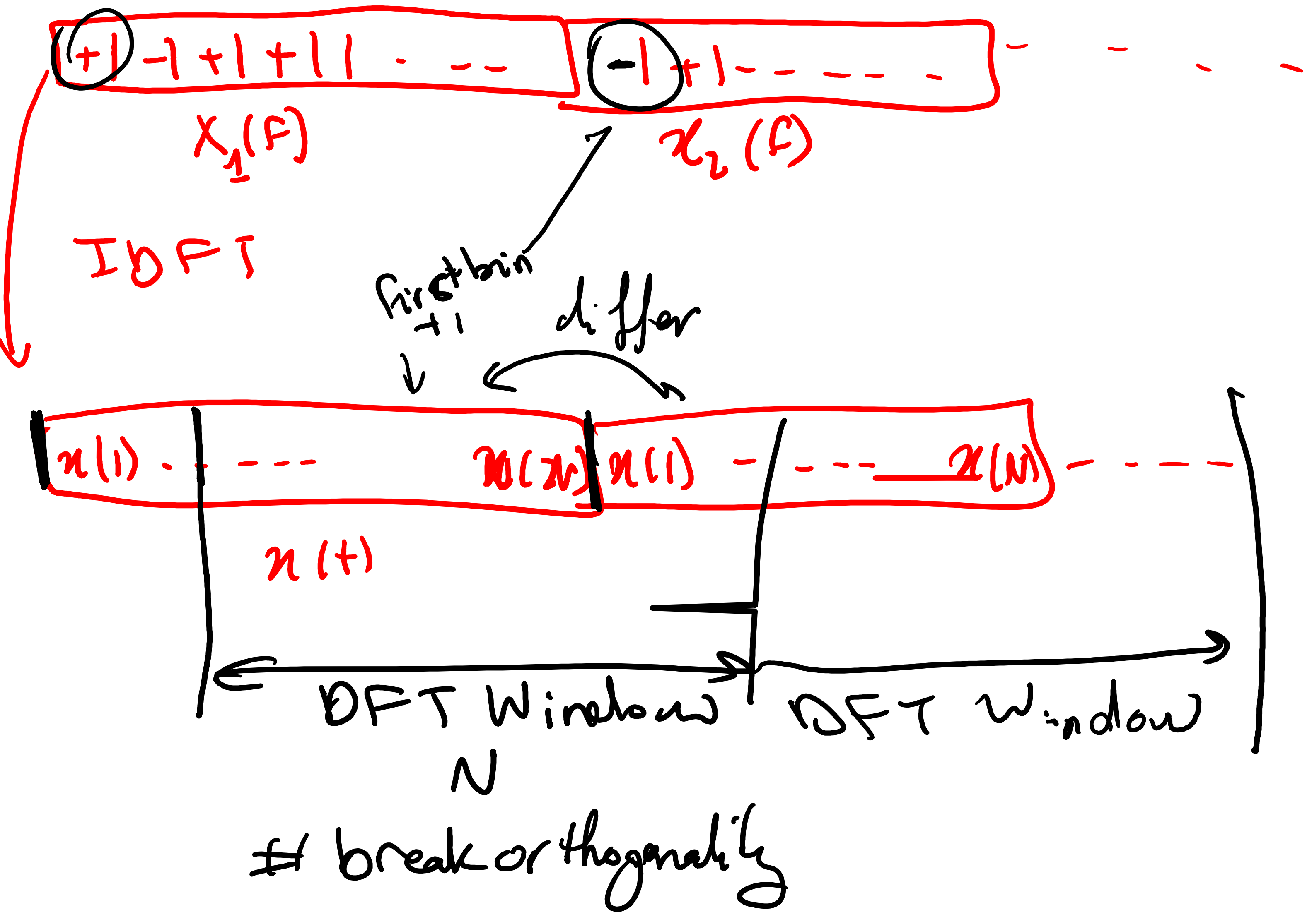
Channel Estimation

Preamble : 1/2 OFDM symbols.

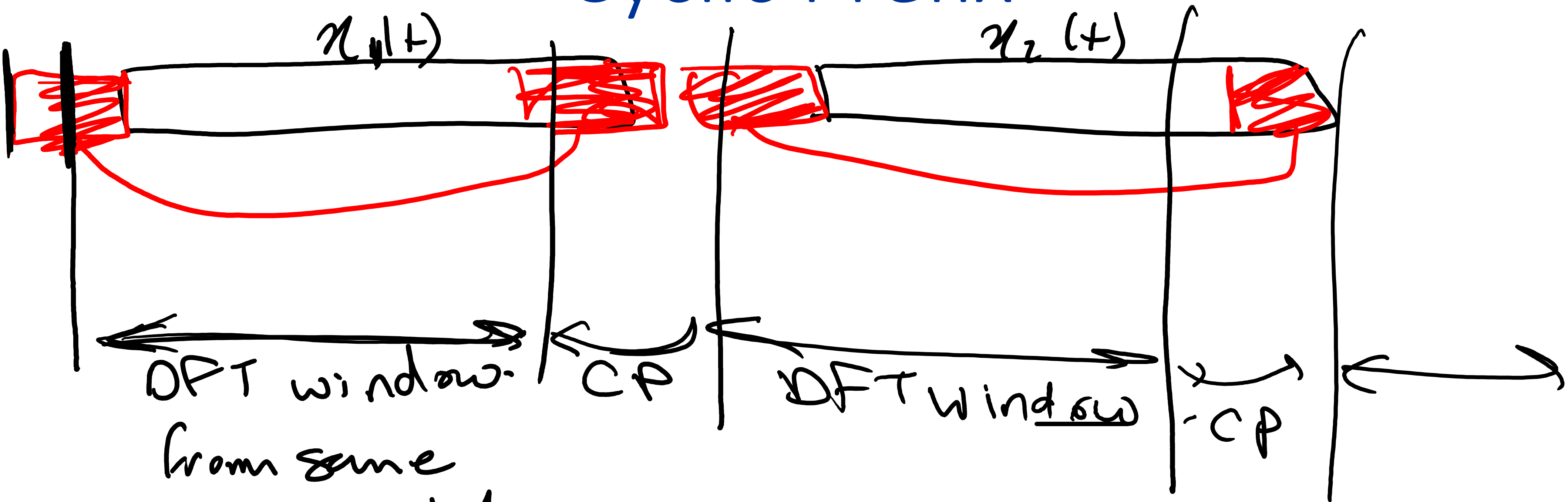
$$\underbrace{+1 \; -1 \; +1 \; +1 \; \cdots}_{\text{know sequens}} \longrightarrow \frac{+h(1) \; -h(2) \; \cdots}{+1 \quad -1 \quad \cdots}$$

