

Linear attenuation coefficients, and Atomic attenuation coefficient

Linear Attenuation Coefficients

- ☞ Monochromatic photons are attenuated exponentially in a uniform target.
- ☞ The number of photons in a pencil beam interact within a small distance dx is

$$dN = -\mu N dx$$

where μ is the linear attenuation coefficient and the solution to the above differential equation is

$$N(x) = N_0 e^{-\mu x}$$

$\exp(-\mu x)$ is simply the probability of a photon penetrating a distance x without interaction.

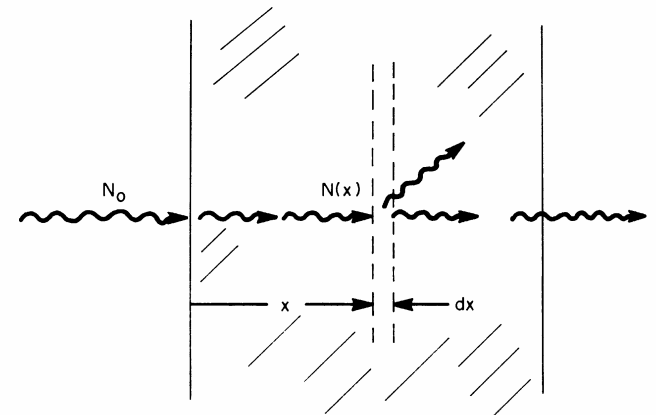


FIGURE 8.6. Pencil beam of N_0 monoenergetic photons incident on slab. The number of photons that reach a depth x without having an interaction is given by $N(x) = N_0 e^{-\mu x}$ where μ is the linear attenuation coefficient.

Linear Attenuation Coefficients

Linear attenuation coefficient is measured using the following setup

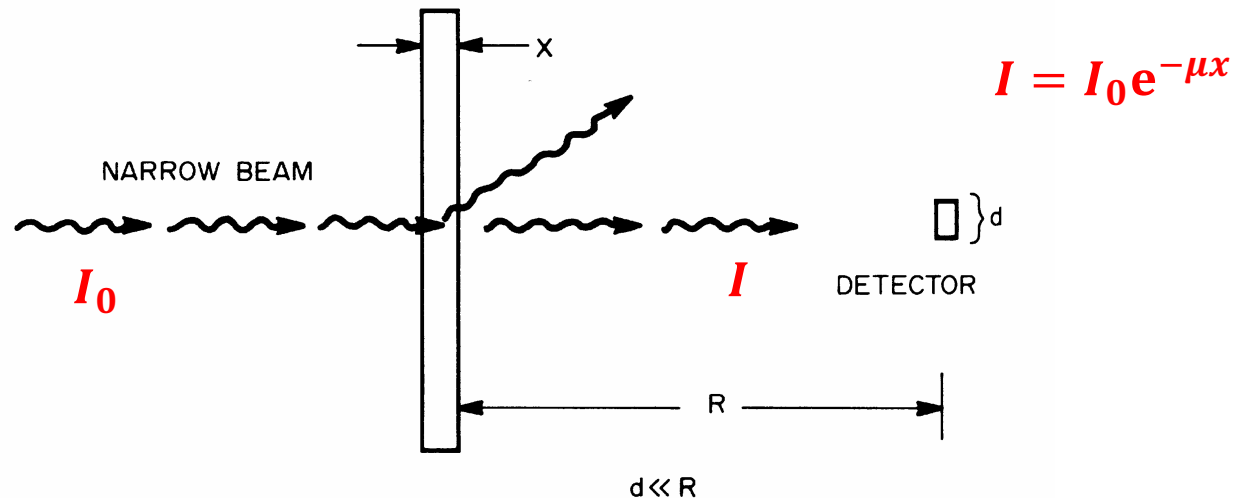


FIGURE 8.7. Illustration of “good” scattering geometry for measuring linear attenuation coefficient μ . Photons from a *narrow* beam that are absorbed or scattered by the absorber do not reach a small detector placed in beam line some distance away.

Using small detector to avoid the effect of Compton scattered photons on the measured linear attenuation coefficient.

Linear Attenuation Coefficients

- ☞ **Linear attenuation coefficient** for a given material comprises the individual contributions from various physical processes,

$$\mu = \tau_{photoelectric} + \sigma_{Compton} + \kappa_{pair}$$

- ☞ **Mass attenuation coefficient** is simply defined as,

$$\mu_m = \mu / \rho \text{ (cm}^2 \text{ / g)}$$

As shown in the discussion of the attenuation of beta and alpha particles, the **mass attenuation coefficient** is used to partially remove the dependence on different atomic compositions and densities, and provides in an unified measure of photon attenuation amongst various materials

Linear Attenuation Coefficients

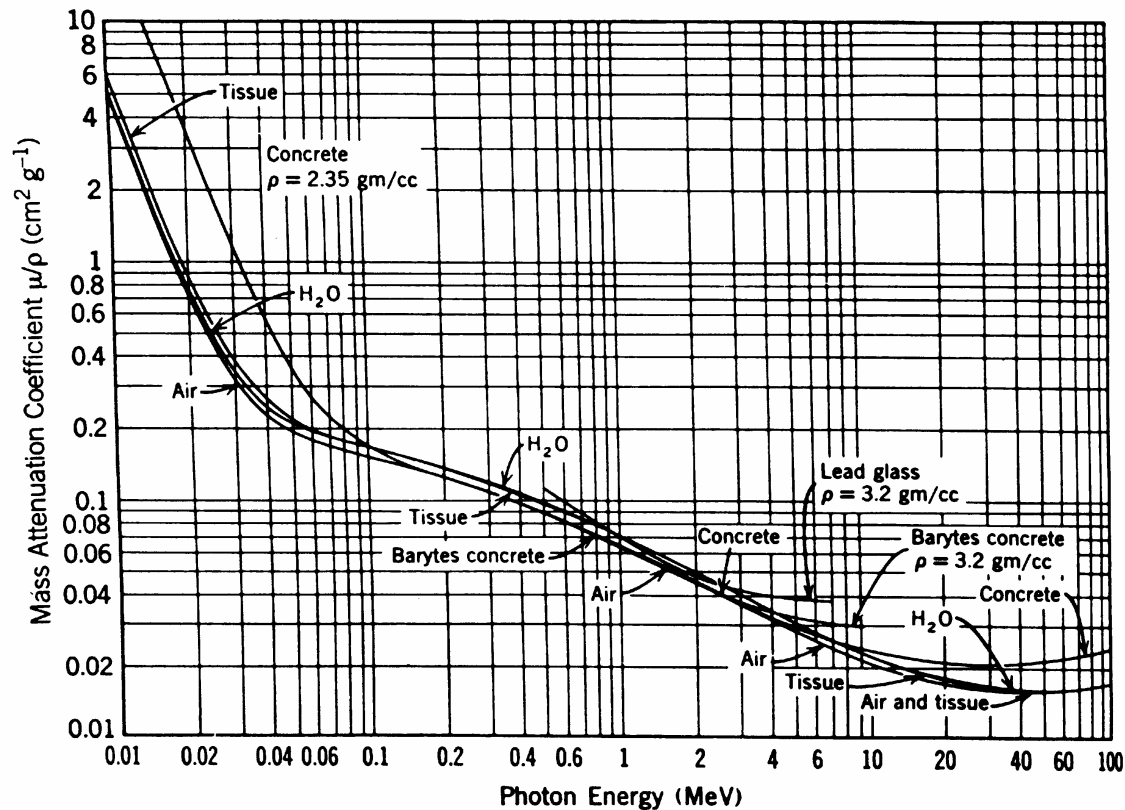


FIGURE 8.9. Mass attenuation coefficients for various materials. [Reprinted with permission from K. Z. Morgan and J. E. Turner, eds., *Principles of Radiation Protection*, Wiley, New York (1967). Copyright 1967 by John Wiley & Sons.]

Atomic Attenuation Coefficients

- ☞ **Atomic attenuation coefficient (cross section per atom)** – the probability of a photon is removed from the beam, when passing normally through a layer of material containing one atom per unit area.

$$\mu_a = \frac{\mu(\text{cm}^{-1})}{N(\text{atoms}/\text{cm}^3)}$$

- ☞ The **atomic attenuation coefficient** is also called the **microscopic cross section** of an atom in the absorber material. It is the microscopic “target area” sustained by an atom in the absorber.

$$\mu_a = \frac{\mu}{N} (\text{cm}^2 \text{ per atom})$$

Atomic Attenuation Coefficients

The numerical values for μ_a have been published for many elements and for a range of quantum energies.

