ECE 598HH: Advanced Wireless Networks and Sensing Systems

Lecture 8: MIMO Part 1

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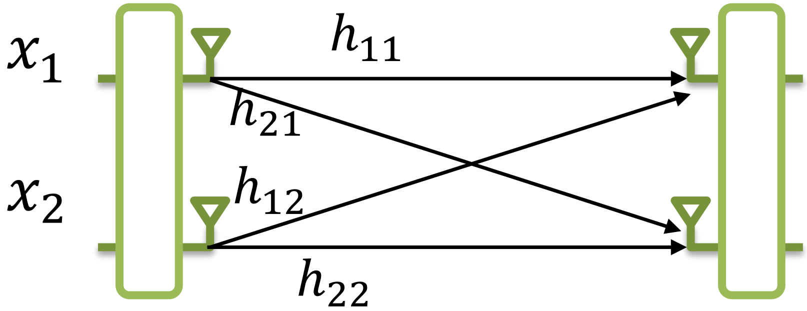
MIMO: Multiple Input Multiple Output

So far: single input single output

This lecture: multiple input multiple output

Increase capacity of channel using multiple transmit and receive antennas.

MIMO: Multiple TX-RX streams



$$y_1 = h_{11}x_1 + h_{12}x_2$$

$$y_2 = h_{21}x_1 + h_{22}x_2$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \Rightarrow \mathbf{y} = \mathbf{Hx} + \mathbf{n}$$

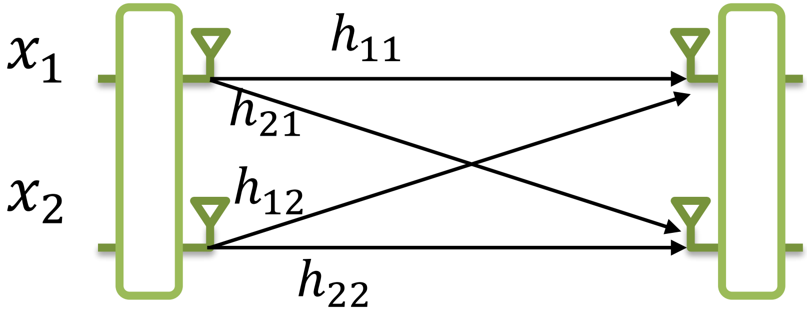
How to recover x_1 and x_2?

Estimate $\mathbf{H}$, compute $\mathbf{H}^{-1}$ and invert the channel!

$$\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{y} = \mathbf{H}^{-1}\mathbf{Hx} + \mathbf{H}^{-1}\mathbf{n} = \mathbf{x} + \mathbf{H}^{-1}\mathbf{n}$$

Transmit 2 packets at the same time!

MIMO: Multiple TX-RX streams



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For N antennas, $\mathbf{H}$ is $N \times N$ matrix → $O(N^3)$

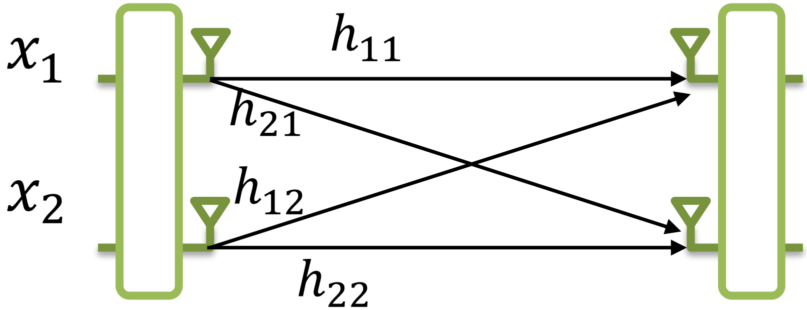
How to recover x_1 and x_2?

Estimate $\mathbf{H}$, compute $\mathbf{H}^{-1}$ and invert the channel!

$$\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{y} = \mathbf{H}^{-1}\mathbf{H}\mathbf{x} + \mathbf{H}^{-1}\mathbf{n} = \mathbf{x} + \mathbf{H}^{-1}\mathbf{n}$$

Noise amplification

MIMO: Vector Representation



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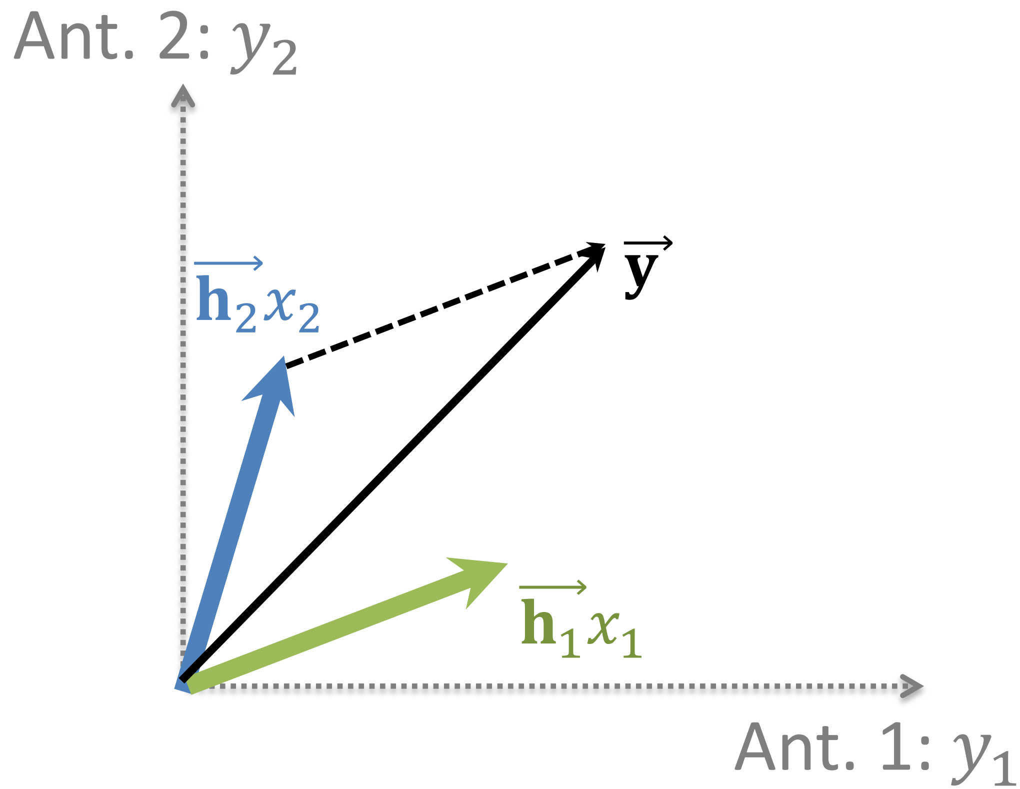
How to recover x_1 and x_2?

$$\mathbf{y} = \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} x_1 + \begin{bmatrix} h_{12} \\ h_{22} \end{bmatrix} x_2 = \overrightarrow{\mathbf{h}_1} x_1 + \overrightarrow{\mathbf{h}_2} x_2$$

MIMO: Antenna Space

$$\vec{\mathbf{y}} = \overrightarrow{\mathbf{h}_1} x_1 + \overrightarrow{\mathbf{h}_2} x_2$$