$= h(1), h(2) \cdots$

FFT Window Synchronization



Cyclic Prefix



from same OFDM symbol

$$x(t) \Rightarrow x(t - \tau \bmod N)$$

$$X(f) \Rightarrow X(f)\, e^{-j 2\pi \frac{f\tau}{N}}$$

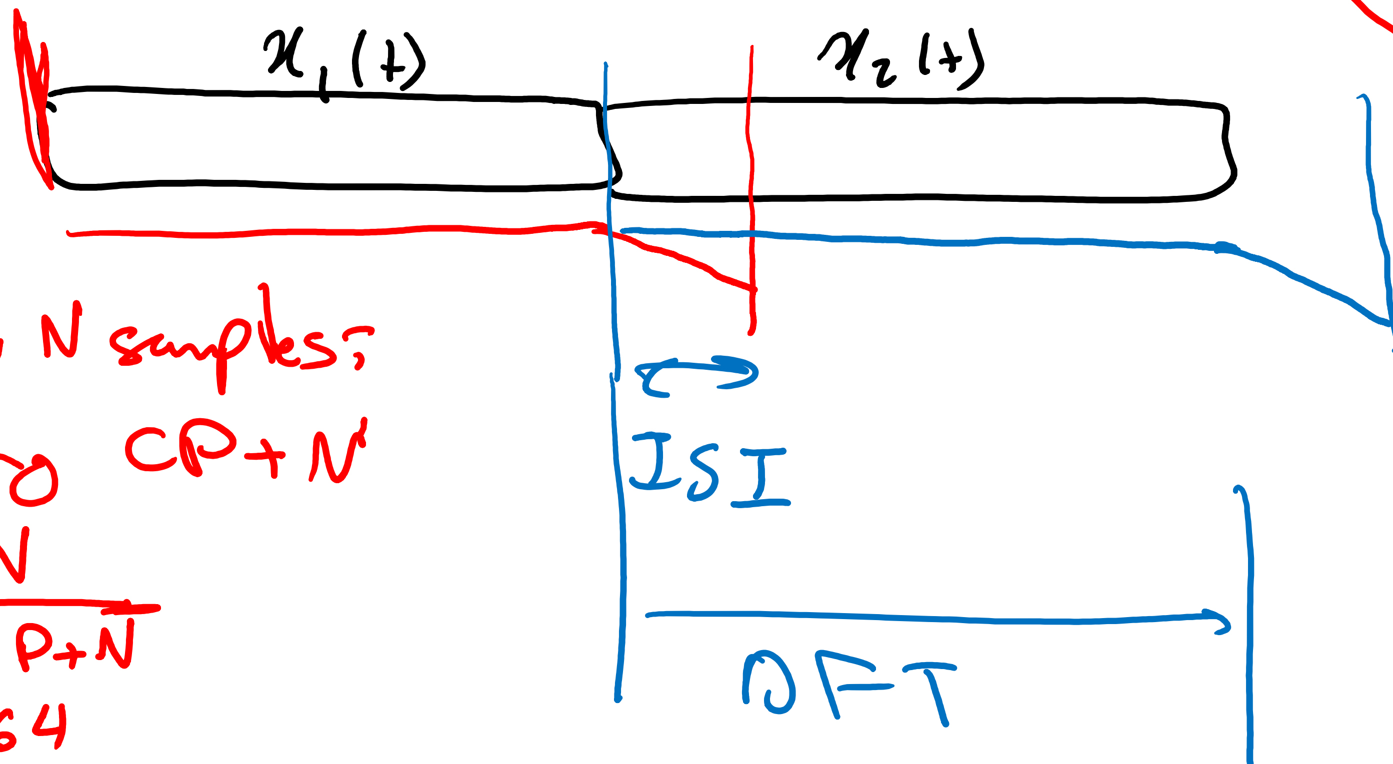
DFT property of circular shift

Cyclic Prefix

synchronized

ISI

Ensures that all samples come from same OFDM symbol

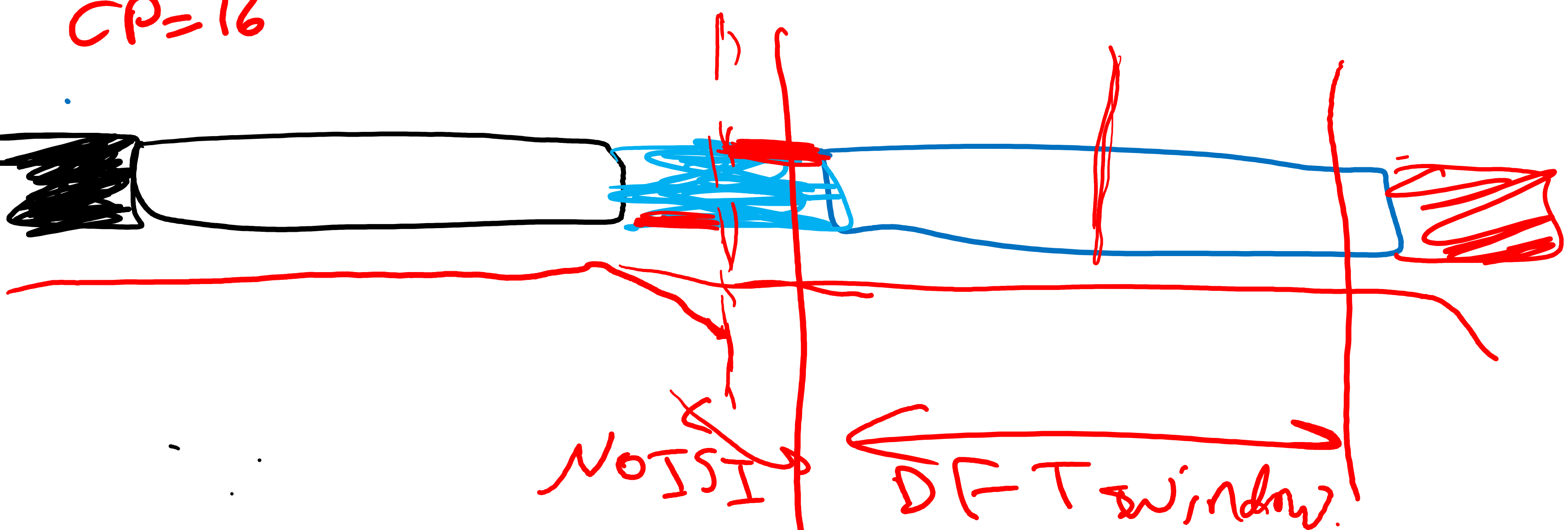


For every N samples; sending CP+N

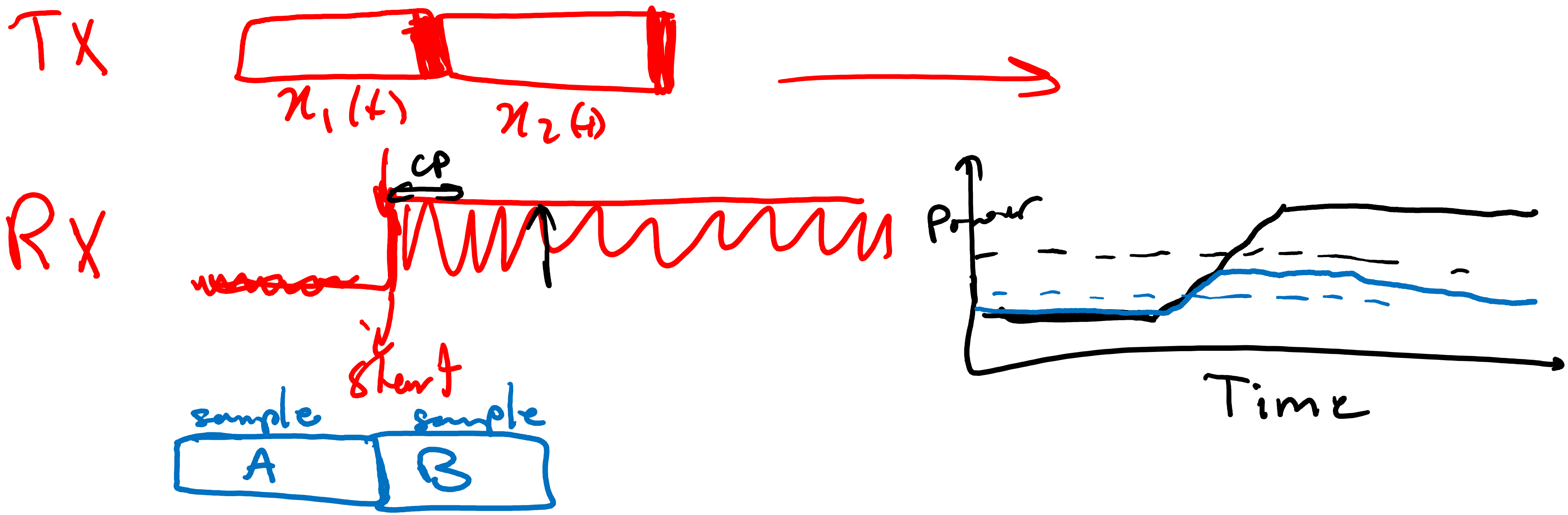
