Neutron Dose Calculation

Neutron dose to tissue:

- Fast neutron dose for elastic scattering.
- Thermal neutron dose
  - neutron capture by H → gamma ray dose.
  - neutron capture by N → dose from the recoil nucleus.

Neutron Sources – Spontaneous Fission

Cf-252 neutron source can be made extremely compact

An engineer tests the prototype Timed Neutron Detector, a device that detects landmines. The neutron source of the landmine detector holds a tiny amount of californium-252. (Photo credit: Pacific Northwest National Lab)

Thermal Neutron Capture Radiation Therapy

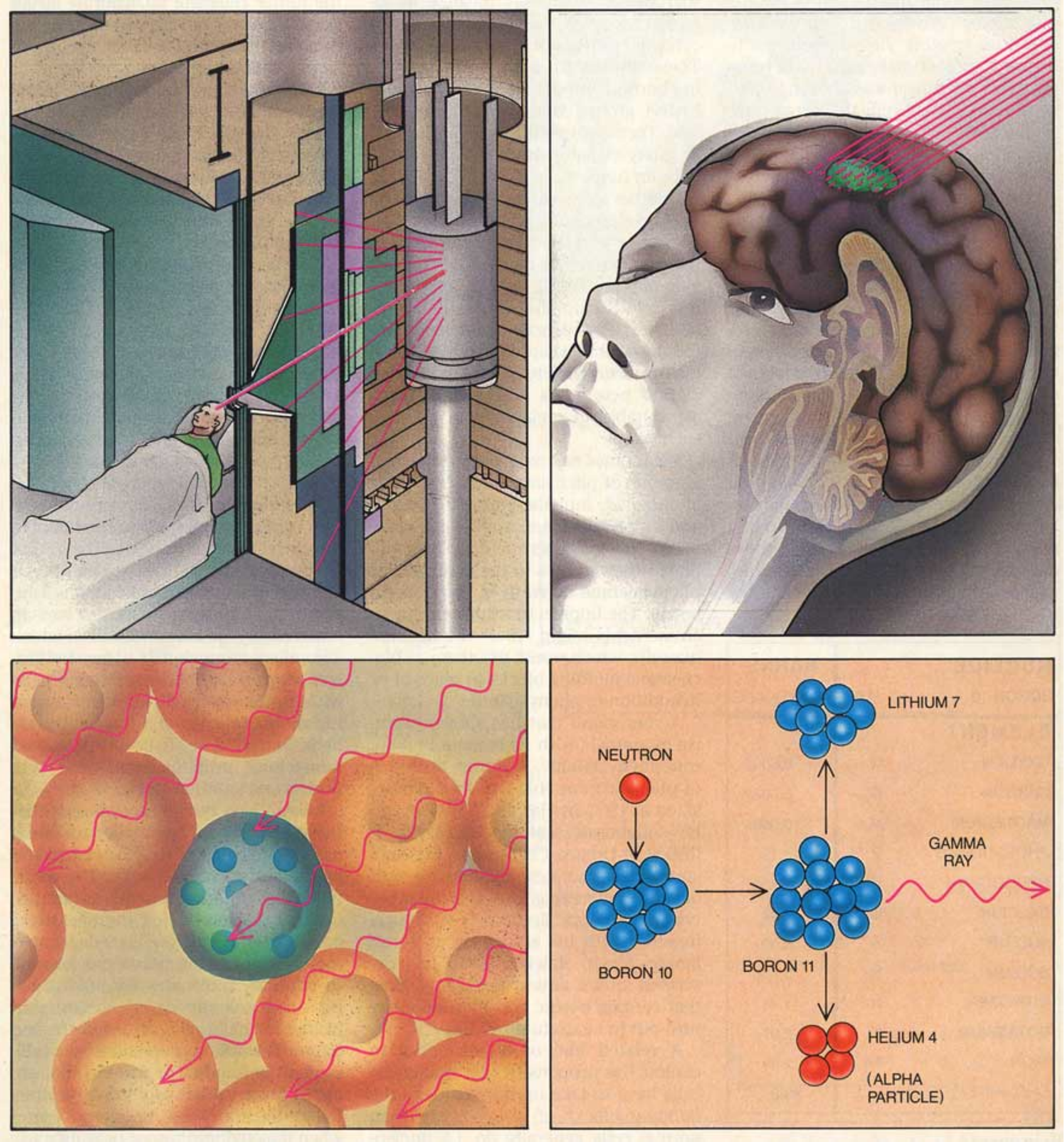


Fig.1 boron neutron capture therapy (BNCT) can be performed at a facility with a nuclear reactor or at hospitals that have developed alternative neutron sources. A beam of epithermal neutrons penetrates the brain tissue, reaching the malignancy. Once