MIMO: Antenna Space

$$\vec{\mathbf{y}} = \vec{\mathbf{h}}_1 x_1 + \vec{\mathbf{h}}_2 x_2$$

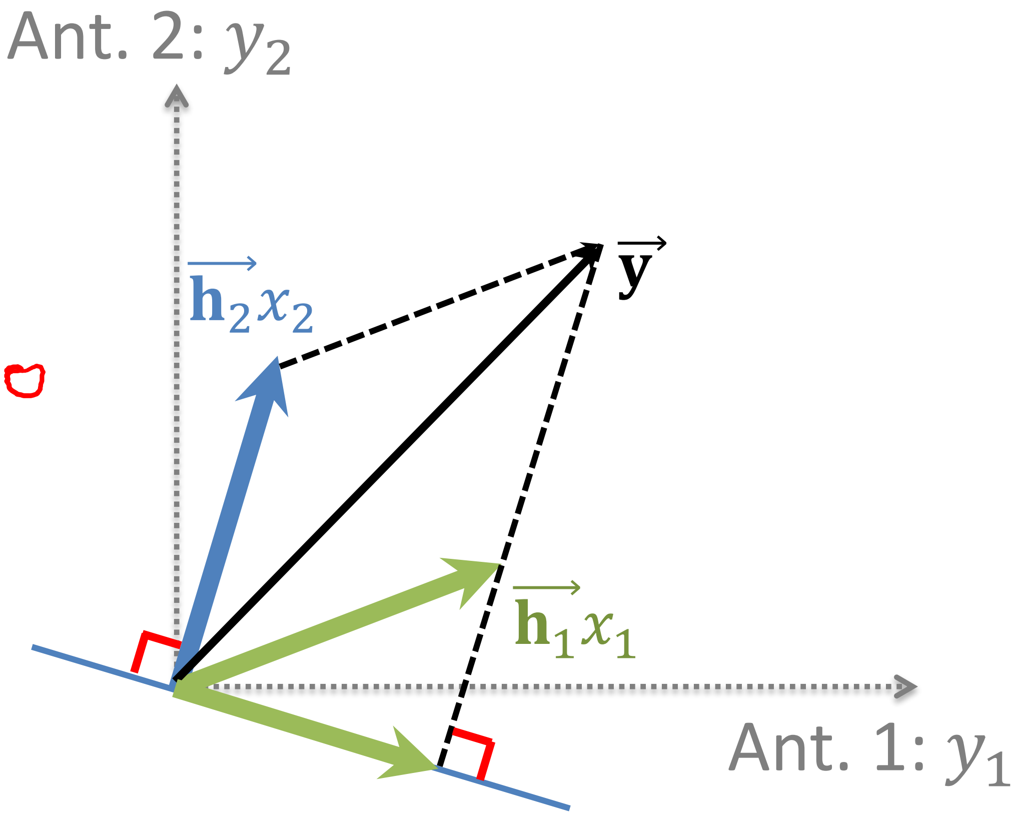
To decode x_1, project on a vector $\vec{\mathbf{h}}_2^{\perp}$ orthogonal to $\vec{\mathbf{h}}_2$

$$\vec{\mathbf{h}}_2^{\perp} \vec{\mathbf{y}} = \vec{\mathbf{h}}_2^{\perp} \vec{\mathbf{h}}_1 x_1 + \vec{\mathbf{h}}_2^{\perp} \vec{\mathbf{h}}_2 x_2$$

$$= \vec{\mathbf{h}}_2^{\perp} \vec{\mathbf{h}}_1 x_1$$

$$\mathbf{y} = \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} x_1 + \begin{bmatrix} h_{12} \\ h_{22} \end{bmatrix} x_2$$

$$\vec{\mathbf{h}}_2^{\perp} = \begin{bmatrix} h_{22} & -h_{12} \end{bmatrix}$$



$$\vec{\mathbf{h}}_2^{\perp} \vec{\mathbf{y}} = h_{22} h_{11} x_1 - h_{12} h_{21} x_1$$
$$+ h_{22} h_{12} x_2 - h_{12} h_{22} x_2$$
$$= (h_{22} h_{11} - h_{12} h_{21})\, x_1$$

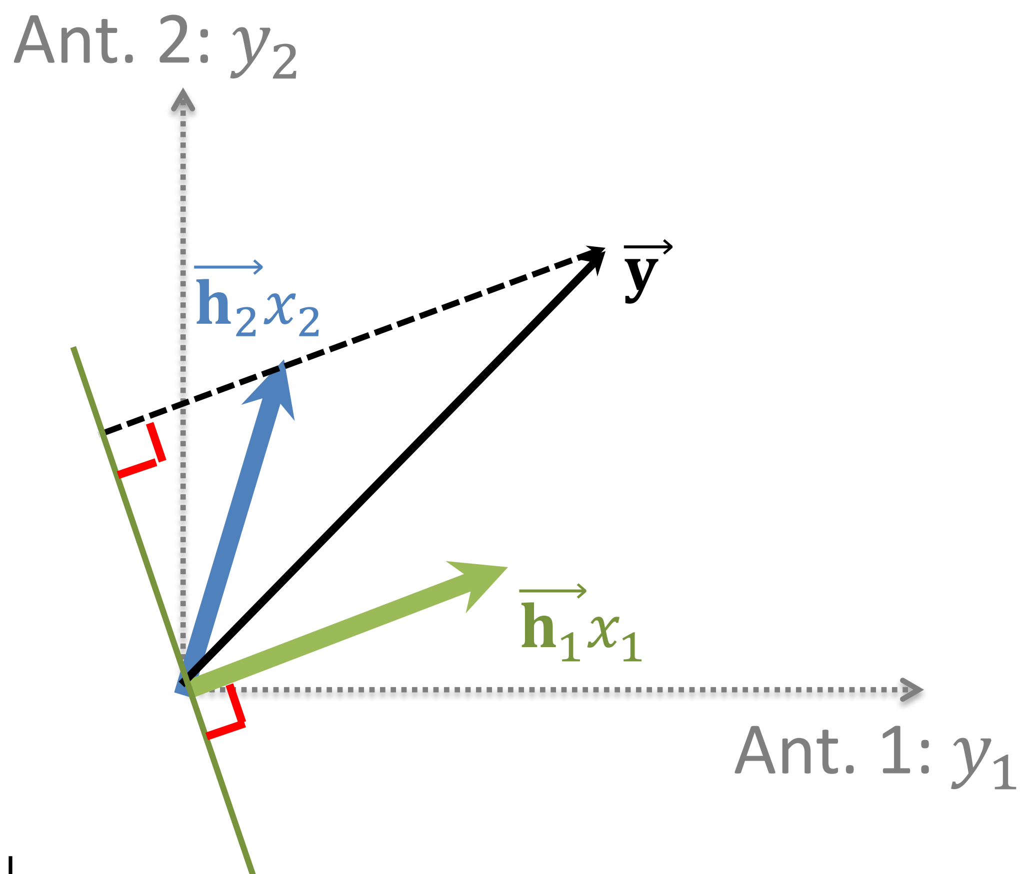
MIMO: Antenna Space

$$\vec{\mathbf{y}} = \vec{\mathbf{h}_1} x_1 + \vec{\mathbf{h}_2} x_2$$

To decode x_2, project on a vector $\vec{\mathbf{h}_1}^{\perp}$ orthogonal to $\vec{\mathbf{h}_1}$

$$\vec{\mathbf{h}_1}^{\perp} \vec{\mathbf{y}} = \vec{\mathbf{h}_1}^{\perp} \vec{\mathbf{h}_1} x_1 + \vec{\mathbf{h}_1}^{\perp} \vec{\mathbf{h}_2} x_2$$

$$= \vec{\mathbf{h}_1}^{\perp} \vec{\mathbf{h}_2} x_2$$



$$\mathbf{y} = \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} x_1 + \begin{bmatrix} h_{12} \\ h_{22} \end{bmatrix} x_2$$

$$\vec{\mathbf{h}_1}^{\perp} = \begin{bmatrix} h_{21} & -h_{11} \end{bmatrix}$$

$$\begin{aligned} \vec{\mathbf{h}_1}^{\perp} \vec{\mathbf{y}} &= h_{21} h_{11} x_1 - h_{21} h_{11} x_1 \\ &\quad + h_{21} h_{12} x_2 - h_{11} h_{22} x_2 \\ &= (h_{21} h_{12} - h_{11} h_{22})\, x_2 \end{aligned}$$