$B \times \frac{N}{CP+N}$

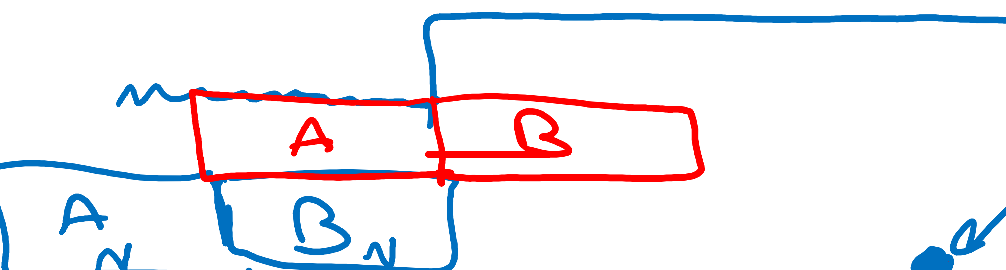
$N = 64$

$CP = 16$

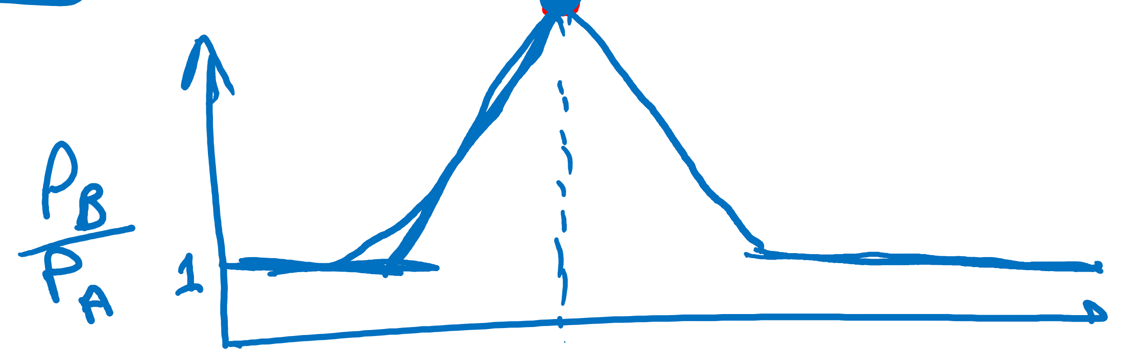


Packet Detection: Sliding Window

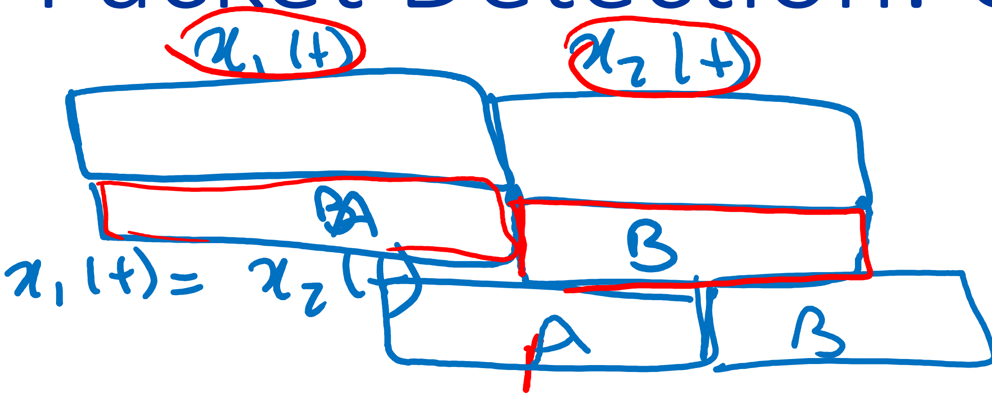


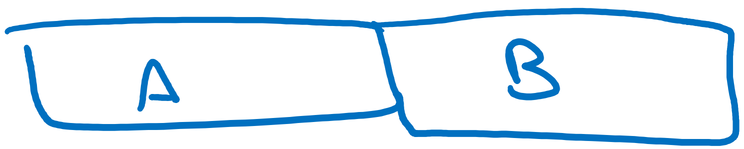


$$\frac{P_B}{P_A} = 1$$



Packet Detection: Cross Correlation



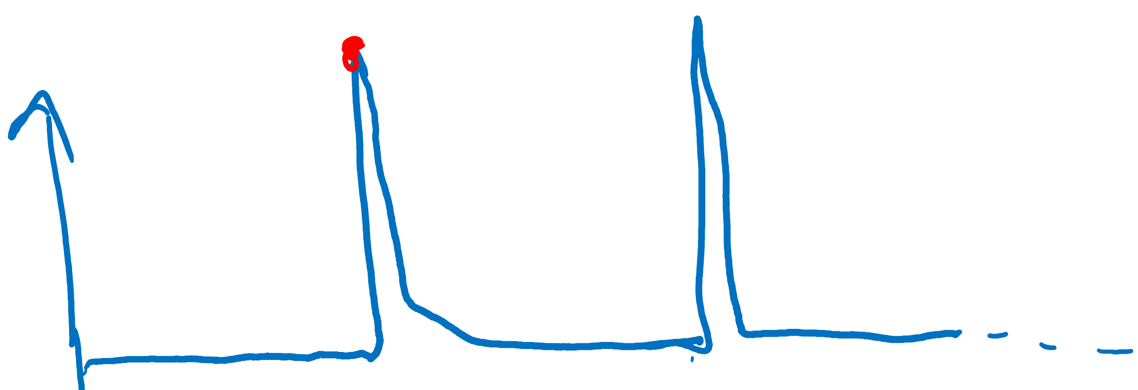


$\sum_N A(t) \cdot B^*(t)$ → outside. $\sum_N n(t) \cdot n^*(t) = \downarrow$

→ B inside A out $\sum_N n(t) \cdot x_1(t) =$