there the epithermal neutrons slow down and these low-energy neutrons combine with boron-10 (delivered beforehand to the cancer cells by drugs or antibodies) to form boron-11, releasing lethal radiation (alpha particles and lithium ions) that can kill the tumor.[1]

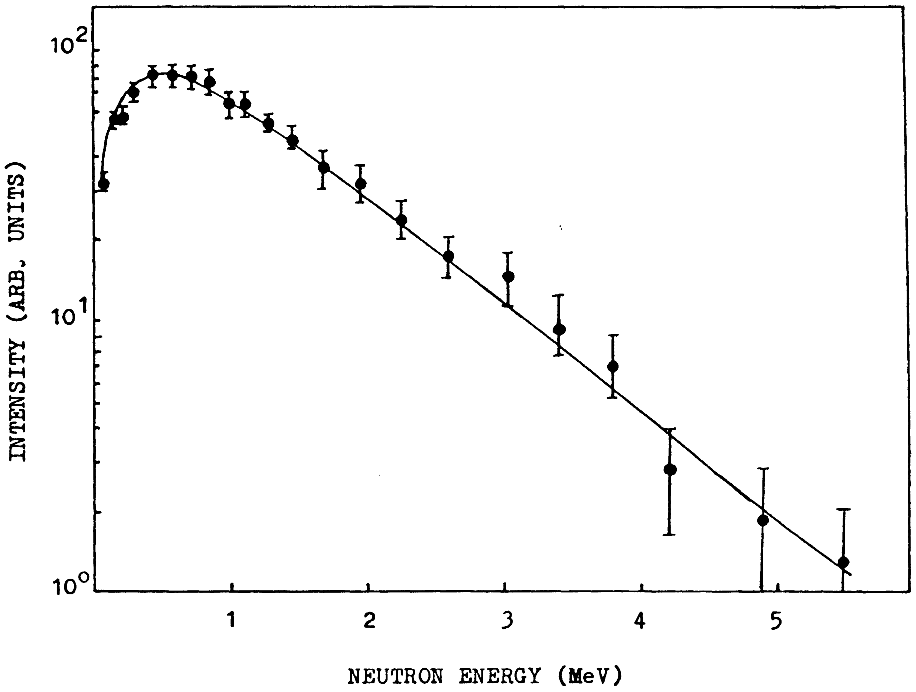
Figure from: http://en.wikipedia.org/wiki/File:NeutronCaptureTherapyImage.jpg

Barth, Rolf F.; Soloway, Albert H.; Fairchild, Ralph G. (1990). "Boron Neutron Capture Therapy for Cancer". Scientific American 263 (4): 100–3, 106–7. Bibcode:1990SciAm.263d.100B. doi:10.1038/scientificamerican1090-100. PMID 2173134.

Neutron Sources – Spontaneous Fission

***Spontaneous fission of tarnsuranic heavy nuclides**, such as ^{252}Cf, produces several fast neutrons, in addition to heavy fission products, prompt fission gamma rays and beta and gamma ray activities.*