TABLE 5.2. Linear Attenuation Coefficients, cm^{-1}

$\rho, \text{g/cm}^3$	Quantum energy, MeV													
	0.1	0.15	0.2	0.3	0.5	0.8	1.0	1.5	2	3	5	8	10	
C	2.25	0.335	0.301	0.274	0.238	0.196	0.159	0.143	0.117	0.100	0.080	0.061	0.048	0.044
Al	2.7	0.435	0.362	0.324	0.278	0.227	0.185	0.166	0.135	0.117	0.096	0.076	0.065	0.062
Fe	7.9	2.72	1.445	1.090	0.838	0.655	0.525	0.470	0.383	0.335	0.285	0.247	0.233	0.232
Cu	8.9	3.80	1.830	1.309	0.960	0.730	0.581	0.520	0.424	0.372	0.318	0.281	0.270	0.271
Pb	11.3	59.7	20.8	10.15	4.02	1.64	0.945	0.771	0.579	0.516	0.476	0.482	0.518	0.552
Air	1.29×10^{-3}	1.95×10^{-4}	1.73×10^{-4}	1.59×10^{-4}	1.37×10^{-4}	1.12×10^{-4}	9.12×10^{-5}	8.45×10^{-5}	6.67×10^{-5}	5.75×10^{-5}	4.6×10^{-5}	3.54×10^{-5}	2.84×10^{-5}	2.61×10^{-5}
H ₂ O	1	0.167	0.149	0.136	0.118	0.097	0.079	0.071	0.056	0.049	0.040	0.030	0.024	0.022
Concrete ^a	2.35	0.397	0.326	0.291	0.251	0.204	0.166	0.149	0.122	0.105	0.085	0.067	0.057	0.054

^aOrdinary concrete of the following composition: 0.56% H, 49.56% O, 31.35% Si, 4.56% Al, 8.26% Ca, 1.22% Fe, 0.24% Mg, 1.71% Na, 1.92% K, 0.12% S.

Source: From National Bureau of Standards Report No. 1003, 1952.

Atomic Attenuation Coefficients

Atomic attenuation coefficient (microscopic cross section per atom) μ_a (σ) can be related to the linear attenuation coefficient (μ) as the following:

$$\mu = N_A \mu_A$$

where

μ : linear attenuation coefficient (cm^{-1})

N_A : atomic density (No. of atoms per cm^3)

μ_A : atomic cross section (cm^{-2})

Attenuation Coefficients

The **linear attenuation coefficient** (μ_l) is related to the atomic attenuation coefficient (microscopic cross section per atom) as the following:

$$\mu_l \text{ cm}^{-1} = \mu_a \frac{\text{cm}^2}{\text{atom}} \times N \frac{\text{atoms}}{\text{cm}^3}$$

The linear attenuation coefficient for a mixture of materials or an alloy is given by

$$\mu_l = \mu_{a1} \times N_1 + \mu_{a2} \times N_2 + \dots = \sum_{n=1}^n \mu_{an} \times N_n, \quad (5.33)$$

↑ Number density of the n'th atom in the compound or mixture

where

μ_n = atomic coefficient of the n th element and
 N_n = number of atoms per cm^3 of the n th element.

P170, <<Introduction to Health Physics>>, Third edition, by Cember.

Attenuation Coefficients of an Alloy or a Compound

EXAMPLE 5.11

Aluminum bronze, an alloy containing 90% Cu (atomic weight = 63.57) and 10% Al (atomic weight = 26.98) by weight, has a density of 7.6 g/cm³. What are the linear and mass attenuation coefficients for 0.4-MeV gamma rays if the cross sections for Cu and Al for this quantum energy are 9.91 and 4.45 b?

$$\mu = N_A \mu_A \quad \text{or} \quad \frac{\mu}{\rho} = \frac{N_0}{A} \mu_A$$

μ : linear attenuation coefficient (cm⁻¹)

ρ : density of the material (g/cm⁻³)

N_A : atomic density (No. of atoms per cm³)

N_0 : Avogadro's number

μ_A : atomic cross section (cm⁻²)

A : atomic weight (g)

Attenuation Coefficients

Solution

From Eq. (5.33), the linear attenuation coefficient of aluminum bronze is

$$\mu_1 = (\mu_a)_{\text{Cu}} \times N_{\text{Cu}} + (\mu_a)_{\text{Al}} \times N_{\text{Al}}.$$

N_{Cu} : the number of Cu atoms per unit volume of the mixture or compound.

The number of Cu atoms per cm^3 in the alloy is

$$N_{\text{Cu}} = \frac{6.02 \times 10^{23} \text{ atoms/mol}}{63.57 \text{ g/mol}} \times (7.6 \times 0.9) \frac{\text{g}}{\text{cm}^3} = 6.5 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3}$$

and for Al, it is given by

$$N_{\text{Al}} = \frac{6.02 \times 10^{23} \text{ atoms/mol}}{27 \frac{\text{g}}{\text{mol}}} \times (7.6 \times 0.1) \frac{\text{g}}{\text{cm}^3} = 1.7 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3}.$$

Attenuation Coefficients

The linear attenuation coefficient therefore is

$$\begin{aligned}\mu_1 &= 9.91 \times 10^{-24} \frac{\text{cm}^2}{\text{atom}} \times 6.5 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3} \\ &\quad + 4.45 \times 10^{-24} \frac{\text{cm}^2}{\text{atom}} \times 1.7 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3} = 0.72 \text{ cm}^{-1}.\end{aligned}$$

The mass attenuation coefficient is, from Eq. (5.28),

$$\mu_m = \frac{\mu_1}{\rho} = \frac{0.72 \text{ cm}^{-1}}{7.6 \text{ g/cm}^3} = 0.095 \frac{\text{cm}^2}{\text{g}}.$$