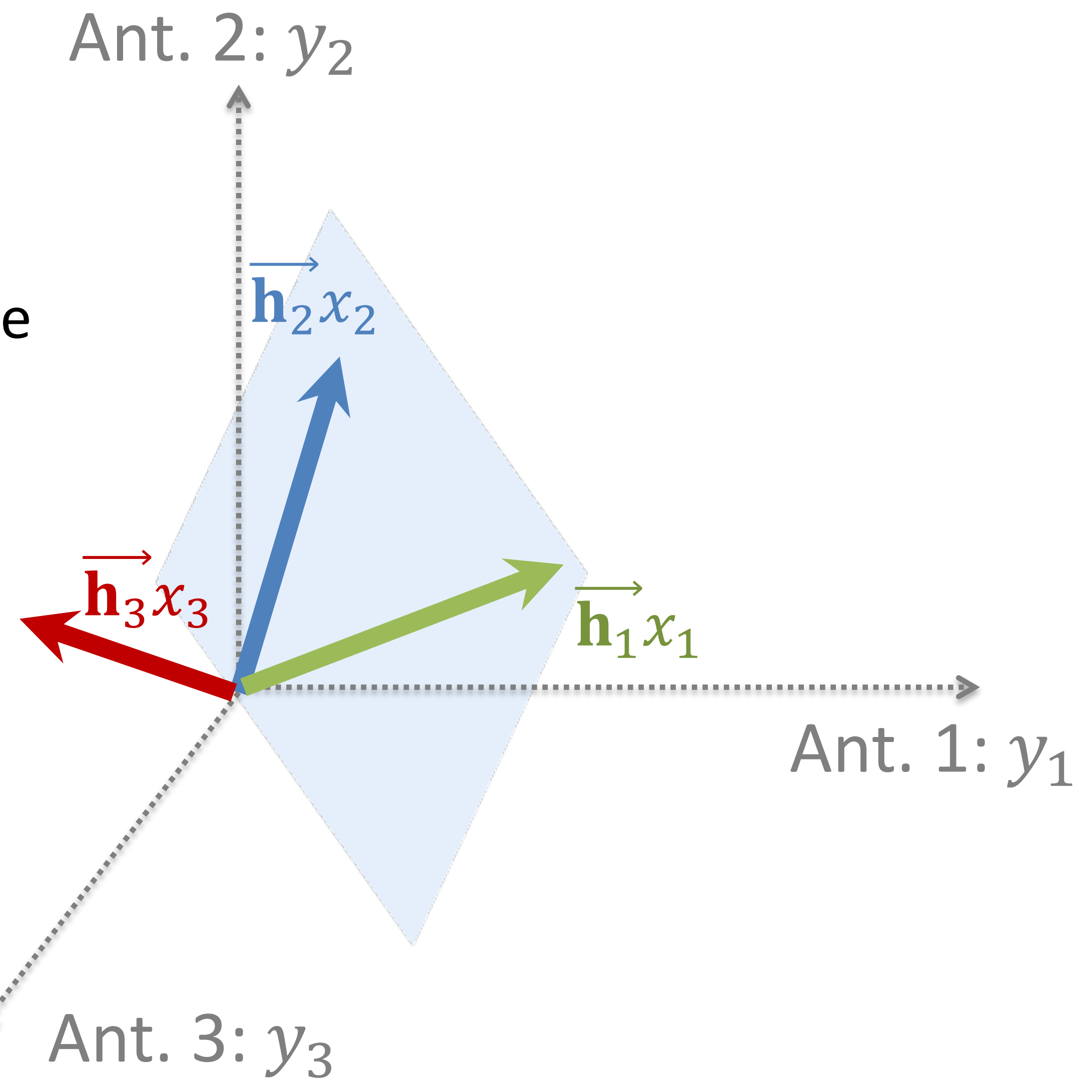
MIMO: Antenna Space

What about MIMO with more antennas?

$$\vec{\mathbf{y}} = \overrightarrow{\mathbf{h}_1} x_1 + \overrightarrow{\mathbf{h}_2} x_2 + \overrightarrow{\mathbf{h}_3} x_3$$

To decode x_1, project on a vector $\overrightarrow{\mathbf{h}_{23}}^{\perp}$ orthogonal to the plane formed by $\overrightarrow{\mathbf{h}_2}$ and $\overrightarrow{\mathbf{h}_3}$

$$\overrightarrow{\mathbf{h}_{23}}^{\perp} = \overrightarrow{\mathbf{h}_2} \times \overrightarrow{\mathbf{h}_3}$$



MIMO: Antenna Space

What about MIMO with more antennas?

$$\overrightarrow{\mathbf{y}} = \overrightarrow{\mathbf{h}_1}x_1 + \overrightarrow{\mathbf{h}_2}x_2 + \overrightarrow{\mathbf{h}_3}x_3$$

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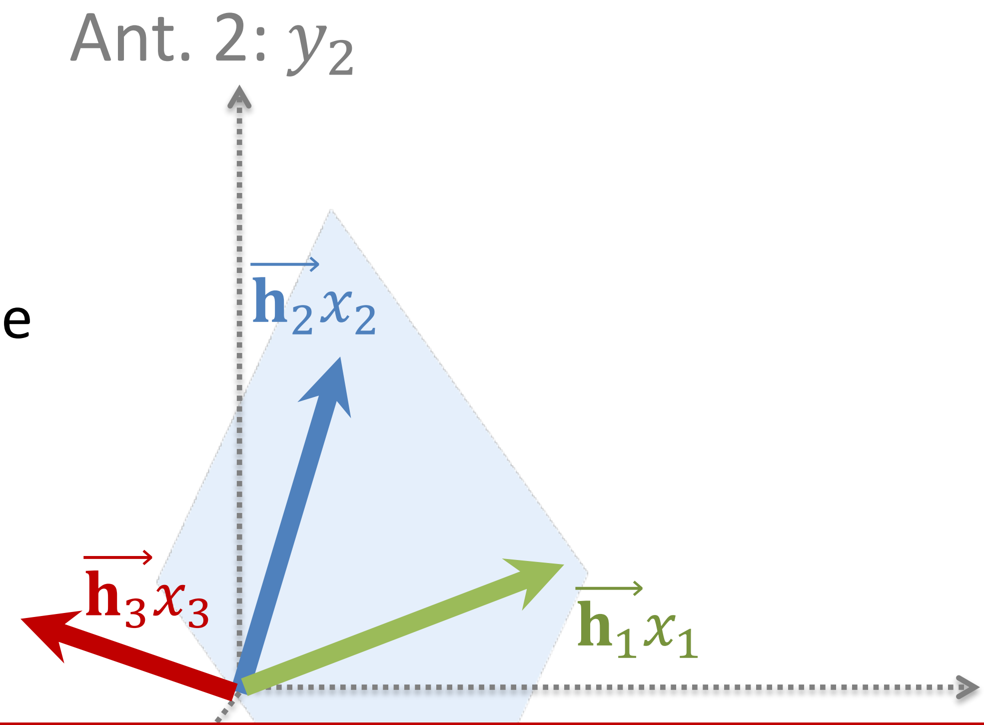
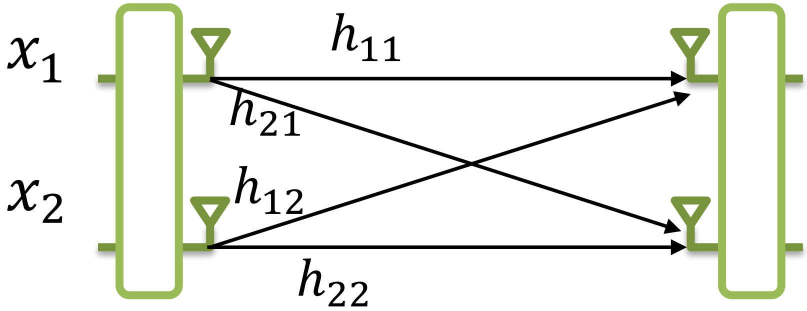


N antenna MIMO, receives signals in N dimensional space
→ Can decode N parallel signals
Decoding complexity scales $O(N^2)$ operations with the number of antennas

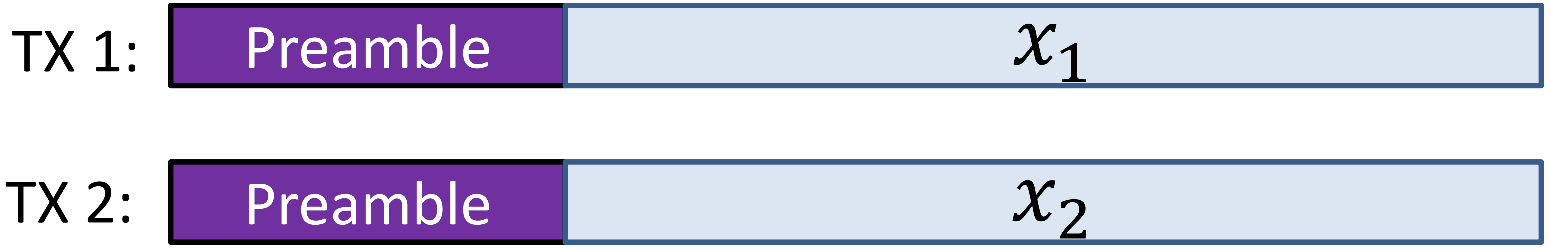
MIMO Channel



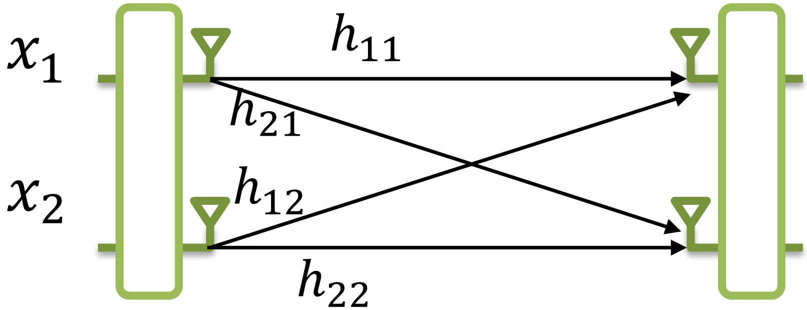
$$y_1 = h_{11}x_1 + h_{12}x_2$$

$$y_2 = h_{21}x_1 + h_{22}x_2$$

How to estimate the channels: $h_{11}, h_{12}, h_{21}, h_{22}$?