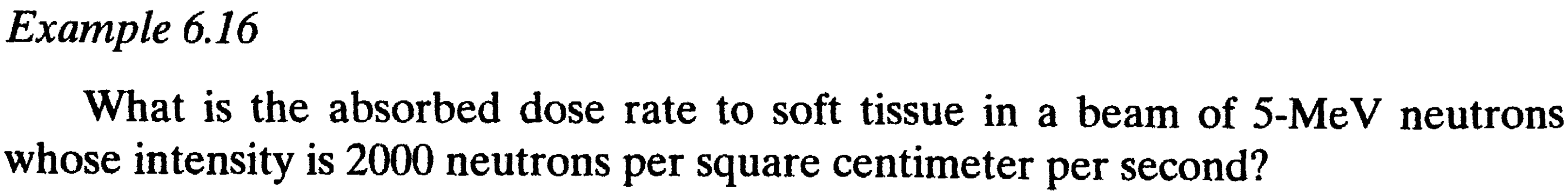
- ***Half-life**: 2.65 years*
- *Neutron yield: 0.116n/s per Bq, or 2.3 × 10^6 n/s per mg*
- ***Neutron energy** peaking at 0.5MeV and extends beyond 10MeV.*



Measured neutron energy spectrum from spontaneous fission of ^{252}Cf

Page 19-27, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Radiation Dose from Fast Neutrons



Example 6.16

What is the absorbed dose rate to soft tissue in a beam of 5-MeV neutrons whose intensity is 2000 neutrons per square centimeter per second?

Radiation Dose from Fast Neutrons

- Neutron dose is deposited through scattering and neutron induced nuclear reactions.
- In cases of elastic scattering, the scattered nuclei dissipate their energy in the immediate vicinity of the primary neutron interaction. The radiation dose absorbed locally in this way is called the **first collision dose**. The scattered neutron is **not** considered after this primary interaction.
- For fast neutrons, the first collision dose rate is given by

$$\dot{D}_n(E) = \frac{\phi(E)E\sum_i N_i\sigma_i f}{1\ \text{J/kg}\cdot\text{Gy}}, \qquad (6.103)$$

where $\phi\,(E)$ = flux of neutrons whose energy is E, in neutrons/cm^2. s,
E = neutron energy, in joules,
N_i = atoms per kilogram of the ith element,
σ_i = scattering across section of the ith element for neutrons of energy E, in barns $\times\ 10^{-24}$ cm^2,
f = mean fractional energy transferred from neutron to scattered atom during collision with neutron.

Elastic Scattering of Neutrons

The **maximum energy** that a neutron of mass m and kinetic energy E_n can transfer to a nucleus of mass M in a single elastic collision given by