→ B & A $\sum x_1(t) \cdot x_2(t) \cdot \nearrow$

correlation



CFO: Carrier Frequency Offset

$x(t) \longrightarrow x(t)\, e^{-j2\pi \boxed{f_c} t} = y(t)$ } up conversion

At RX: $y(t) \times e^{j2\pi f_c t} = x(t)$ } down conversion

$y(t) \times e^{j2\pi(f_c+\Delta f_c)t} = x(t)\, e^{j2\pi \Delta f_c t}\, \underbrace{e^{j2\pi \frac{d}{\lambda}}}_{h}$

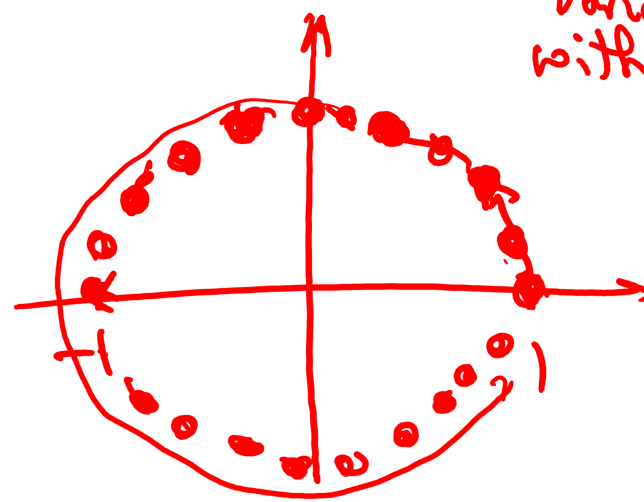
variable with time

h

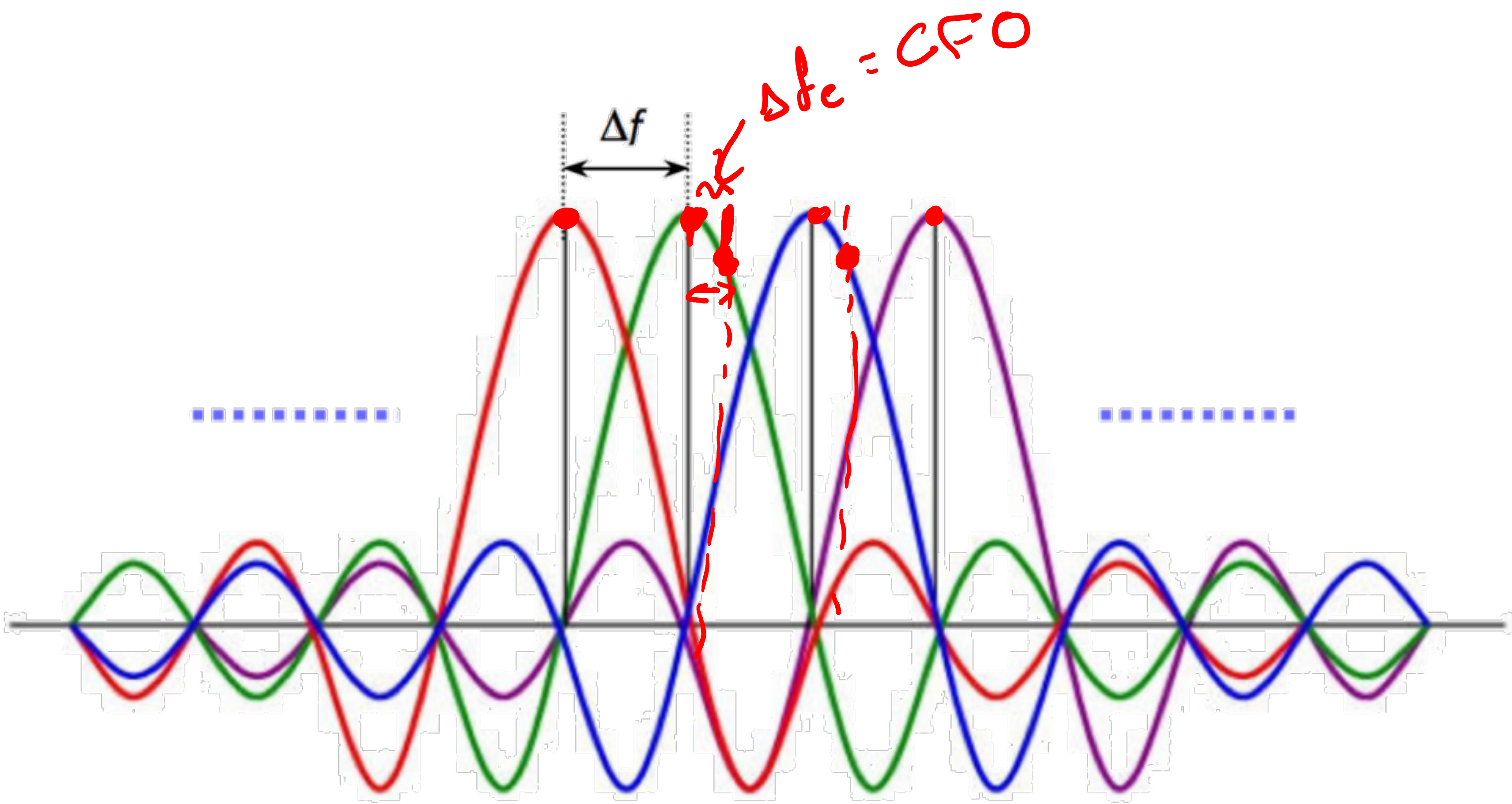
constant with time.