Attenuation Coefficients

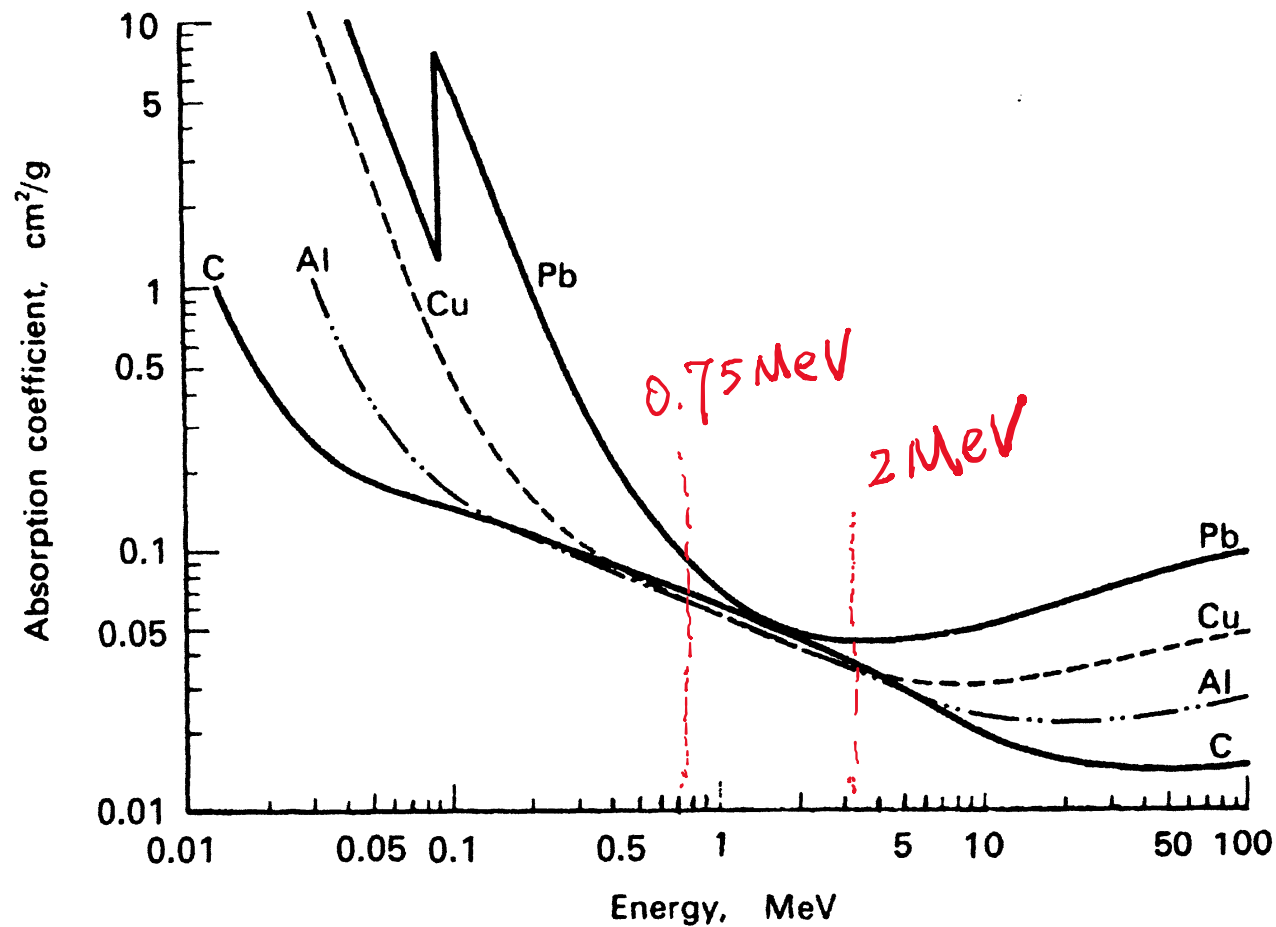


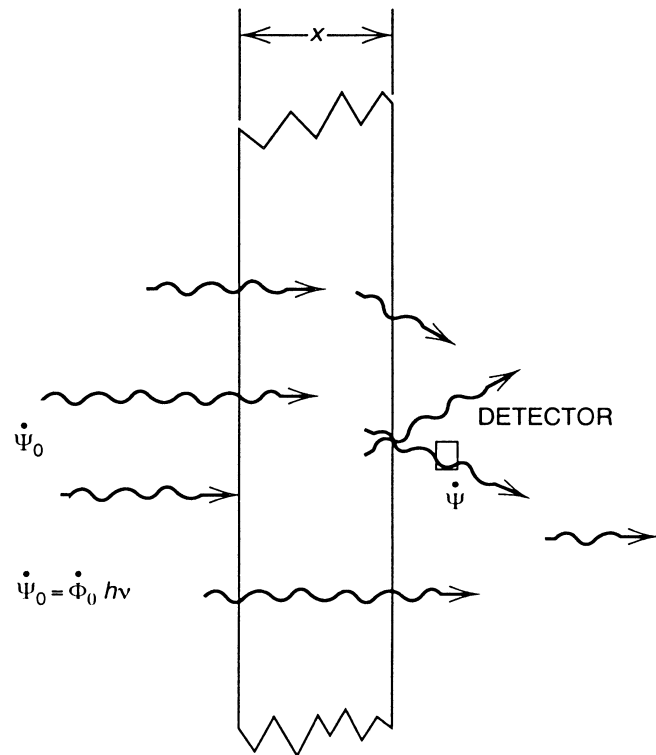
FIGURE 5.12. Curves illustrating the systematic variation of attenuation coefficient with atomic number of absorber and with quantum energy.

Photon Attenuation Coefficients

- ☞ For photon energies between 0.75 to 5Mev, almost all materials have, on mass attenuation coefficient basis, about the same gamma ray attenuation properties. The shielding properties of a given material is approximately proportional to the density of the material.
- ☞ For lower or higher energies, absorbers of high atomic number is much better than those of low atomic number.

**Energy transfer coefficient
&
Energy absorption coefficient**

Energy Transfer and Energy Absorption Coefficients



For dosimetry applications, we are often interested in

- ☞ How much energy is **transferred** to the absorber?
and
- ☞ How much of the energy transferred is eventually **absorbed** in the shielding?

To answer these questions,

- ☞ It is important to consider the **secondary radiations** resultant from the interactions of the incident gamma rays and the absorber.
- ☞ Under certain conditions, energy absorption in shielding material can be modeled with exponential function ...

Energy-Transfer and Energy-Absorption Coefficients

The photon fluence Φ : the number of photons cross a unit area perpendicular to the beam.

The photon fluence rate: the number of photons per unit area per unit time.

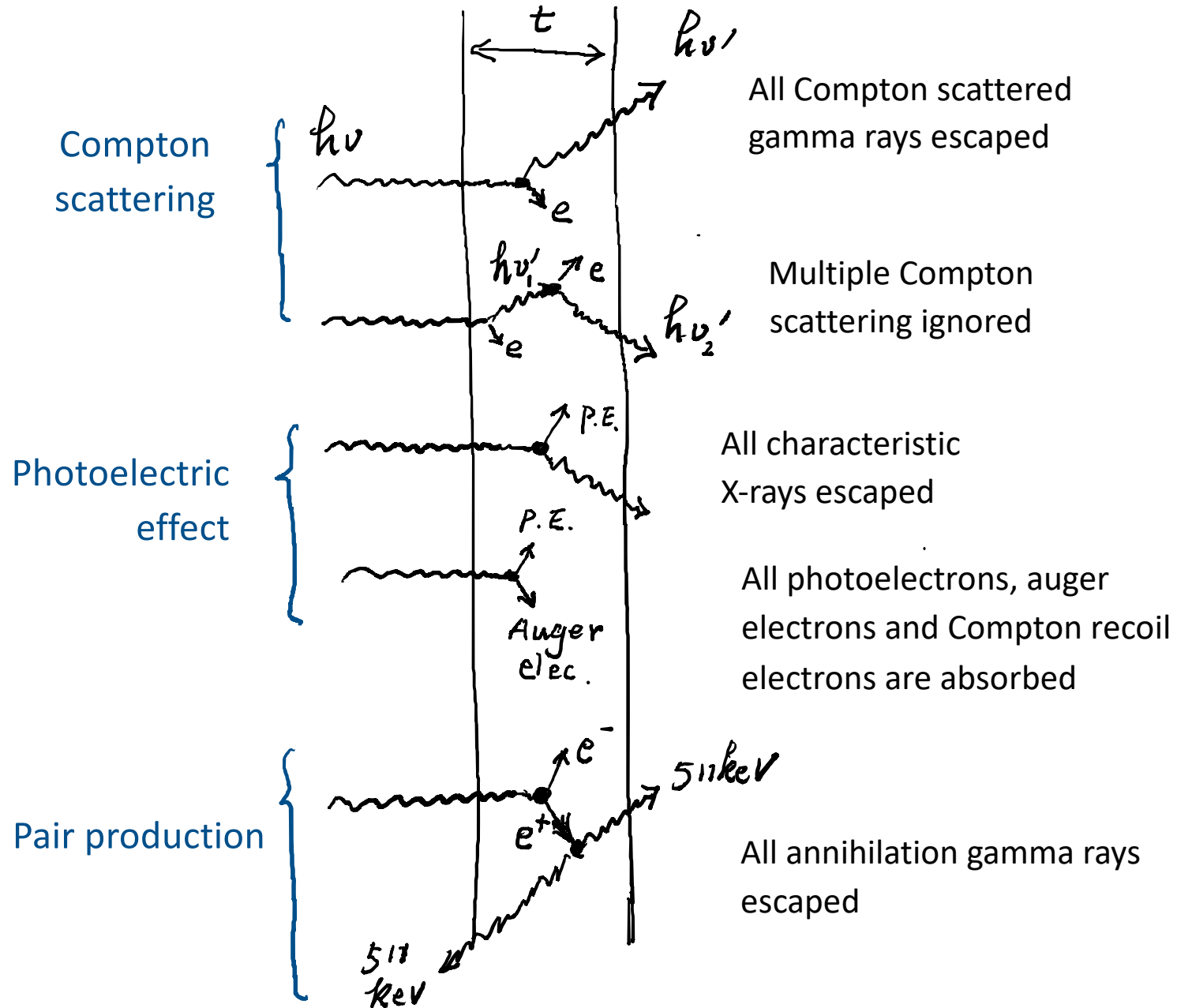
$$\dot{\Phi} = \frac{d\Phi}{dt} (m^{-2}s^{-1})$$

The energy fluence Ψ (Jm^{-2}): the amount of energy passes per unit area perpendicular to the beam.