MIMO Channel



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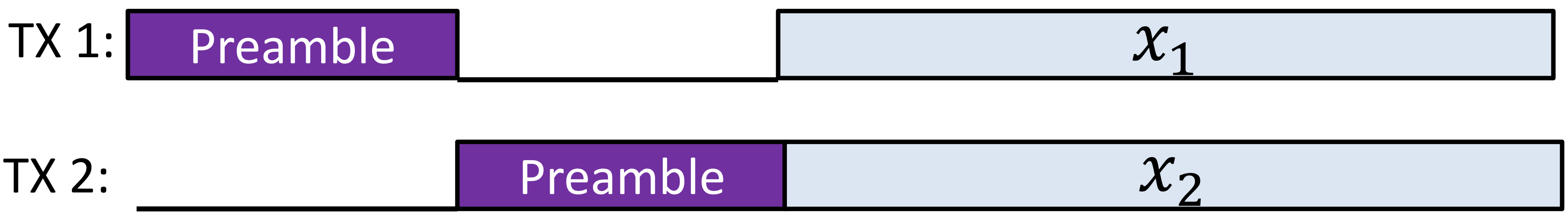


MIMO Gains

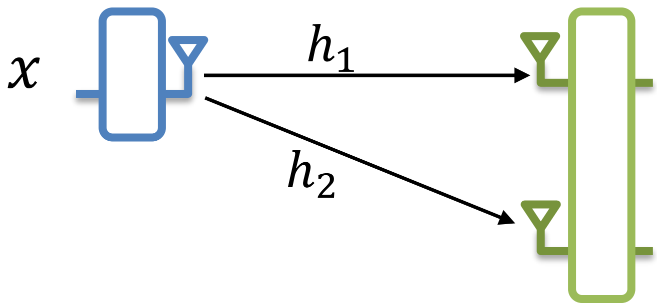
Multiplexing Gain:

- Send multiple packets at the same time
- $N \times N$ MIMO → $N\times$ more packets

Diversity Gain:

- Send/Receive the same packet on multiple antennas
- Increase SNR of the received packets → transmit at higher data rates

Receiver Diversity



$y_1 = h_1 x + n_1$

$y_2 = h_2 x + n_2$

How to best decode x?

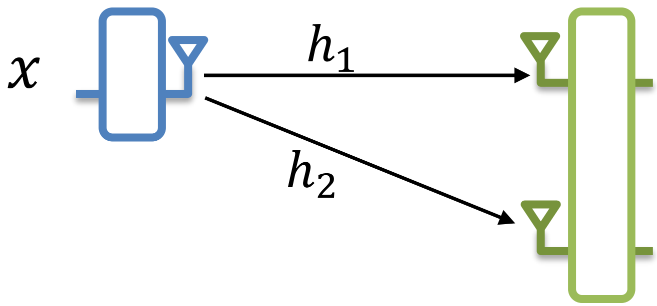
Option 1: Add the received signals

$$y_1 + y_2 = h_1 x + n_1 + h_2 x + n_2$$

$$= (h_1 + h_2) x + n_1 + n_2$$

Channels can sum up destructively! $h_1 + h_2 \approx 0$

Receiver Diversity



$$y_1 = h_1 x + n_1$$

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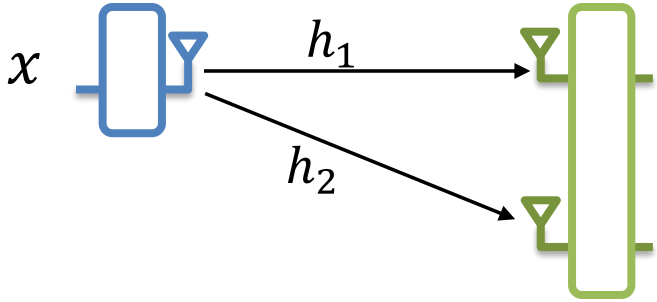
How to best decode x?

Option 1: Add the received signals

Option 2: Decode independently

Sub-optimal!

Receiver Diversity



$$y_1 = h_1 x + n_1$$

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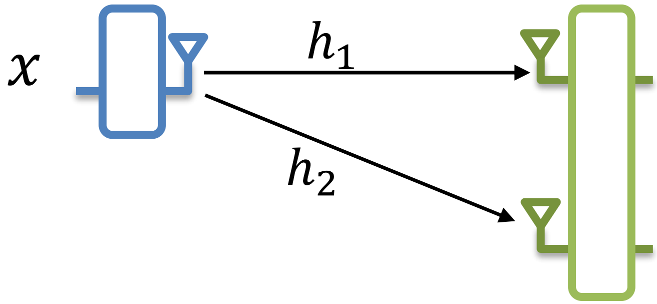
How to best decode x?

Option 1: Add the received signals

Option 2: Decode independently

Optimal Solution: Maximum Ratio Combining (MRC)

Maximum Ratio Combining



$$y_1 = h_1 x + n_1$$

$$y_2 = h_2 x + n_2$$

$$\alpha_1 y_1 + \alpha_2 y_2 = h_1^* y_1 + h_2^* y_2 = h_1^* h_1 x + h_1^* n_1 + h_2^* h_2 x + h_2^* n_2$$

$$= (|h_1|^2 + |h_2|^2)x + h_1^* n_1 + h_2^* n_2$$

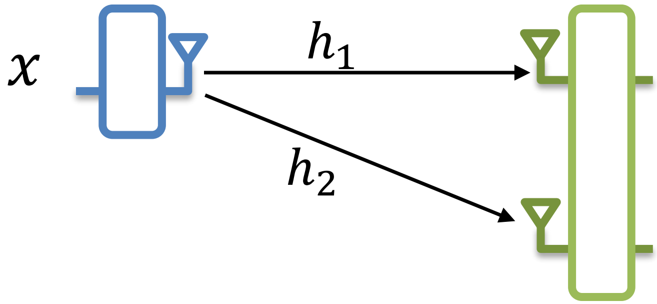
Let $P = \mathbb{E}[|x|^2]$ and $\sigma^2 = \mathbb{E}[|n_1|^2] = \mathbb{E}[|n_2|^2]$

$$Signal\ Power = \mathbb{E}[|(|h_1|^2 + |h_2|^2)x|^2]$$

$$= (|h_1|^2 + |h_2|^2)^2\ \mathbb{E}[|x|^2]$$

$$= (|h_1|^2 + |h_2|^2)^2\ P$$

Maximum Ratio Combining



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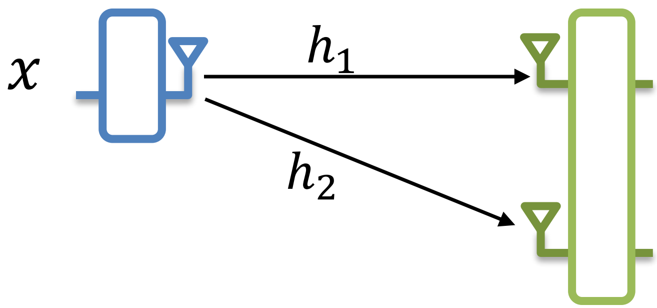
Let $P = \mathbb{E}[|x|^2]$ and $\sigma^2 = \mathbb{E}[|n_1|^2] = \mathbb{E}[|n_2|^2]$

$$Signal\ Power = (|h_1|^2 + |h_2|^2)^2\, P$$

$$Noise\ Power = \mathbb{E}[|h_1^* n_1 + h_2^* n_2|^2] = \mathbb{E}[|h_1^* n_1|^2] + \mathbb{E}[|h_2^* n_2|^2]$$

$$= |h_1|^2 \mathbb{E}[|n_1|^2] + |h_2|^2 \mathbb{E}[|n_2|^2] = (|h_1|^2 + |h_2|^2)\sigma^2$$

Maximum Ratio Combining



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$$= (|h_1|^2 + |h_2|^2) x + h_1^* n_1 + h_2^* n_2$$

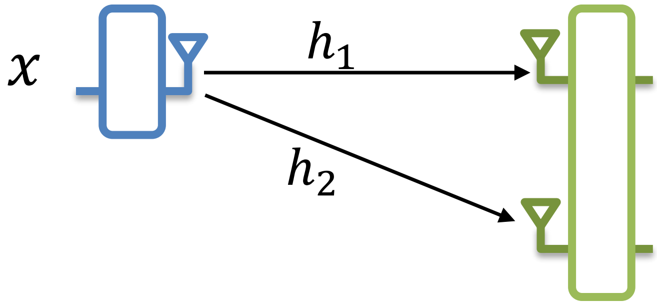
Let $P = \mathbb{E}[|x|^2]$ and $\sigma^2 = \mathbb{E}[|n_1|^2] = \mathbb{E}[|n_2|^2]$

$$Signal\ Power = (|h_1|^2 + |h_2|^2)^2\, P$$

$$Noise\ Power = (|h_1|^2 + |h_2|^2)\sigma^2$$

$$SNR = \frac{(|h_1|^2 + |h_2|^2)^{\cancel{2}}\, P}{(|h_1|^2 + |h_2|^2)\sigma^2} = (|h_1|^2 + |h_2|^2)\frac{P}{\sigma^2}$$

