1 Treating a Perturbing Potential

For a perturbing potential A, the free energy is given by

$$e^{-\beta F(\lambda)} = \int dR\, e^{-\beta V - \beta\lambda A} \tag{1}$$

$$F(\lambda) = F(0) + \lambda\langle A\rangle_0 - \frac{\beta\lambda^2}{2}[\langle A^2\rangle_0 - \langle A\rangle_0^2] + O(\lambda^3) \tag{2}$$

$$F(\lambda) = F(0) + \int_0^\lambda d\lambda'\, \langle A\rangle'_\lambda \tag{3}$$

If B is a property of the system,

$$B(\lambda) = B(0) - \beta\lambda[\langle AB\rangle_0 - \langle A\rangle_0\langle B\rangle_0] + O(\lambda^2) \tag{4}$$

For example, let $A = \rho_{\mathbf{k}}$ and $B = \rho_{-\mathbf{k}}$ (density-density response). Then

$$\left.\frac{d\rho_{-\mathbf{k}}}{d\lambda}\right|_0 = -\beta\langle|\rho_{\mathbf{k}}|^2\rangle| = -\beta N S_{\mathbf{k}} \tag{5}$$

2 Diffusion and velocity-velocity correlation.

The diffusion equation is

$$\frac{\partial\rho}{\partial t} = D\nabla^2\rho(\mathbf{r}, t) \tag{6}$$

D can be determined from the mean-square displacement, or, equivalently, the velocity-velocity correlation time.

$$D = \lim_{t\to\infty}\frac{1}{6t}\langle[\mathbf{r}(t) - \mathbf{r}(0)]^2\rangle \tag{7}$$

$$= \frac{1}{3}\int_0^\infty dt\, \mathbf{v}(t)\cdot\mathbf{v}(0) \tag{8}$$

3 Treating a Dynamic Perturbing Potential

For an external field $Ae^{-i\omega t}$, denote the response of B as $\chi_{BA}(\omega)e^{-i\omega t}$. The fluctuation-dissipation theorem says

$$\chi_{BA}(\omega) = \beta\int_0^\infty dt\, e^{i\omega t}\langle B(t)\frac{dA(0)}{dt}\rangle \tag{9}$$

The energy absorption of the system is

$$\frac{dE}{dt} = \beta\left|\frac{\omega}{2}\right|^2\int_0^\infty dt\, \cos(\omega t)\langle A(0)A(t)\rangle. \tag{10}$$

The dynamic structure factor (density-density response) is given by

$$S_{\mathbf{k}}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, F_{\mathbf{k}}(t)e^{i\omega t}, \qquad \text{where} \qquad F_{\mathbf{k}}(t) = \frac{1}{2}\langle\rho_{\mathbf{k}}(t)\rho_{-\mathbf{k}}(0)\rangle \tag{11}$$