$$E_{\max} = E_n \frac{4Mm}{(M+m)^2}$$

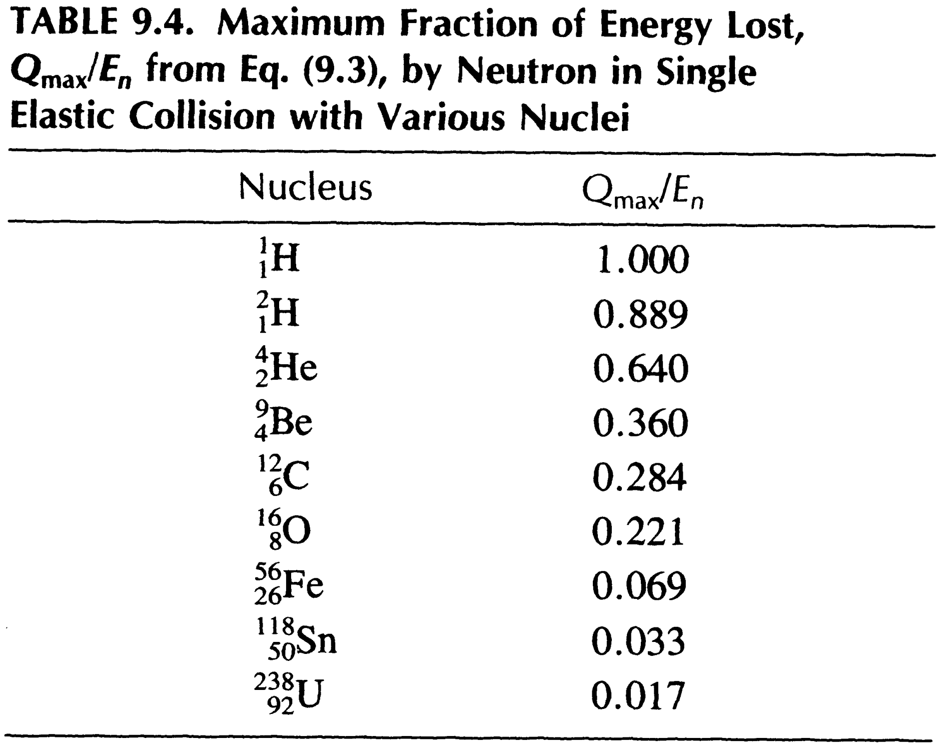


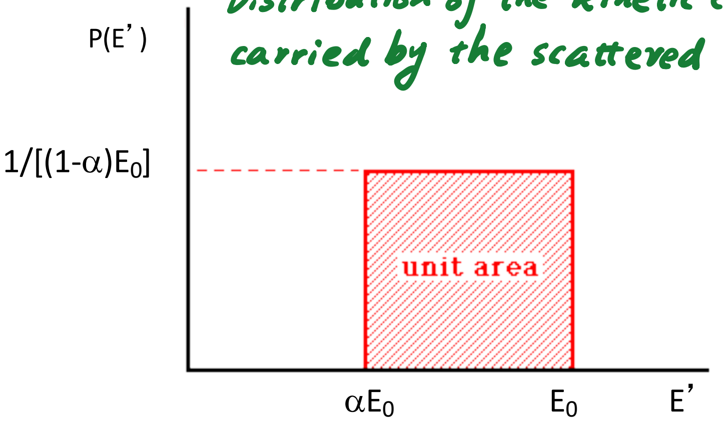
TABLE 9.4. Maximum Fraction of Energy Lost, $Q_{\max}/E_n$ from Eq. (9.3), by Neutron in Single Elastic Collision with Various Nuclei

Nucleus	$Q_{\max}/E_n$
${}^{1}_{1}\mathrm{H}$	1.000
${}^{2}_{1}\mathrm{H}$	0.889
${}^{4}_{2}\mathrm{He}$	0.640
${}^{9}_{4}\mathrm{Be}$	0.360
${}^{12}_{6}\mathrm{C}$	0.284
${}^{16}_{8}\mathrm{O}$	0.221
${}^{56}_{26}\mathrm{Fe}$	0.069
${}^{118}_{50}\mathrm{Sn}$	0.033
${}^{238}_{92}\mathrm{U}$	0.017

Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p213

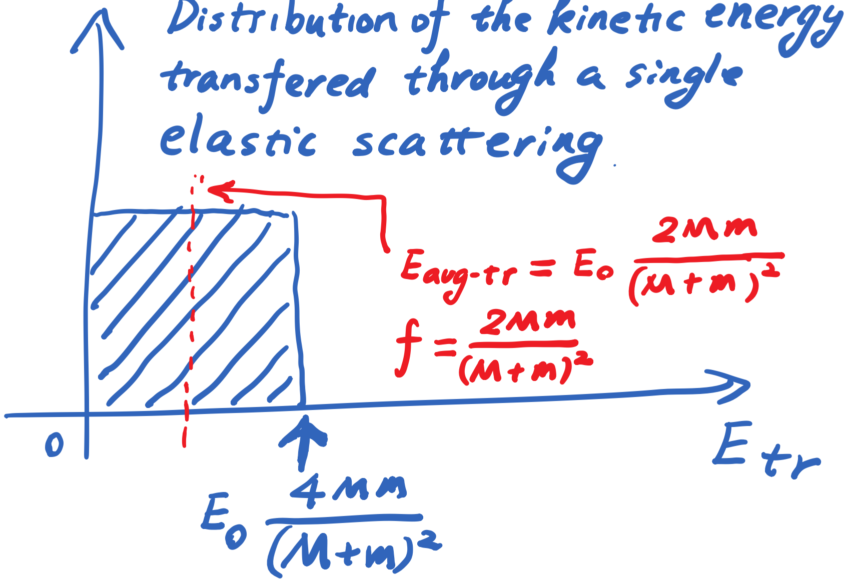
Energy Distributions of Scattered Neutrons