The energy fluence rate ($Jm^{-2}s^{-1}$): the amount of energy transfer per unit area per unit time.

$$\dot{\Psi} = \frac{d\Psi}{dt} (Jm^{-2}s^{-1})$$

Energy Transfer by a Gamma Ray Beam

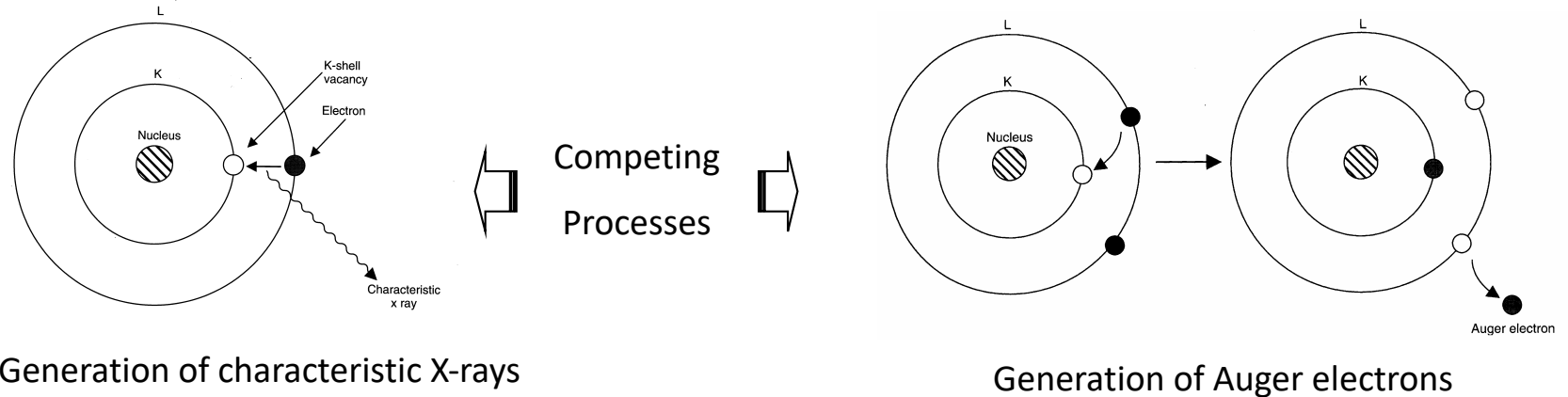


What is Energy Transfer Coefficients?

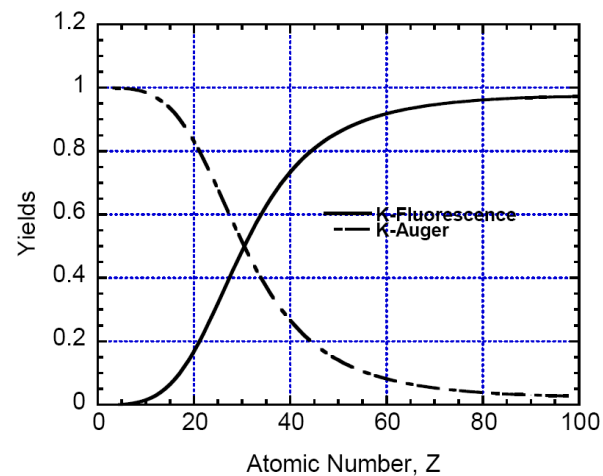
For a parallel beam of monochromatic gamma rays transmitting through a unit distance in an absorbing material, **the energy-transfer coefficient is the fraction of energy that was originally carried by the incident gamma ray beam and transferred into the kinetic energy of secondary electron** inside the absorber.

Relaxation Process after Photoelectric Effect

The excited atoms will **de-excite** through one of the following processes:



Auger electron emission dominates in **low-Z** elements. **Characteristic X-ray** emission dominates in **higher-Z** elements.



Energy-Transfer Coefficients through Photoelectric Effect

Based on these considerations, we can define the **mass energy transfer coefficient**, which is the fraction of photons that interact by photoelectric absorption per $\text{g}\cdot\text{cm}^{-2}$,

$$\frac{\tau_{tr}}{\rho} = \frac{\tau}{\rho} \left(1 - \frac{\delta}{h\nu} \right)$$

δ is the fraction of the gamma ray energy got converted into characteristic x-rays following the photoelectric interaction.

Note that the photoelectrons and Auger electrons may also lead to secondary photon emission through Bremsstrahlung. So the energy transfer coefficient defined above does not fully describe **energy absorption** in the slab. We will return to this point later.

Energy-Transfer Coefficients Through Compton Scattering

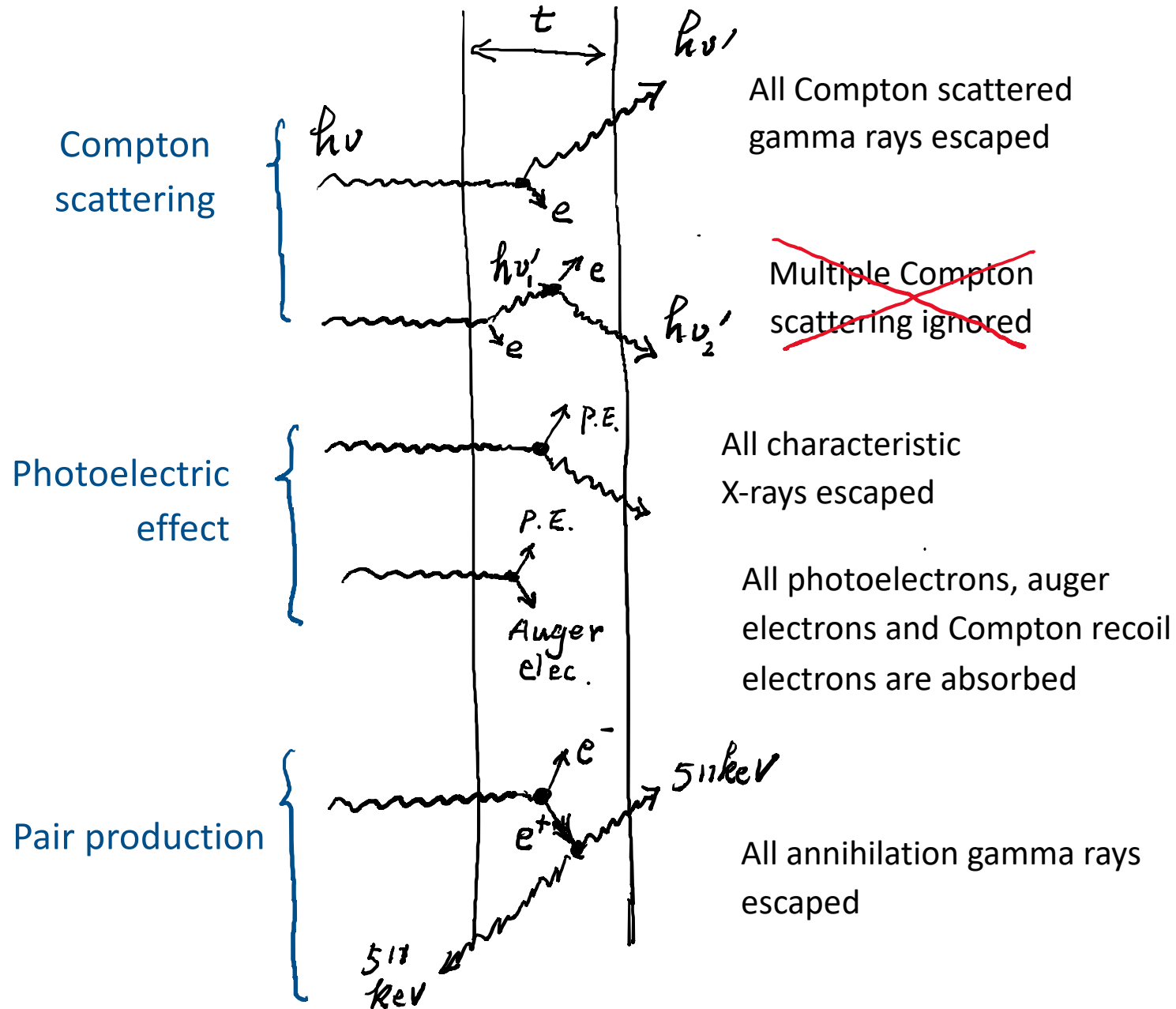
For Compton scattering of monoenergetic photons, the mass energy transfer coefficient

$$\frac{\sigma_{tr}}{\rho} = \frac{\sigma}{\rho} \left(\frac{E_{avg}}{h\nu} \right)$$

The factor $E_{avg}/h\nu$ is the fraction of the incident photon energy that is converted into the initial kinetic energy of Compton electrons.