Maximum Ratio Combining



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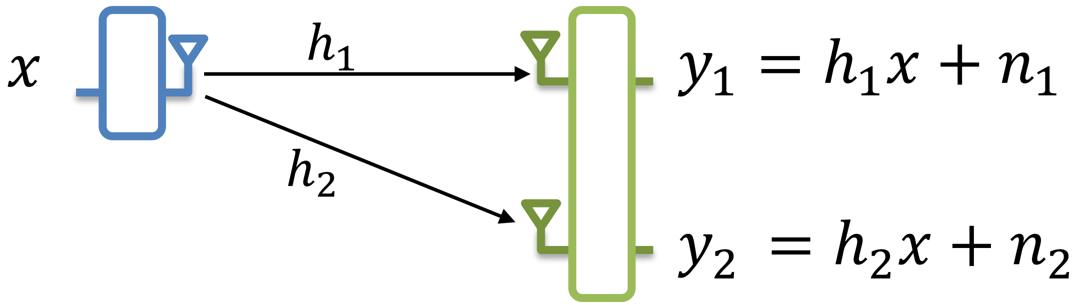
$$= (|h_1|^2 + |h_2|^2) x + h_1^* n_1 + h_2^* n_2$$

With Receiver Diversity: $SNR = (|h_1|^2 + |h_2|^2)\frac{P}{\sigma^2}$

Single Receiver: $SNR = |h_1|^2 \frac{P}{\sigma^2}$

Receiver Diversity Gain



With Receiver Diversity: $SNR = (|h_1|^2 + |h_2|^2)\frac{P}{\sigma^2}$

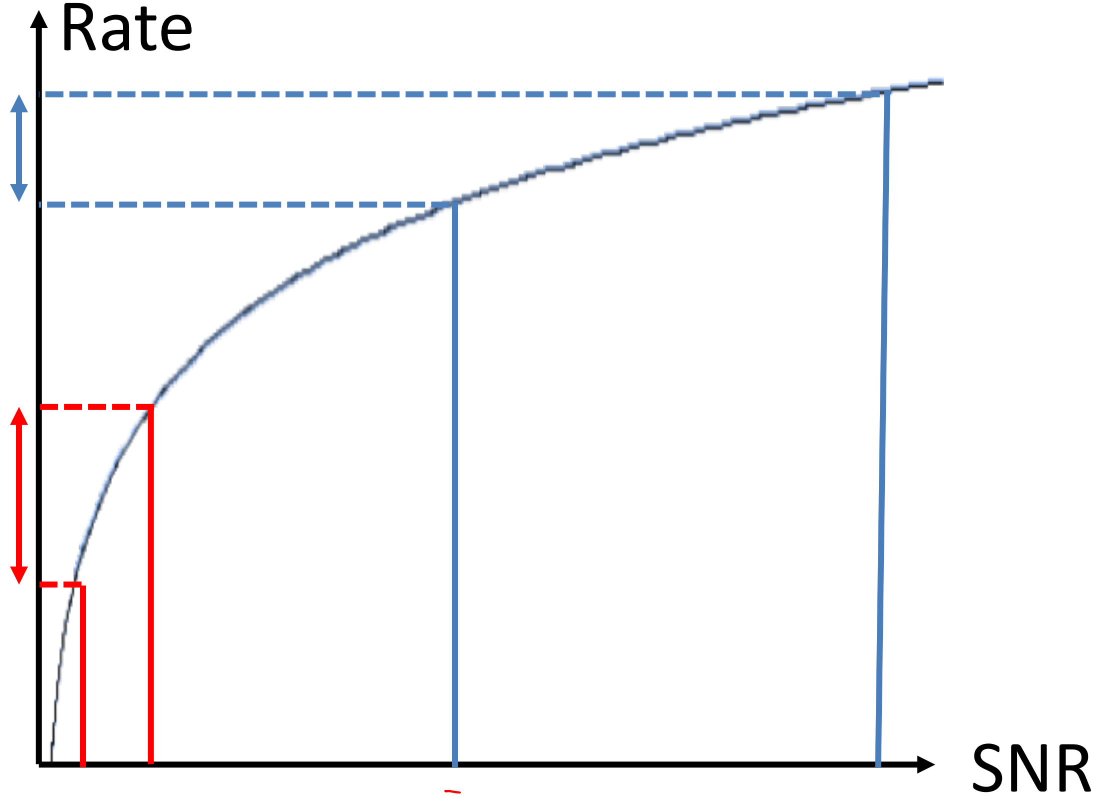
Single Receiver: $SNR = |h_1|^2 \frac{P}{\sigma^2}$

- $|h_1|^2 \approx |h_2|^2$ → Can double SNR!

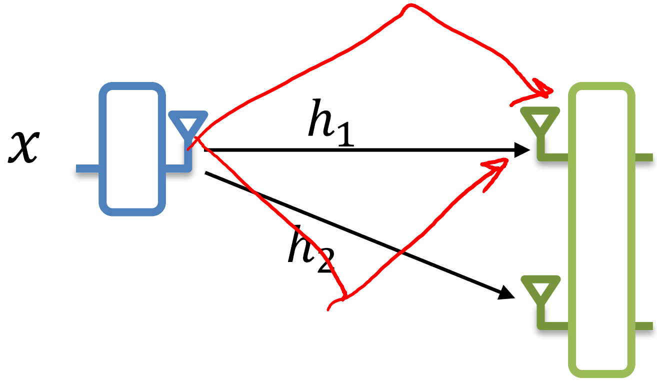
Receiver Diversity Gain

Do we care about doubling the SNR?

$$Capacity \propto \log(SNR)$$



Receiver Diversity Gain



$y_1 = h_1 x + n_1$

$y_2 = h_2 x + n_2$

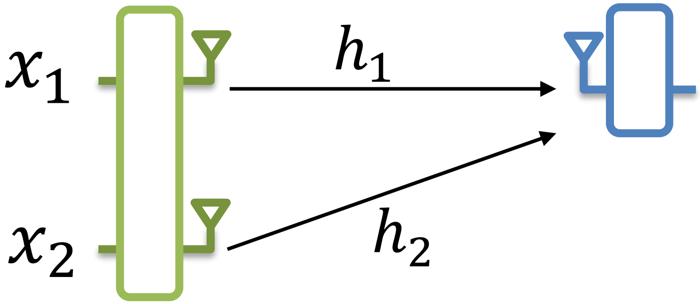
With Receiver Diversity: $SNR_{2RX} = (|h_1|^2 + |h_2|^2)\frac{P}{\sigma^2}$

Single Receiver: $SNR_{1RX} = |h_1|^2 \frac{P}{\sigma^2}$

- $|h_1|^2 \approx |h_2|^2$ → Can double SNR!
- fading $|h_1|^2 \ll |h_2|^2$ → Huge Gain in SNR
- $|h_1|^2 \gg |h_2|^2$ → Little Gain in SNR

It is unlikely that both antennas experience channel fading.

Transmitter Diversity



$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

Option 1: transmit the same thing x on both antennas

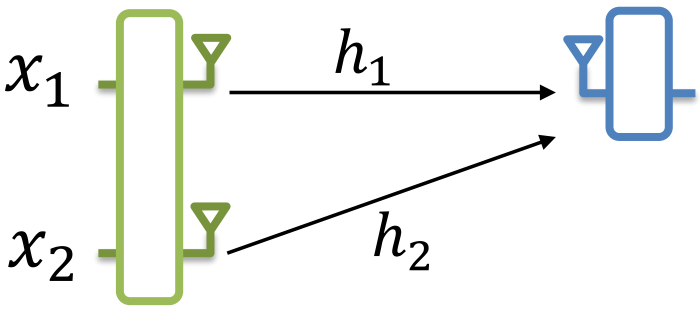
$$y = (h_1 + h_2)x + n$$

Channels can sum up destructively! $h_1 + h_2 \approx 0$

Total transmit power $= 2P$ → Doubled TX power

→ Why not use 1 TX with $2P$

Transmitter Diversity



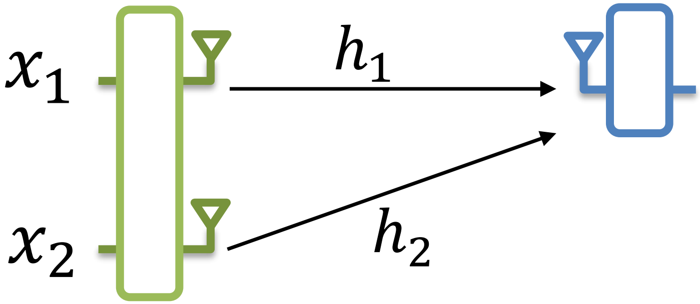
$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

Option 1: transmit the same thing x on both antennas

- Must ensure signals sum up constructively.
- Must ensure total TX power $= \mathbb{E}[|x_1|^2] + \mathbb{E}[|x_2|^2]$
  $= \mathbb{E}[|x|^2] = P$