+1   −1

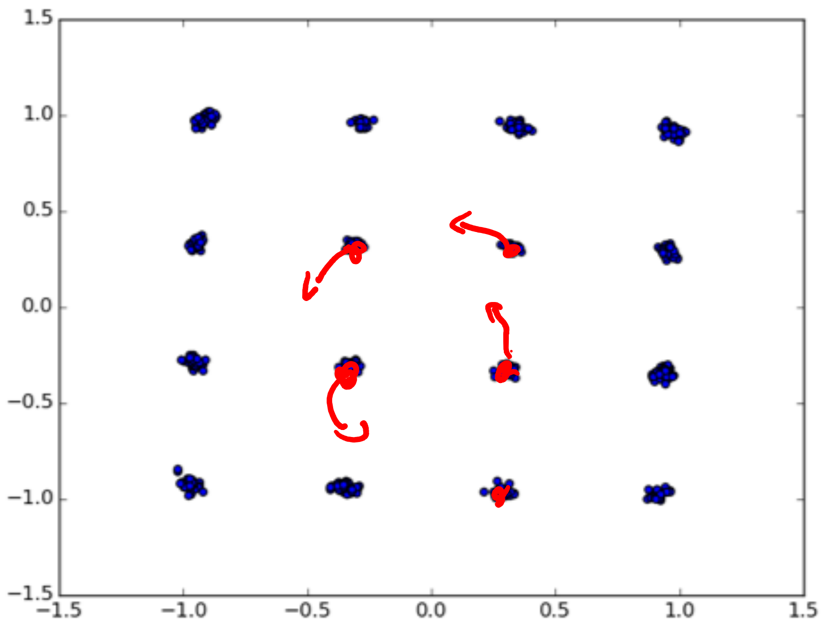
$e^{j2\pi \Delta f_c}$ $e^{j2\pi 2\Delta f_c}$. - - - . . -

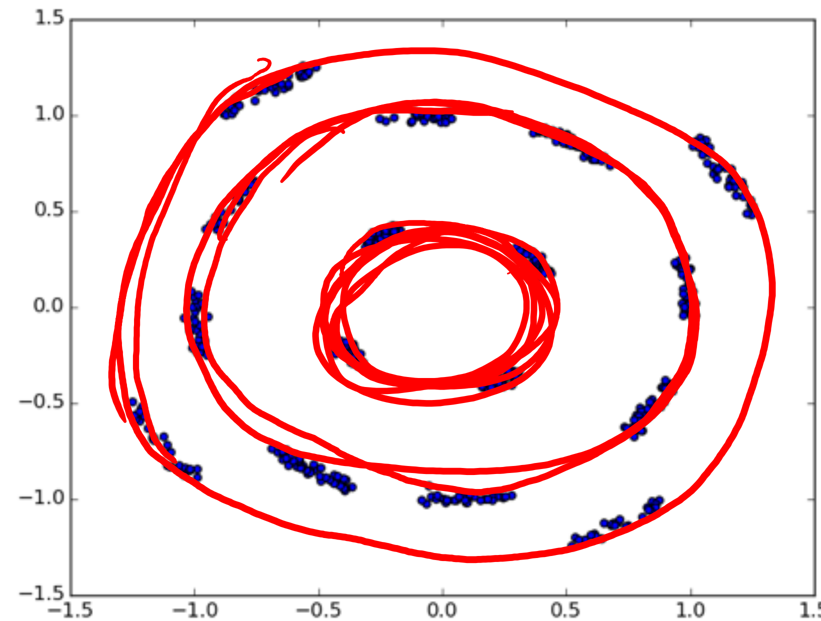


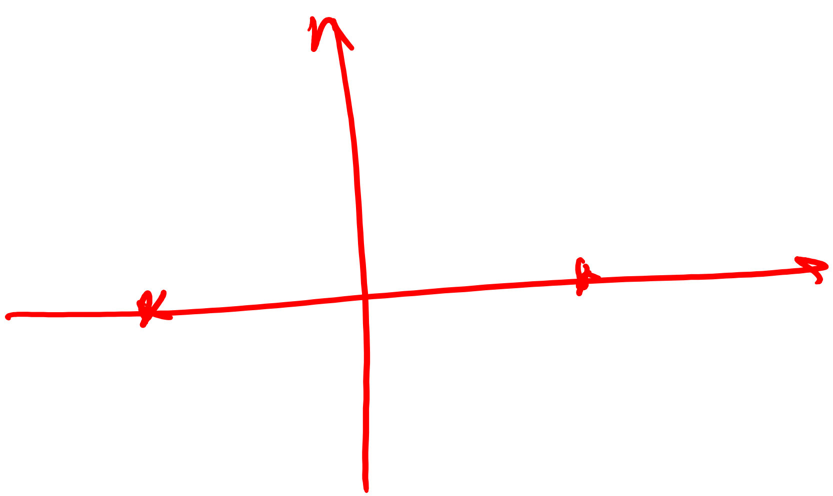
CFO: Carrier Frequency Offset

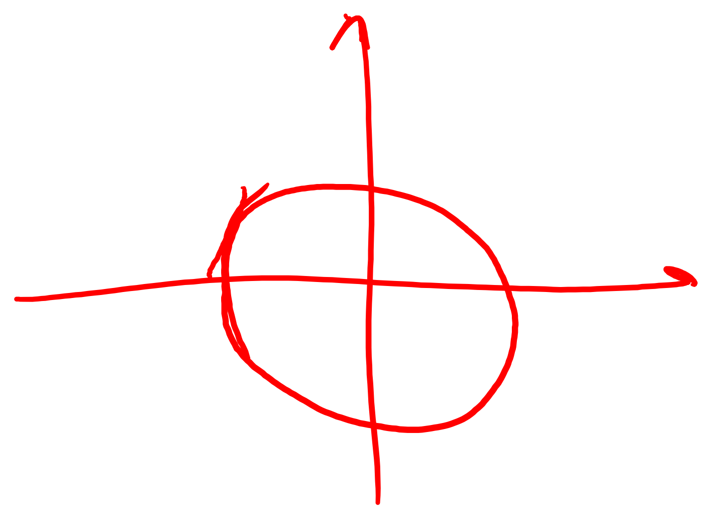


CFO: Carrier Frequency Offset









CFO Estimation and Correction

$x_1(t) + n(t)$ $\quad$ $x_2(t) + n(t)$ $\quad$ $x_1(t) = x_2(t)$

$$y_1(t) = x_1(t)\, e^{-j2\pi \Delta f_c \frac{t}{N}}$$

$$y_2(t) = x_2(t)\, e^{-j2\pi \Delta f_c \frac{(t+N)}{N}}$$

$$A = \sum_{t=1}^{N} y_1(t) \cdot y_2^*(t) = \sum_t x_1(t)\, x_2^*(t)\, e^{-j2\pi \Delta f_c \frac{t}{N}} \cdot e^{j2\pi \Delta f_c \frac{(t+N)}{N}}$$

$$A = \sum_t |x_1(t)|^2 e^{j2\pi \Delta f_c} + n(t)$$

$$\measuredangle A = 2\pi \Delta f_c \quad \} \text{ coarse estimate.}$$

CFO Estimation and Correction

$\widetilde{\Delta f_c}$

correct: take all symbols $\times\, e^{\frac{j 2\pi \widetilde{\Delta f_c}\, t}{N}}$