As with the photoelectric effect, the above mass energy transfer coefficient does not take into account the subsequent photon emission due to bremsstrahlung.

Energy Transfer by a Gamma Ray Beam



Energy Distribution of Compton Recoil Electrons

Klein-Nishina formula can be used to calculate the expected energy spectrum of recoil electrons as the following:

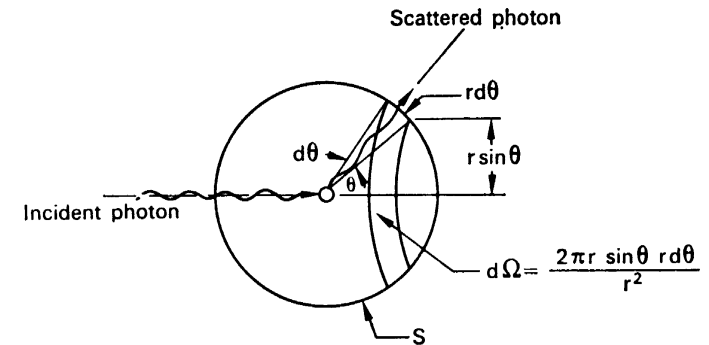
$$\frac{d\sigma}{dE_{recoil}} = \frac{d\sigma}{d\Omega} \frac{d\Omega}{d\theta} \frac{d\theta}{dE_{recoil}}$$

and the probability that a recoil electron possesses an energy between $E_{recoil} - \Delta E/2$ and $E_{recoil} + \Delta E/2$ is given by

$$\propto \Delta E \cdot \frac{d\sigma}{dE_{recoil}}$$

Energy Distribution of Compton Recoil Electrons

$$\frac{d\sigma}{dE_{recoil}} = \frac{d\sigma}{d\Omega} \frac{d\Omega}{d\theta} \frac{d\theta}{dE_{recoil}}$$



The three partial derivative terms on the right-hand side of the equation can be derived from the following relationships,

Klein-Nishina Formula

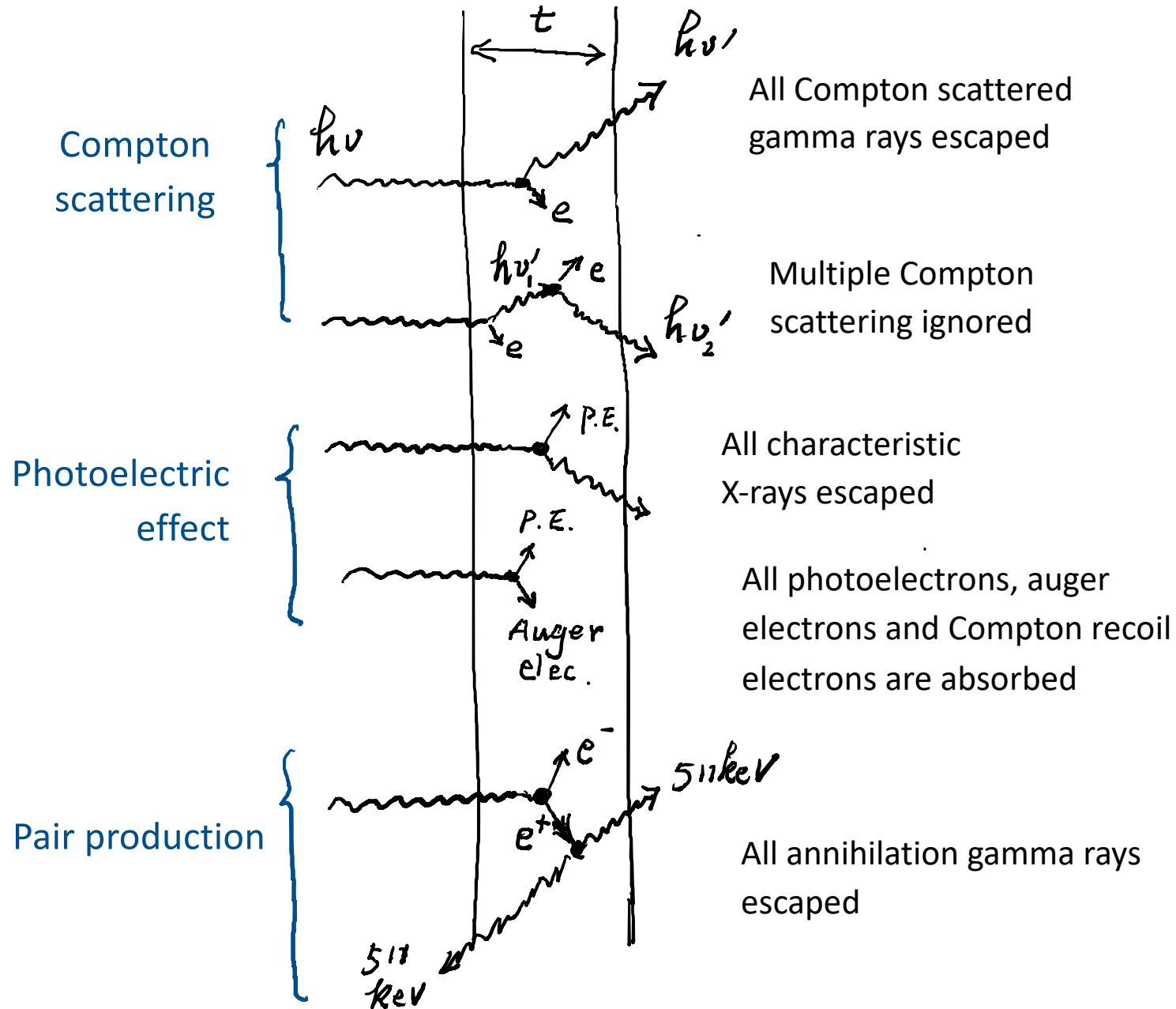
$$\frac{d\sigma}{d\Omega}(\theta) = r_e^2 \left(\frac{1}{1 + \alpha(1 - \cos \theta)} \right)^2 \left(\frac{1 + \cos^2 \theta}{2} \right) \left(1 + \frac{\alpha^2(1 - \cos \theta)^2}{(1 + \cos^2 \theta)[1 + \alpha(1 - \cos \theta)]} \right)$$

$$E_{recoil} = hv - hv' = hv - \frac{hv}{1 + \frac{hv}{m_0c^2}(1 - \cos \theta)} \Rightarrow$$

$$\frac{d\theta}{dE_{recoil}} = - \frac{m_0c^2}{E_{recoil}^2 \sin \theta} = - \frac{m_0c^2}{\left[hv - \frac{hv}{1 + \frac{hv}{m_0c^2}(1 - \cos \theta)} \right]^2 \sin \theta}$$

$$d\Omega = 2\pi \sin \theta d\theta \Rightarrow \frac{d\Omega}{d\theta} = 2\pi \sin \theta$$

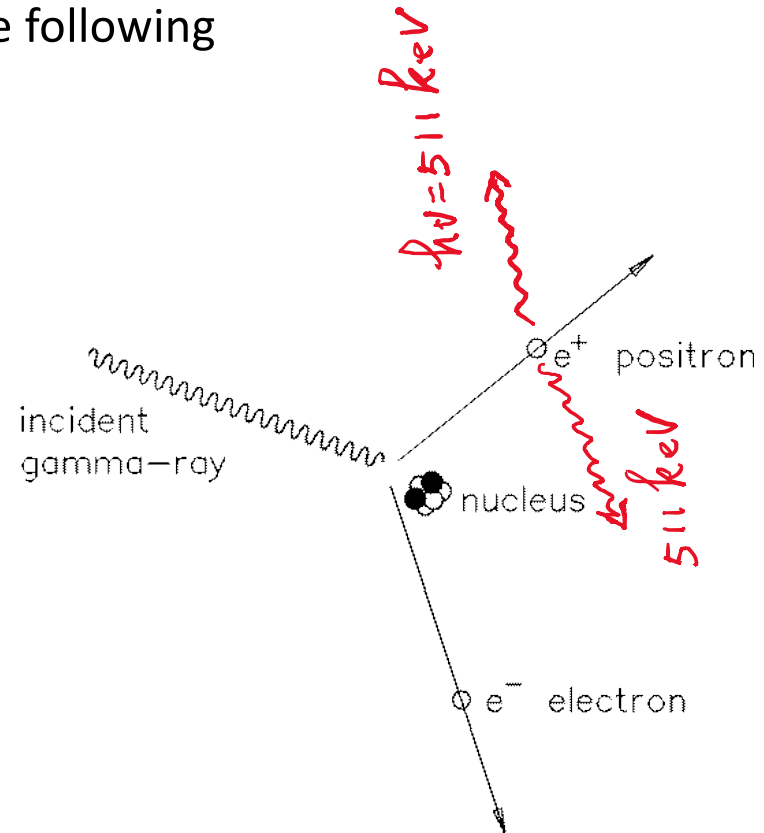
Energy Transfer by a Gamma Ray Beam



Energy-Transfer Coefficient through Pair Production

For **pair production process**, the initial energy carried by the electron-positron pair is $h\nu - 2mc^2$. Therefore the mass energy transfer coefficient for pair production is related to the mass attenuation coefficient as the following

$$\frac{\kappa_{tr}}{\rho} = \frac{\kappa}{\rho} \left(1 - \frac{2mc^2}{h\nu} \right)$$