Transmitter Diversity



$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

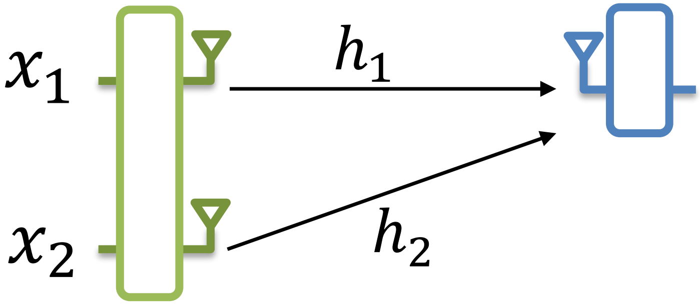
Option 2: Maximum Ratio Combining (MRC)

$$\left.\begin{aligned} x_1 &= \alpha_1 x \\ x_2 &= \alpha_2\, x \end{aligned}\right\} y = (\alpha_1 h_1 + \alpha_2 h_2)x + n$$

Set: $\alpha_1 = h_1^*, \alpha_2 = h_2^*$ $\rightarrow y = (h_1^* h_1 + h_2^* h_2)x + n$

$$= (|h_1|^2 + |h_2|^2)x + n$$

Transmitter Diversity



$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

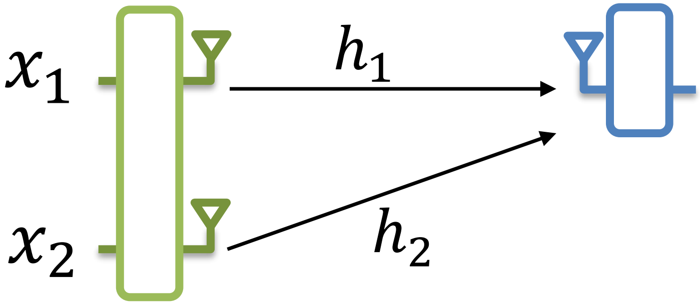
Option 2: Maximum Ratio Combining (MRC)

$$x_1 = \frac{h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x \qquad x_2 = \frac{h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x$$

$$Total\ TX\ Power = \mathbb{E}[|x_1|^2] + \mathbb{E}[|x_1|^2]$$

$$= \frac{|h_1|^2}{|h_1|^2 + |h_2|^2} P + \frac{|h_2|^2}{|h_1|^2 + |h_2|^2} P = P$$

Transmitter Diversity



$y = h_1 x_1 + h_2 x_2 + n$

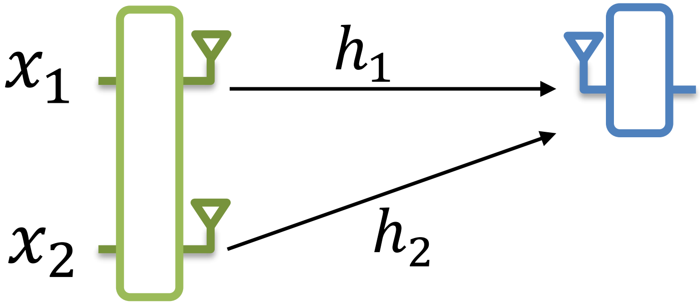
What should we transmit on each antenna?

Option 2: Maximum Ratio Combining (MRC)

$$x_1 = \frac{h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x \qquad x_2 = \frac{h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x$$

$$y = \frac{h_1 h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x + \frac{h_2 h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x + n$$

Transmitter Diversity



$$y = h_1 x_1 + h_2 x_2 + n$$

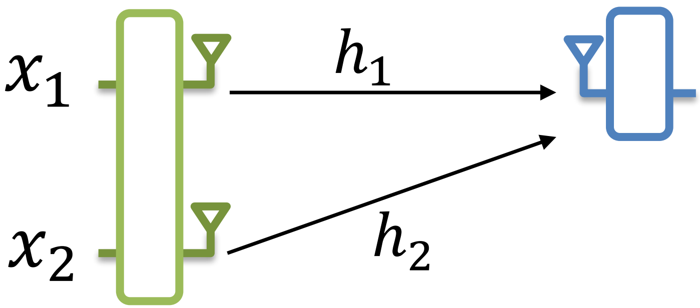
What should we transmit on each antenna?

Option 2: Maximum Ratio Combining (MRC)

$$x_1 = \frac{h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x \qquad x_2 = \frac{h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}\, x$$

$$y = \frac{|h_1|^2 + |h_2|^2}{\sqrt{|h_1|^2 + |h_2|^2}}\, x + n = \left(\sqrt{|h_1|^2 + |h_2|^2}\right) x + n$$

Transmitter Diversity



$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

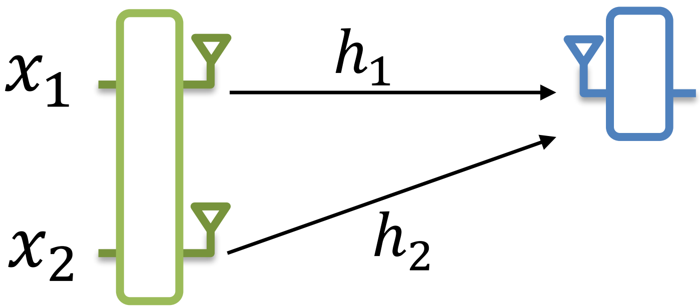
Option 2: Maximum Ratio Combining (MRC)

$$y = \left(\sqrt{|h_1|^2 + |h_2|^2}\right) x + n$$

$$SNR = (|h_1|^2 + |h_2|^2)\frac{P}{\sigma^2}$$ → Similar SNR Gain to RX Diversity

Caveat: MRC at TX Requires Feedback from the Receiver!

Transmitter Diversity

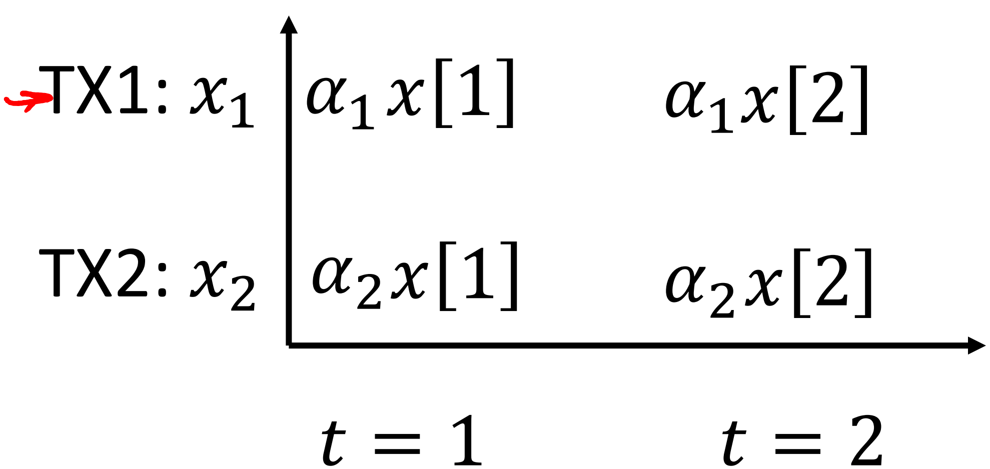


$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

Solution: Use Space-Time Codes

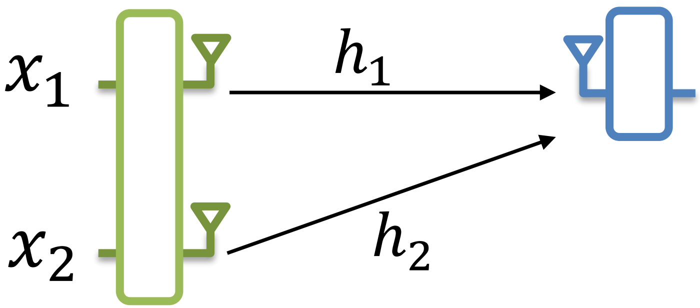
MRC codes across space only (Requires Channel Feedback):



$$\alpha_1 = \frac{h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}}$$

$$\alpha_2 = \frac{h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}$$

Transmitter Diversity



$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

Solution: Use Space-Time Codes

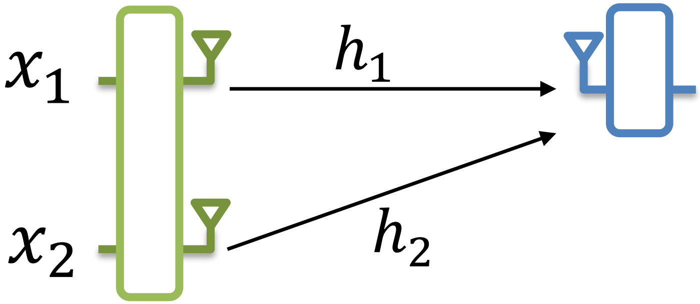
Alamouti Codes:

	$t = 1$	$t = 2$
TX1: x_1	$x[1]$	$-x^*[2]$
TX2: x_2	$x[2]$	$x^*[1]$

$$y[1] = h_1 x[1] + h_2 x[2]$$

$$y[2] = -h_1 x^*[2] + h_2 x^*[1]$$

Transmitter Diversity



$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

Solution: Use Space-Time Codes

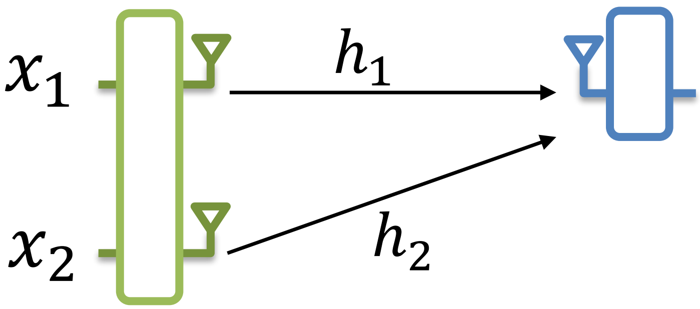
Alamouti Codes:

$$y^*[2] = -h_1^* x[2] + h_2^* x[1]$$

$$y[1] = h_1 x[1] + h_2 x[2] \qquad y[2] = -h_1 x^*[2] + h_2 x^*[1]$$

$$h_1^* y[1] + h_2 y^*[2] = h_1^* h_1 x[1] + h_1^* h_2 x[2]$$
$$- h_2 h_1^* x[2] + h_2 h_2^* x[1]$$
$$= (|h_1|^2 + |h_2|^2) x[1]$$

Transmitter Diversity



$y = h_1 x_1 + h_2 x_2 + n$

What should we transmit on each antenna?

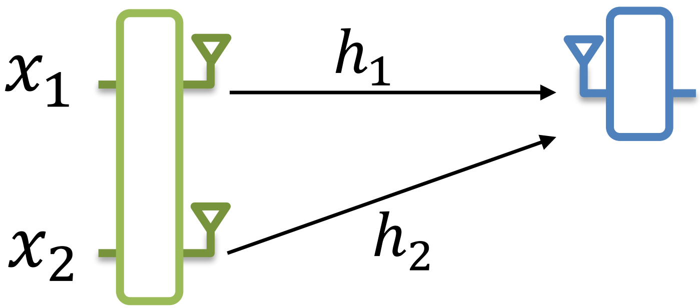
Solution: Use Space-Time Codes

Alamouti Codes:

$y[1] = h_1 x[1] + h_2 x[2]$ $y[2] = -h_1 x^*[2] + h_2 x^*[1]$

$$h_2^* y[1] - h_1 y^*[2] = h_2^* h_1 x[1] + h_2^* h_2 x[2]$$
$$+ h_1 h_1^* x[2] - h_1 h_2^* x[1]$$
$$= (|h_1|^2 + |h_2|^2) x[2]$$

Transmitter Diversity



$$y = h_1 x_1 + h_2 x_2 + n$$

What should we transmit on each antenna?

Solution: Use Space-Time Codes

Alamouti Codes:

	$t = 1$	$t = 2$
TX1: x_1	$\frac{x[1]}{\sqrt{2}}$	$\frac{-x^*[2]}{\sqrt{2}}$
TX2: x_2	$\frac{x[2]}{\sqrt{2}}$	$\frac{x^*[1]}{\sqrt{2}}$

$$h_1^* y[1] + h_2 y^*[2]$$

$$= (|h_1|^2 + |h_2|^2)\frac{x[1]}{\sqrt{2}}$$

$$h_2^* y[1] - h_1 y^*[2]$$

$$= (|h_1|^2 + |h_2|^2)\frac{x[2]}{\sqrt{2}}$$

MIMO Gains

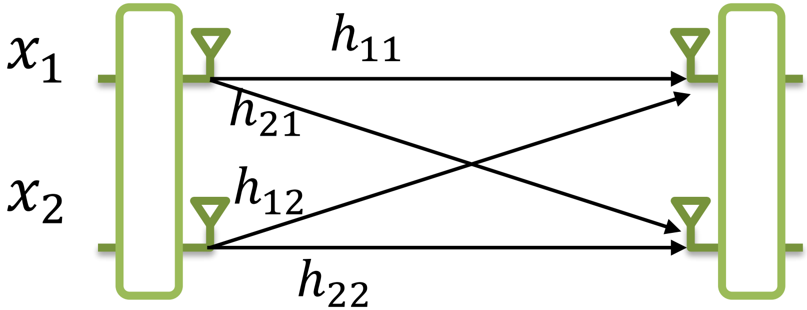
Multiplexing Gain:

- Send multiple packets at the same time
- $N \times N$ MIMO → $\times N$ more packets
- Data Rate: $\propto N \log(SNR/N)$

Diversity Gain:

- Increase SNR of the received packets
- $N \times N$ MIMO → $\times \log N^2$ data rate
- Data Rate $\propto \log(SNR \times N^2)$

MIMO



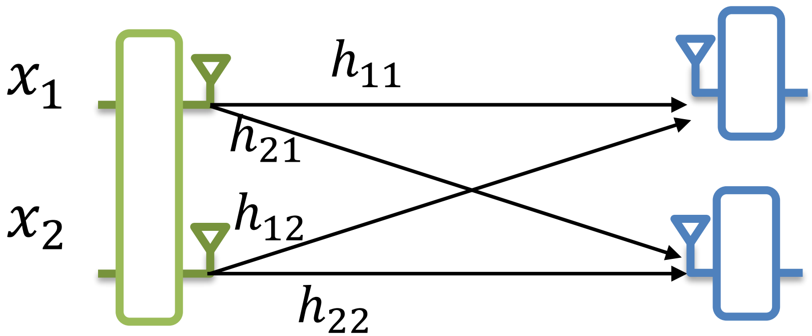
$$y_1 = h_{11}x_1 + h_{12}x_2$$

$$y_2 = h_{21}x_1 + h_{22}x_2$$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

$$\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{y} = \mathbf{H}^{-1}\mathbf{H}\mathbf{x} + \mathbf{H}^{-1}\mathbf{n} = \mathbf{x} + \mathbf{H}^{-1}\mathbf{n}$$

Multi-User MIMO



$$y_1 = h_{11}x_1 + h_{12}x_2$$

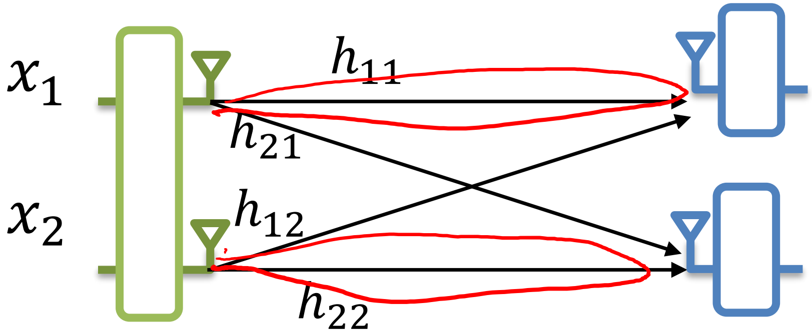
$$y_2 = h_{21}x_1 + h_{22}x_2$$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

$$\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{y} = \mathbf{H}^{-1}\mathbf{H}\mathbf{x} + \mathbf{H}^{-1}\mathbf{n} = \mathbf{x} + \mathbf{H}^{-1}\mathbf{n}$$

Does not work. Receivers do not have access to each other's signals

Precoding



$y_1 = h_{11}x_1 + h_{12}x_2$

$y_2 = h_{21}x_1 + h_{22}x_2$

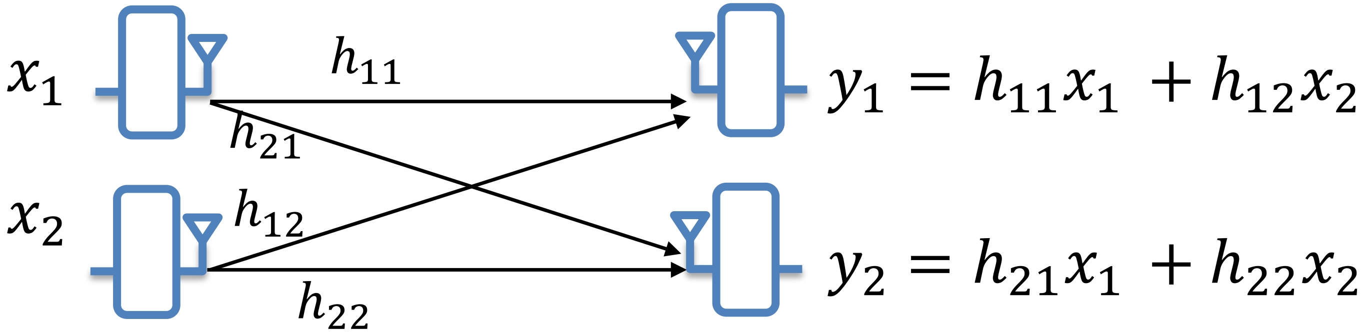
Send: $\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{x}$

Receive: $\mathbf{y} = \mathbf{H}\tilde{\mathbf{x}} + \mathbf{n} = \mathbf{H}\mathbf{H}^{-1}\mathbf{x} + \mathbf{n} = \mathbf{x} + \mathbf{n}$

Also known as Beamforming or Zero-Forcing

- Channel Feedback!
- Only Downlink traffic

What about distributed transmitters?