First 2: use for CFO estimation

All remaining symbols ⇒ correct CFO

left with

$x(t)\, e^{-\frac{j2\pi \delta f_c t}{N}}$

$\Delta f_c = \underbrace{\widetilde{\Delta f_c}}_{\text{coarse CFO}} + \underbrace{\delta f_c}_{\text{residual CFO}}$

$t \uparrow \Rightarrow \delta f_c t \uparrow$

Sampling Frequency Offset

Bandwidth B $\Rightarrow$ Sampling $T = \frac{1}{B}$ sec

$B = 1$ MHz $\Rightarrow$ sample every $T = 1\mu$sec

$$x(t) = \sum_{f_i} \underbrace{X(f_i)}_{\text{mod bits}} e^{j2\pi f_i t} \Bigg\} \Rightarrow 1 \text{ sample every } T \text{ sec}$$

$$x(t_0 + nT) = \sum_{f_i} X(f_i)\, e^{j2\pi f_i (t_0 + nT)}$$

$$\left.\begin{aligned} T_{RX} &= T' \\ T_{TX} &= T \end{aligned}\right\} \Delta T$$

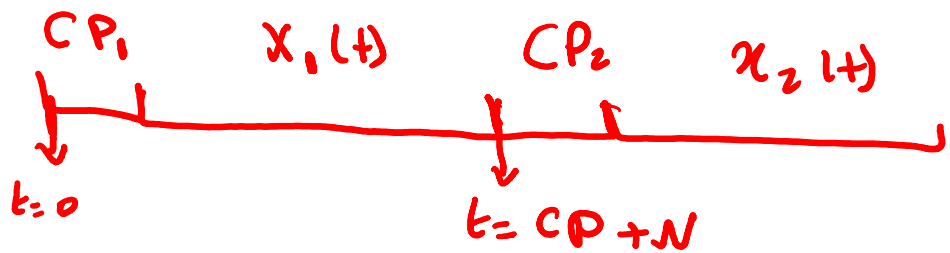
At RX : $T_{RX} \neq T_{TX}$ ← 1μse

T_{RX}: 1μse $+ 1$ n sec

$$x(t_0 + nT) = \sum_{f_i} X(f_i)\, e^{j2\pi f_i (t_0 + nT + n\Delta T)}$$

Phase Tracking

Residual CFO   SFO



CFO: $\sum_i X(f_i)\, e^{j\frac{2\pi f_i t}{N} + \frac{2\pi \delta f_c t}{N}}$

$$\sum_i X(f_i)\, e^{j\frac{2\pi f_i (t+CP+N)}{N} + \frac{2\pi \delta f_c (t+CP+N)}{N}}$$

FFT

$$X(f_i) = \pm 1\, e^{\underbrace{j\frac{2\pi f_i (CP+N)}{N}}_{\text{channel } H} + \frac{2\pi \delta_c (CP+N)}{N}}$$

$\forall f_i \Rightarrow$ Residual CFO adds a phase $\frac{2\pi \delta f_c (CP+N)}{N}$

Phase Tracking

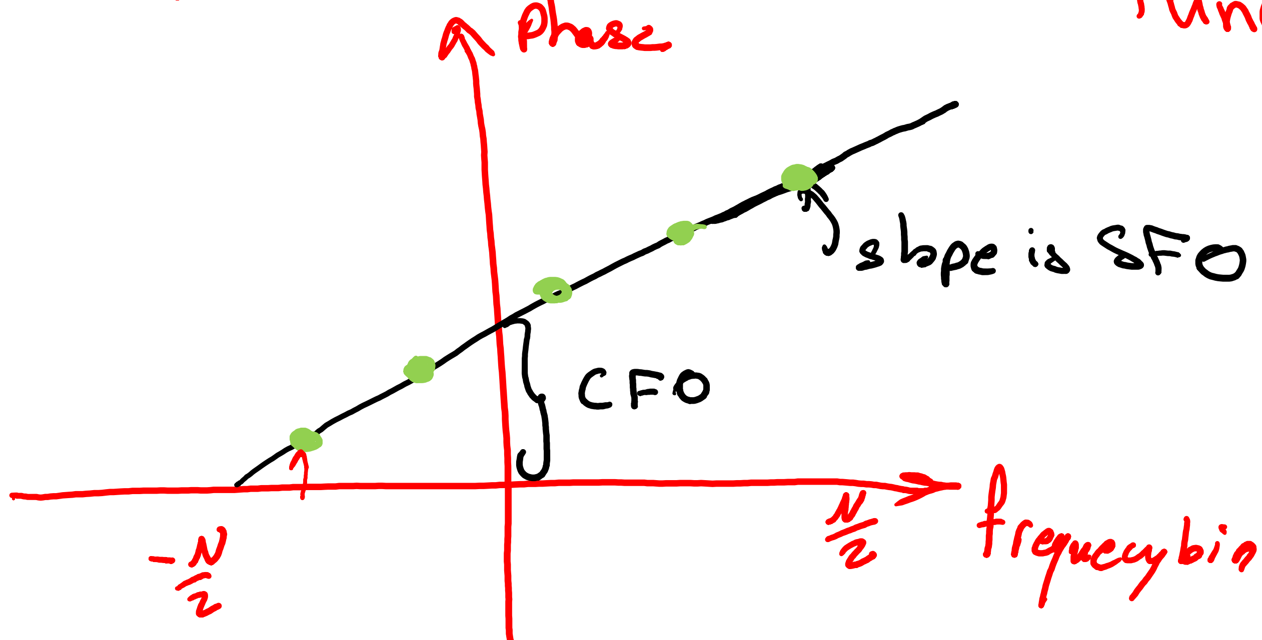
$$\overbrace{\qquad}^{CP_1}\overbrace{\qquad\qquad\qquad}^{x_1(t)}\overbrace{\qquad}^{CP_2}\overbrace{\qquad\qquad\qquad}^{x_2(t)}$$

