Normal distributions

The distribution of a sum of *independent* normally distributed random variables also follows a normal distribution. This is a rather particular result for normally distributed variables; see here for a detailed proof of this result. In particular, consider two independent random variables X and Y, where $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$ (i.e., means μ_X and μ_Y and standard deviations σ_X and σ_Y). Then, their sum $Z = X+Y$ is also normally distributed, where $Z \sim N(\mu_X+\mu_Y, \sigma_X^2+\sigma_Y^2)$. The most straightforward proof of this is the geometric one (see link above).

Moving from the *probability density* to the *cumulative distribution function*. Briefly (see here for more detailed information), the probability density of the normal distribution is

$$f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(x-\mu)^2}{\sigma^2}\right]$$

This probability density tells you that if you want to know the probability that a random sample $X \sim N(\mu, \sigma^2)$ is between the values $x_0 < x_1$, it is given by

$$P(x_0 \leq X \leq x_1) = \int_{x_0}^{x_1} f(t|\mu, \sigma^2)\ dt$$

If we take the special case where $x_0 \to -\infty$, and then we have the *cumulative distribution function*,

$$P(X \leq x) = \int_{-\infty}^{x} f(t|\mu, \sigma^2)\ dt = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$$

where erf is the Error function. The cumulative distribution function goes to 0 as $x \to -\infty$ and to 1 as $x \to \infty$.