Energy-Transfer Coefficients

The **total mass energy transfer coefficient** is given by

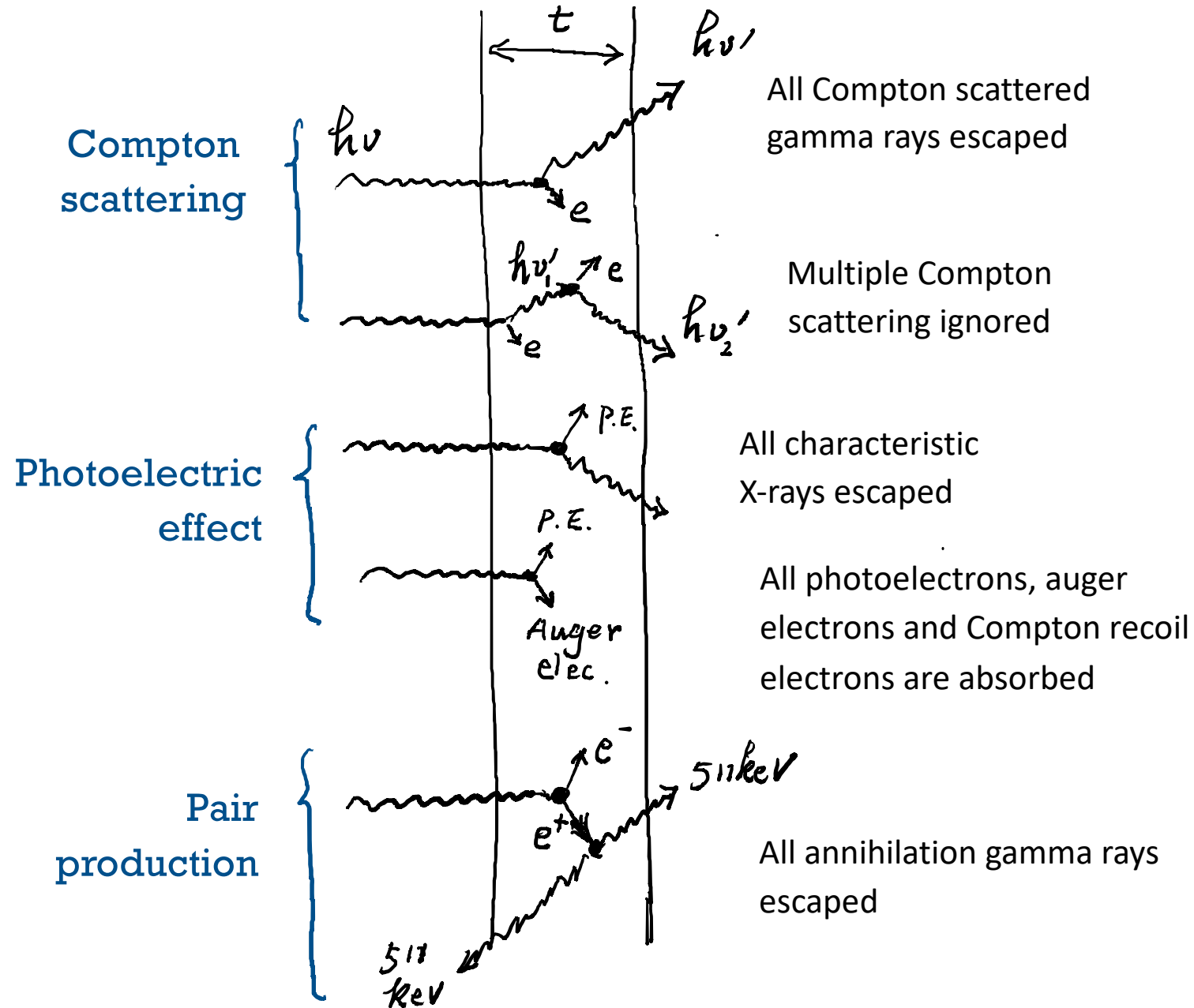
The fraction of energy that is carried away by characteristic x-rays following photoelectric effect.

The fraction of energy that is carried away by the two 511keV gamma rays generated by the annihilation of the positron.

$$\frac{\mu_{tr}}{\rho} = \frac{\tau}{\rho} \left(1 - \frac{\delta}{h\nu} \right) + \frac{\sigma}{\rho} \left(\frac{E_{avg}}{h\nu} \right) + \frac{\kappa}{\rho} \left(1 - \frac{2mc^2}{h\nu} \right)$$

The fraction of energy that is transferred to recoil electron through Compton scattering.

Energy Transfer by a Gamma Ray Beam



What is **Energy Absorption Coefficients**?

The **total mass energy transfer coefficient** is given by

$$\frac{\mu_{tr}}{\rho} = \frac{\tau}{\rho} \left(1 - \frac{\delta}{h\nu} \right) + \frac{\sigma}{\rho} \left(\frac{E_{avg}}{h\nu} \right) + \frac{\kappa}{\rho} \left(1 - \frac{2mc^2}{h\nu} \right)$$

Consider the fraction of energy that may be carried away by the subsequent bremsstrahlung photons, one can define the **mass energy-absorption coefficient** as

$$\frac{\mu_{en}}{\rho} = \frac{\mu_{tr}}{\rho} (1 - g)$$

where g is the average fraction of energy of the initial kinetic energy transferred to electrons that is subsequently emitted as bremsstrahlung photons.

Energy Loss by Bremsstrahlung

- For beta particles to stop in a given medium, the **fraction of energy loss** by **Bremsstrahlung** process is approximately given by

$$f_{\beta} = 3.5 \times 10^{-4} Z E_m,$$

where f_{β} = the fraction of the incident beta energy converted into photons,
 Z = atomic number of the absorber,
 E_m = maximum energy of the beta particle, MeV.

Mass Energy Absorption Coefficient

Mass attenuation coefficient

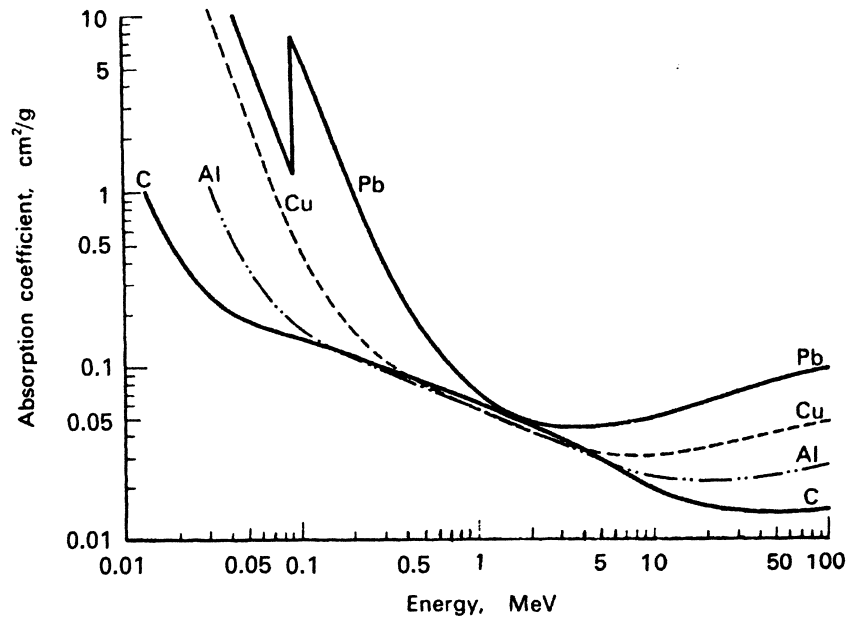


FIGURE 5.12. Curves illustrating the systematic variation of attenuation coefficient with atomic number of absorber and with quantum energy.