① $$\sum_i X(f_i)\, e^{j2\pi f_i (t + nT + n\Delta T)}$$

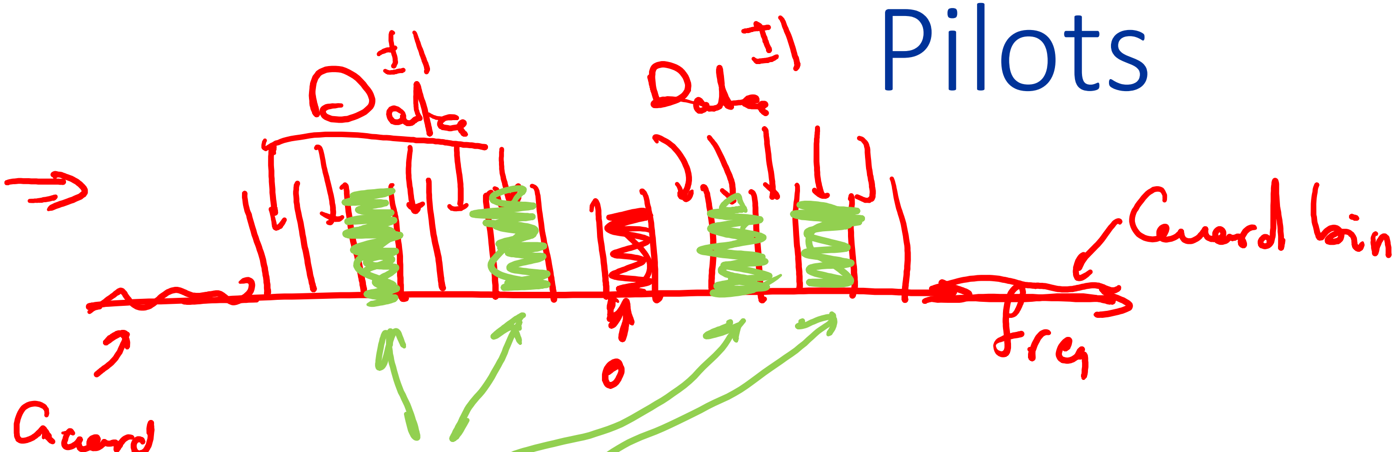
② $$\sum_i X(f_i)\, e^{j2\pi f_i (t + nT + n\Delta T + (CP+N)T + (CP+N)\Delta T)}$$

$$\Downarrow$$

$$X(f_i) = \pm 1 \quad e^{j2\pi f_i (CP+N)T} \quad e^{j2\pi \underbrace{f_i (CP+N)\Delta T}_{\text{Function of } f_i}}$$



Pilots

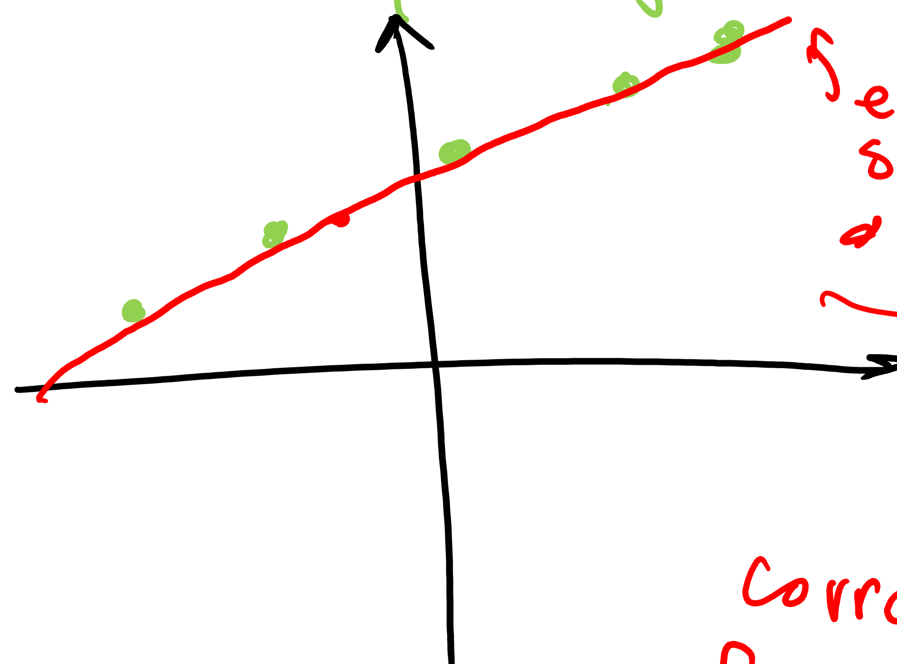


Send known bits : Pilots {BPSK}

Receive Pilots:

$$\angle \frac{\text{Pilots}}{\text{known bits}}$$

linear regression



estimate slope & y-intercept

correct phase for all data bins.

How many subcarriers?

N ??

WiFi : N = 64
CP : 16

Smaller better ⇒ computation easier (smaller FFTs)