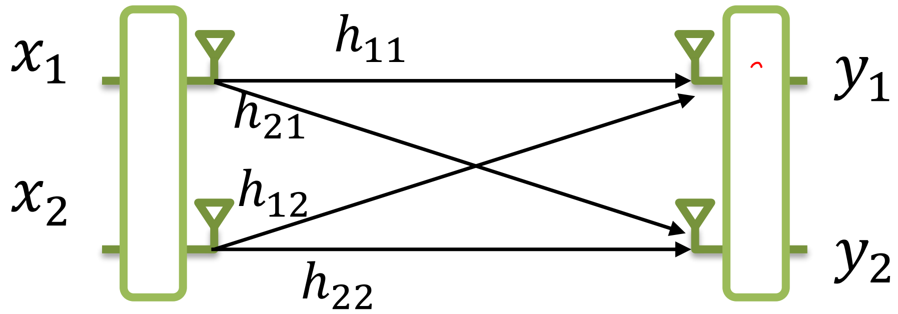
Send: $\tilde{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{x}$

Receive: $\mathbf{y} = \mathbf{H}\tilde{\mathbf{x}} + \mathbf{n} = \mathbf{H}\mathbf{H}^{-1}\mathbf{x} + \mathbf{n} = \mathbf{x} + \mathbf{n}$

Does not work. Transmitters do not have access to each other’s signals

False: what if transmitters are APs connected over Ethernet?

Clock Synchronization in MIMO

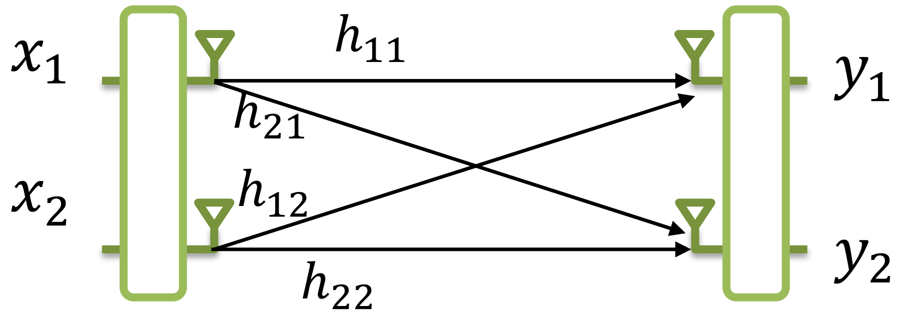


$$y_1 = h_{11}x_1e^{-j2\pi\Delta f_{11}t} + h_{12}x_2e^{-j2\pi\Delta f_{12}t}$$

$$y_2 = h_{21}x_1e^{-j2\pi\Delta f_{21}t} + h_{22}x_2e^{-j2\pi\Delta f_{22}t}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Clock Synchronization in MIMO

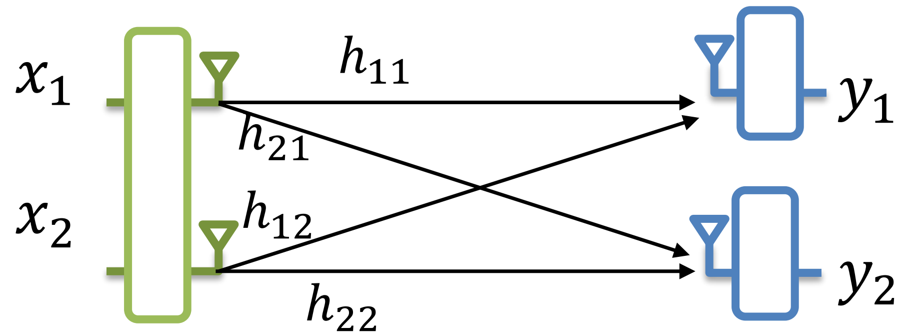


$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11}\, e^{-j2\pi\Delta f_{11} t} & h_{12} e^{-j2\pi\Delta f_{12} t} \\ h_{21} e^{-j2\pi\Delta f_{21} t} & h_{22} e^{-j2\pi\Delta f_{22} t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

On chip MIMO: $\Delta f_{11} = \Delta f_{21} = \Delta f_{12} = \Delta f_{22} = \Delta f$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} e^{-j2\pi\Delta f t} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Clock Synchronization in MIMO

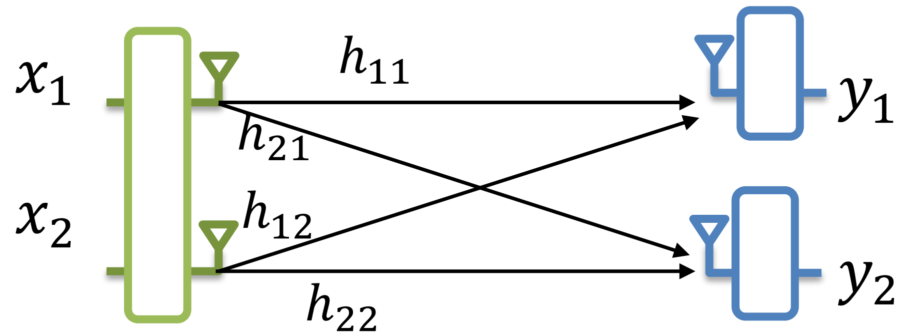


$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11}\, e^{-j2\pi\Delta f_{11}t} & h_{12}e^{-j2\pi\Delta f_{12}t} \\ h_{21}e^{-j2\pi\Delta f_{21}t} & h_{22}e^{-j2\pi\Delta f_{22}t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

MU-MIMO: $\Delta f_{11} = \Delta f_{12}$ and $\Delta f_{21} = \Delta f_{22}$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11}\, e^{-j2\pi\Delta f_{1}t} & h_{12}e^{-j2\pi\Delta f_{1}t} \\ h_{21}e^{-j2\pi\Delta f_{2}t} & h_{22}e^{-j2\pi\Delta f_{2}t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Clock Synchronization in MIMO

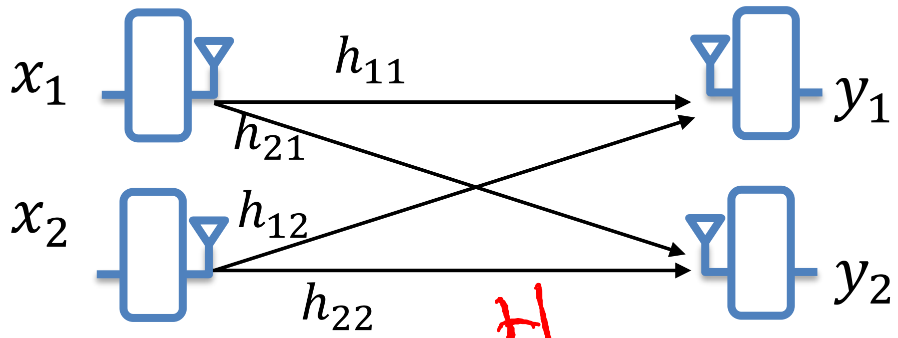


$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11}\, e^{-j2\pi\Delta f_{11}t} & h_{12}e^{-j2\pi\Delta f_{12}t} \\ h_{21}e^{-j2\pi\Delta f_{21}t} & h_{22}e^{-j2\pi\Delta f_{22}t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

MU-MIMO: $\Delta f_{11} = \Delta f_{12}$ and $\Delta f_{21} = \Delta f_{22}$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} e^{-j2\pi\Delta f_1 t} & 0 \\ 0 & e^{-j2\pi\Delta f_2 t} \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Clock Synchronization in MIMO



$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11}\, e^{-j2\pi\Delta f_{11} t} & h_{12} e^{-j2\pi\Delta f_{12} t} \\ h_{21} e^{-j2\pi\Delta f_{21} t} & h_{22} e^{-j2\pi\Delta f_{22} t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Distributed MIMO: $\Delta f_{11} \neq \Delta f_{12} \neq \Delta f_{21} \neq \Delta f_{22}$

Next Lecture!