Mass energy absorption coefficient

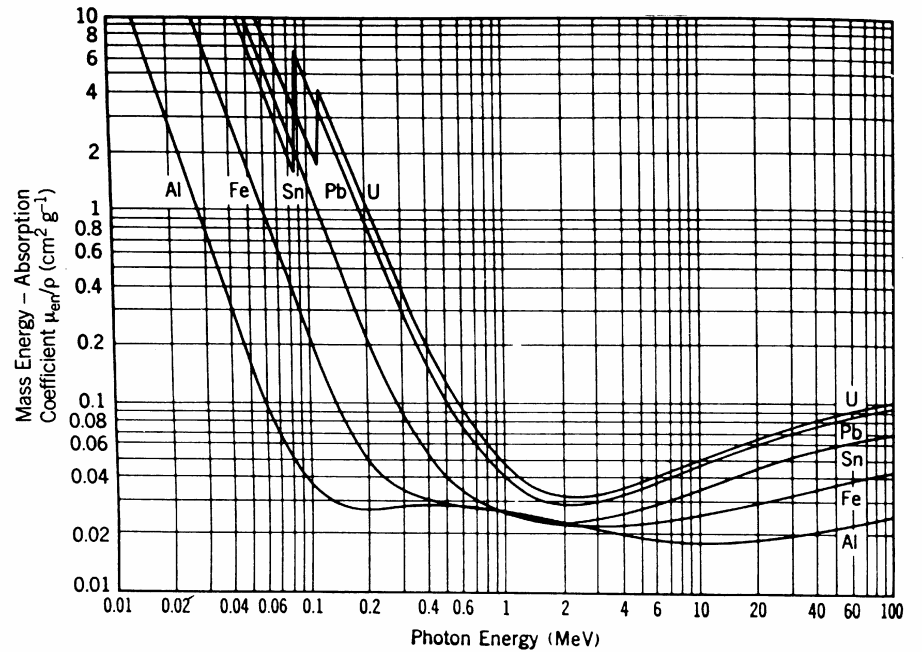


FIGURE 8.11. Mass energy-absorption coefficients for various elements. [Reprinted with permission from K. Z. Morgan and J. E. Turner, eds., *Principles of Radiation Protection*, Wiley, New York (1967). Copyright 1967 by John Wiley & Sons.]

Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p195

Mass Energy Absorption Coefficient

Mass attenuation coefficient

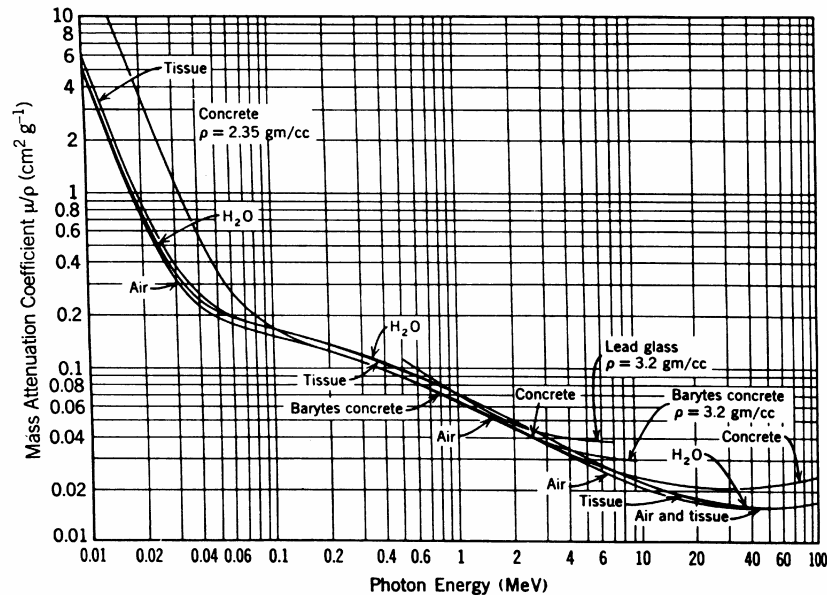


FIGURE 8.9. Mass attenuation coefficients for various materials. [Reprinted with permission from K. Z. Morgan and J. E. Turner, eds., *Principles of Radiation Protection*, Wiley, New York (1967). Copyright 1967 by John Wiley & Sons.]

Mass energy absorption coefficient

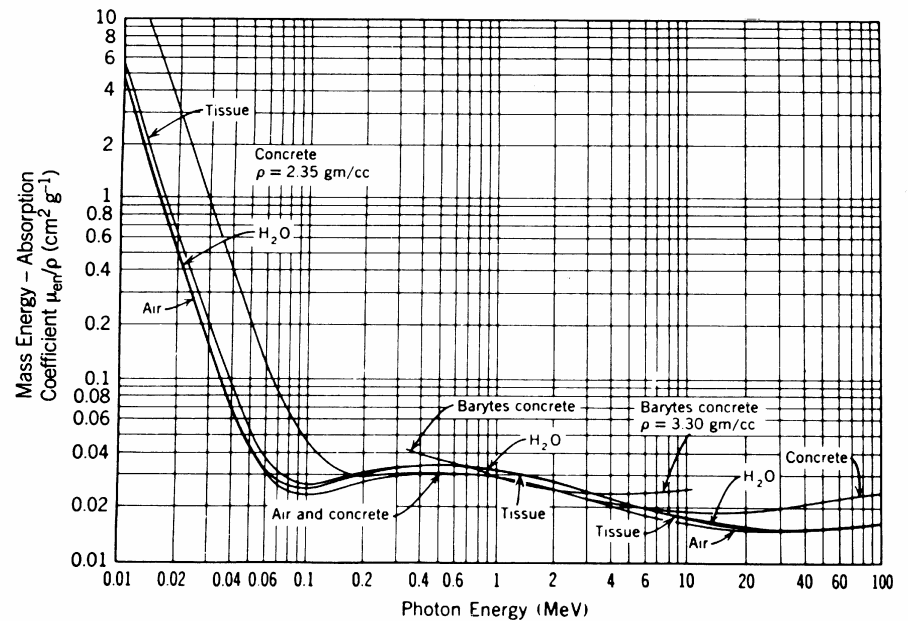


FIGURE 8.12. Mass energy-absorption coefficients for various materials. [Reprinted with permission from K. Z. Morgan and J. E. Turner, eds., *Principles of Radiation Protection*, Wiley, New York (1967). Copyright 1967 by John Wiley & Sons.]

Energy-Transfer and Energy Absorption Coefficients

TABLE 8.3. Mass Attenuation, Mass Energy-Transfer, and Mass Energy-Absorption Coefficients ($\text{cm}^2 \text{g}^{-1}$) for Photons in Water and Lead

Photon Energy (MeV)	Water			Lead		
	μ/ρ	μ_{tr}/ρ	μ_{en}/ρ	μ/ρ	μ_{tr}/ρ	μ_{en}/ρ
0.01	5.33	4.95	4.95	131.	126.	126.
0.10	0.171	0.0255	0.0255	5.55	2.16	2.16
1.0	0.0708	0.0311	0.0310	0.0710	0.0389	0.0379
10.0	0.0222	0.0163	0.0157	0.0497	0.0418	0.0325
100.0	0.0173	0.0167	0.0122	0.0931	0.0918	0.0323

Source: Based on P. D. Higgins, F. H. Attix, J. H. Hubbell, S. M. Seltzer, M. J. Berger, and C. H. Sibata, *Mass Energy-Transfer and Mass Energy-Absorption Coefficients, Including In-Flight Positron Annihilation for Photon Energies 1 keV to 100 MeV*, NISTIR 4680, National Institute of Standards and Technology, Gaithersburg, MD (1991).

- ☞ As expected, bremsstrahlung is relatively unimportant for photon energy of less than $\sim 10\text{MeV}$, whilst it accounts for a significant difference between the mass energy-transfer coefficient and the mass energy absorption coefficient.