Smaller worse ⇒ N=16, CP=16 ⇒ overhead of CP 50%

Larger better ⇒ overhead: N=128, CP16 ⇒ ⇓

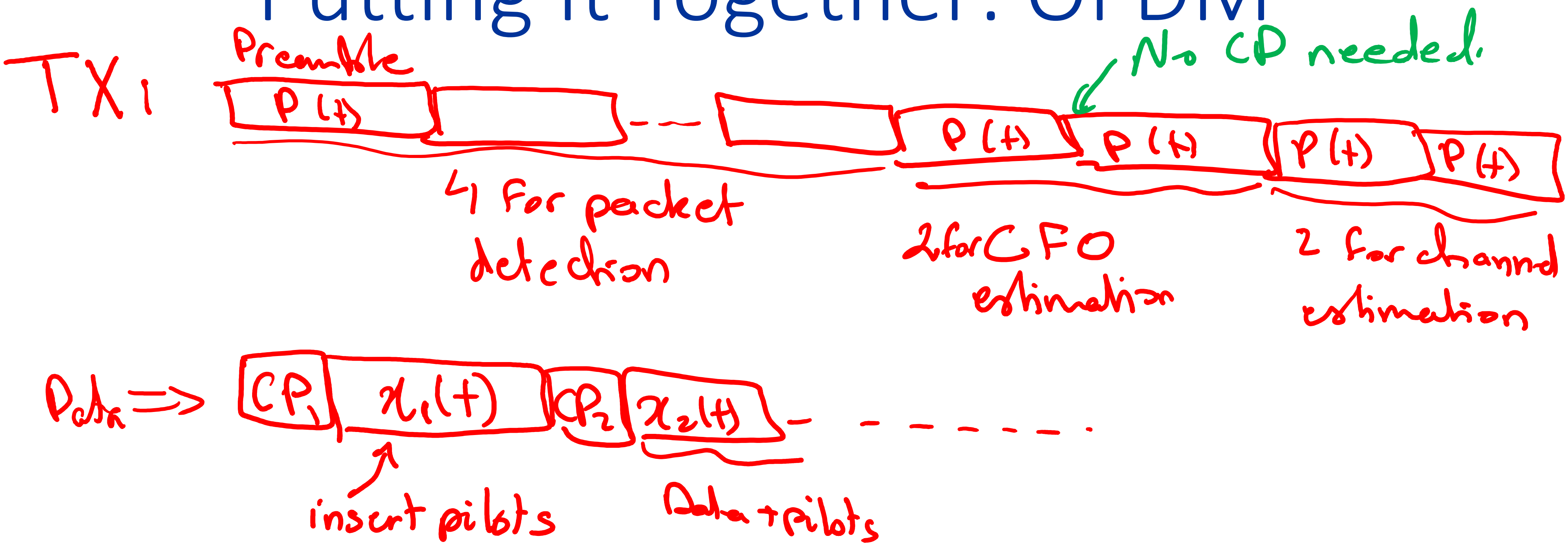
Larger worse ⇒ computation

⇒ wrapping ~~around~~ 2π

⇒ bin size $= \frac{B}{N}$ ⇒ so small

want $\frac{B}{N} \gg$ CFO

Putting It Together: OFDM



RX: • waste 2-4 symbols on packet detection

• use 2 ⇒ CFO estimation ⇒ correct coarse CFO ∀ remaining symbols

• use 1-2 ⇒ estimate H

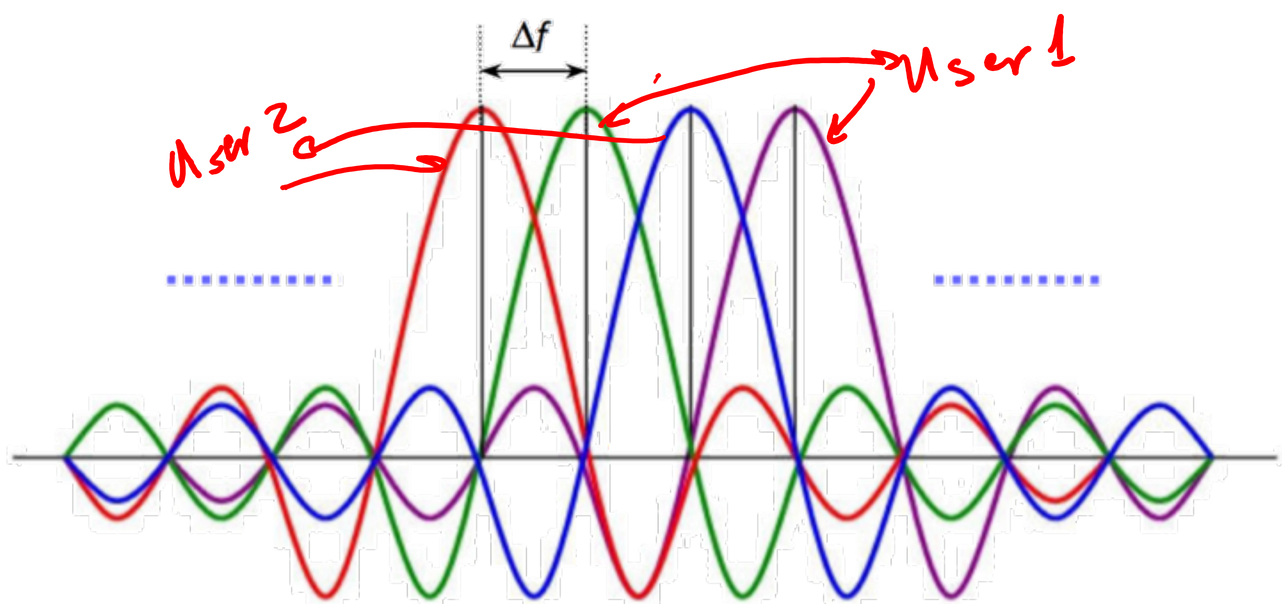
Putting It Together: OFDM

RX : Data :

1. Remove CP
2. Take FFT
3. Correct for channel H
4. Estimate residual CFO & SFO from pilots
5. Correct for residual CFO & SFO
6. Add residual CFO & SFO to $H \times e^{j 2\pi \epsilon_{CFO} + SFO}$
7. Decode bits

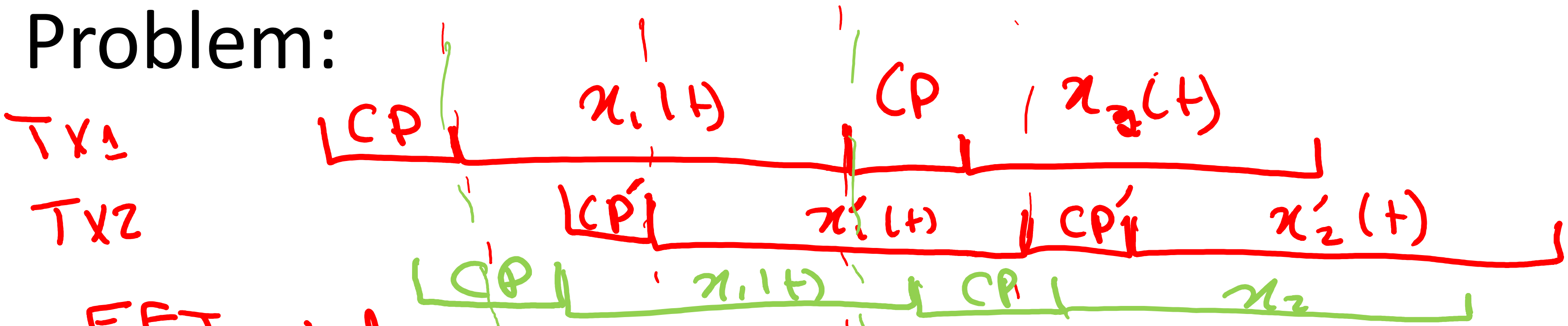
OFDMA: Orthogonal Frequency Division Multiple Access

- Use OFDM: Assign different subcarriers to different users.
- More efficient than FDMA since no guard bands are needed
- Requires Time Synchronization



FICA: Fine Grained Channel Access

- Problem:



FFT window: will not correspond to the same TX symbols.

- Solution:

Make CP very large

Huge Overhead. ⇒ Make symbol large

Increase N

FICA: Fine Grained Channel Access

- Cons: - phase tracking wraps around 2π

  - bins becomes < CFO

    same bandwidth, use a much larger N

    $\text{bin width} = \frac{B}{N} < CFO$

    - Inter carrier iterference
    - shift bins around.

  - Computation