Comparison Between Linear Attenuation Coefficient and Energy Absorption Coefficient

The fraction of energy carried by the photons being absorbed in material.

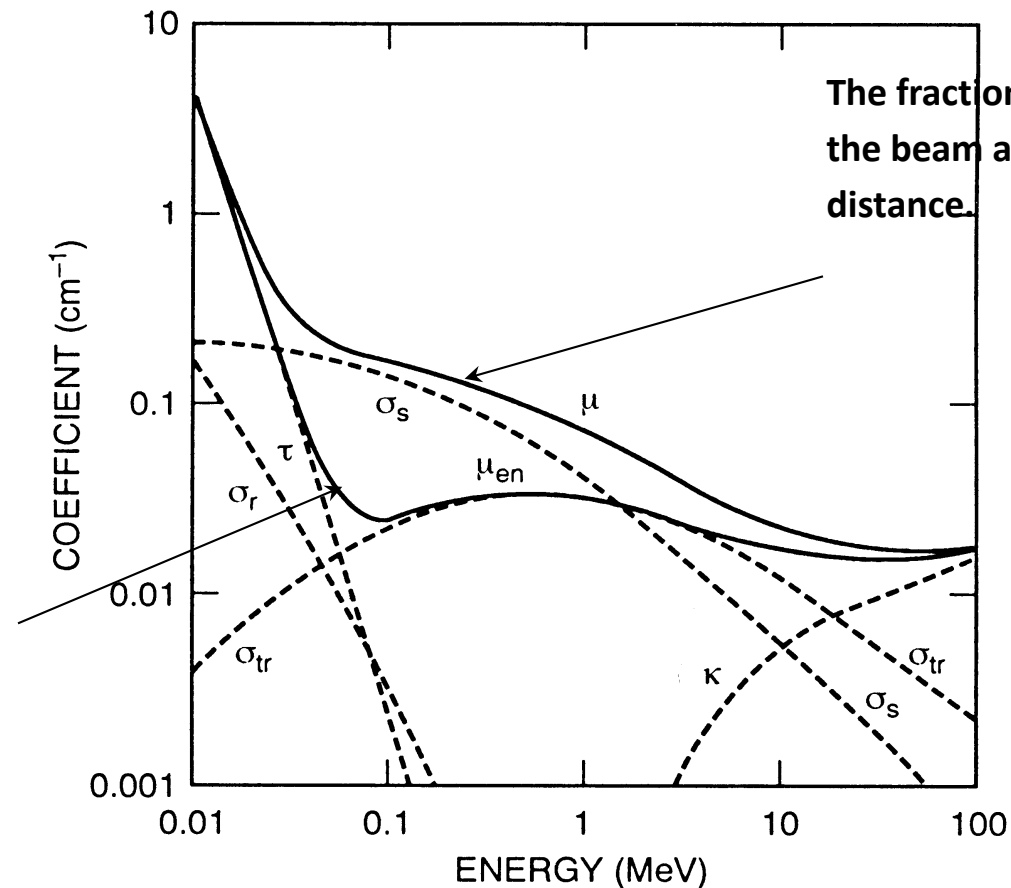


FIGURE 8.13. Linear attenuation and energy-absorption coefficients as functions of energy for photons in water.

Calculation of Energy Transfer and Energy Absorption

For simplicity, we consider an idealized case, in which

- ☞ Photons are assumed to be monoenergetic and in broad parallel beam.
- ☞ Multiple Compton scattering of photons is negligible.
- ☞ Virtually all fluorescence and bremsstrahlung photons escape from the absorber.
- ☞ All secondary electrons (Auger electrons, photoelectrons and Compton electrons) generated are stopped in the slab.

Under these conditions, the transmitted energy intensity (the amount of energy transmitted through a unit area within each second) can be given by

$$\dot{\Psi} = \dot{\Psi}_0 e^{-\mu_{en}x}$$

Calculation of Energy Transfer and Energy Absorption

Assuming $\mu_{en}x \ll 1$, which is consistent with the thin slab approximation and the energy fluence rate carried by the incident gamma ray beam is $\dot{\Psi}_0 (J \cdot cm^{-2} \cdot s^{-1})$. Then the energy absorbed in the thin slab per second over a unit cross section area is given by

$$\dot{\Psi}_0 \mu_{en} x \quad (J \cdot cm^{-2} \cdot s^{-1})$$

The rate of energy absorbed in the slab of area $A (cm^2)$ and thickness x is

$$A \dot{\Psi}_0 \mu_{en} x \quad (J \cdot s^{-1})$$

Given the density of the material is ρ , the rate of energy absorption per unit mass (**Dose Rate**) in the slab is

$$\dot{D} = \frac{A(cm^2) \cdot \dot{\Psi}_0 (J \cdot cm^{-2} \cdot s^{-1}) \cdot \mu_{en}(cm^{-1}) \cdot x (cm)}{\rho(g \cdot cm^{-3}) \cdot A(cm^2) \cdot x(cm)},$$

$$\text{Dose rate in the absorber: } \dot{D} = \dot{\Psi}_0 \frac{\mu_{en}}{\rho} (J \cdot g^{-3} \cdot s^{-1})$$

Calculation of Energy Transfer and Energy Absorption

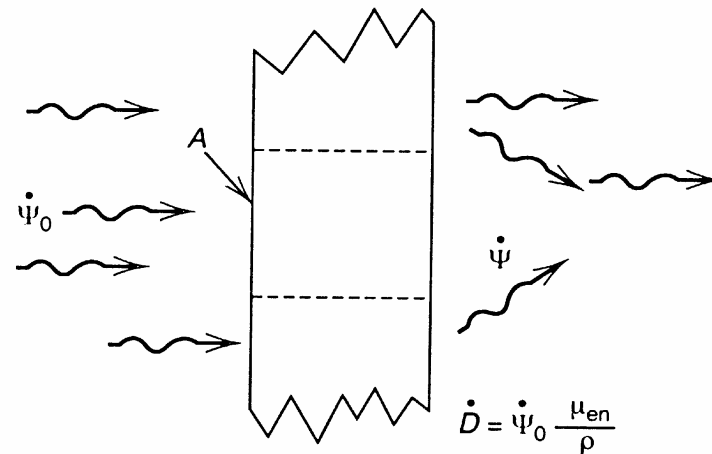


FIGURE 8.14. Rate of energy absorption per unit mass in thin slab (dose rate, \dot{D}) is equal to the product of the incident intensity and mass energy-absorption coefficient.

- ☞ The thin slab geometry discussed is approached in practice only by various degrees of approximation.
- ☞ Non-uniformity and finite width of real beams are two examples that deviate from the ideal.

Calculation of Energy Transfer and Energy Absorption

Example

A ^{137}Cs source is stored in a laboratory. The photon fluence rate in air at a point in the neighborhood of the source is $5.14 \times 10^7 \text{ m}^{-2} \text{ s}^{-1}$. Calculate the rate of energy absorption per unit mass (dose rate) in the air at that point.

Solution

The desired quantity is given by Eq. (8.61). The mass energy-absorption coefficient of air for the photons emitted by ^{137}Cs ($h\nu = 0.662 \text{ MeV}$, Appendix D) is, from Fig. 8.12, $\mu_{\text{en}}/\rho = 0.030 \text{ cm}^2 \text{ g}^{-1}$. The incident fluence rate is $\dot{\Phi} = 5.14 \times 10^7 \text{ m}^{-2} \text{ s}^{-1}$, and so the energy fluence rate is

$$\dot{\Psi} = \dot{\Phi} h\nu = 3.40 \times 10^7 \text{ MeV m}^{-2} \text{ s}^{-1} = 3.40 \times 10^3 \text{ MeV cm}^{-2} \text{ s}^{-1}. \quad (8.66)$$

Thus, Eq. (8.61) gives

$$\dot{D} = \dot{\Psi} \frac{\mu_{\text{en}}}{\rho} = 102 \text{ MeV g}^{-1} \text{ s}^{-1}. \quad (8.67)$$

Expressed in SI units,

$$\dot{D} = \frac{102 \text{ MeV}}{\text{g s}} \times 1.60 \times 10^{-13} \frac{\text{J}}{\text{MeV}} \times 10^3 \frac{\text{g}}{\text{kg}} \quad (8.68)$$

$$= 1.63 \times 10^{-8} \text{ J kg}^{-1} \text{ s}^{-1} = 0.0587 \text{ mGy h}^{-1}. \quad (8.69)$$