Distribution of the kinetic energy carried by the scattered neutrons



$$\alpha = \frac{(M-m)^2}{(M+m)^2}$$

Distribution of the kinetic energy transferred through a single elastic scattering



$$E_{avg\text{-}tr} = E_0 \frac{2Mm}{(M+m)^2}$$

$$f = \frac{2Mm}{(M+m)^2}$$

The fraction of energy carried by the scattered neutron is

$$\frac{E'}{E_0} = \frac{M^2 + m^2 + 2Mm\cos\theta}{(M+m)^2}$$

The distribution of the energy of the scattered neutrons is given by

$$p(E) = \frac{1}{1-\alpha}\frac{1}{E_0}, \ \mathrm{E} \subset [\alpha E_0, E_0].$$

Radiation Dose from Fast Neutrons

- For isotropic scattering, the average fraction of energy transferred in a elastic scattering with a nucleus of atomic mass number M is

$$f = \frac{2M}{(M+1)^2}$$

- The composition of soft tissue is shown below

TABLE 6.12. Synthetic Tissue Composition

Element	% Mass	N, atoms/kg	f
Oxygen	71.39	2.69×10^{25}	0.111
Carbon	14.89	6.41×10^{24}	0.142
Hydrogen	10.00	5.98×10^{25}	0.500
Nitrogen	3.47	1.49×10^{24}	0.124
Sodium	0.15	3.93×10^{22}	0.080
Chlorine	0.10	1.70×10^{22}	0.053

Source: Adapted from G. L. Brownell, W. H. Ellett, and A. R. Reddy, Absorbed Fractions for Photon Dosimetry. *J Nuclear Medicine*, Supplement No. 1, MIRD Pamphlet No. 3, February 1968. By permission.

The scattering cross sections of each of the tissue elements for 5-MeV neutrons are listed below:

ELEMENT	**σ, x 10^{-24}cm^2**	**$N_i\sigma_i f_i$**
O	1.55	4.628
C	1.65	1.502
H	1.50	4.485×10^1
N	1.00	1.848×10^{-1}
Na	2.3	7.231×10^{-3}
Cl	2.8	2.523×10^{-3}
		$\sum N_i\sigma_i f_i = 5.117 \times 10^1 \frac{\text{cm}^2}{\text{kg}}$

$$\dot{D}_n(E) = \frac{\phi(E)E\sum_i N_i\sigma_i f}{1\ \text{J/kg}\cdot\text{Gy}},$$

where $\phi(E)$ = flux of neutrons whose energy is E, in neutrons/cm^2. s,
E = neutron energy, in joules,
N_i = atoms per kilogram of the ith element,
σ_i = scattering across section of the ith element for neutrons of energy E, in barns $\times 10^{-24}$ cm^2,
f = mean fractional energy transferred from neutron to scattered atom during collision with neutron.

Radiation Dose from Fast Neutrons

Example 6.16

What is the absorbed dose rate to soft tissue in a beam of 5-MeV neutrons whose intensity is 2000 neutrons per square centimeter per second?

Substituting the appropriate values into Eq. (6.103) yields

$$\dot{D}_n = \frac{2 \times 10^3\ n/\text{cm}^2 \cdot \text{s} \times 5\ \text{MeV}/n \times 1.6 \times 10^{-13}\ \text{J/MeV} \times 51.17\ \text{cm}^2/\text{kg}}{1\ \text{J/kg} \cdot \text{Gy}}$$

$$= 8.19 \times 10^{-8}\ \text{Gy/s}\left(8.19 \times 10^{-6}\ \text{rad/s}\right),$$

or

$$8.19 \times 10^{-8}\ \text{Gy/s} \times 10^6\ \mu\text{Gy/Gy} \times 3.6 \times 10^3\ \text{s/h}$$

$$= 295\ \mu\text{Gy/h}\ (29.5\ \text{mrad/h}).$$

Dose from Thermal Neutron Capture

$$ {}_{0}^{1}n + {}_{11}^{23}Na \rightarrow {}_{11}^{24}Na + {}_{0}^{0}\gamma $$

- Cross section for thermal neutron is 0.534 barns.
- Q=0.626MeV.
- ^{24}Na undergo radioactive decay with the emission of two gamma rays, having energies of 2.75MeV and 1.37MeV per disintegration.
- Since ^{23}Na is a normal constituent of blood, activation of blood sodium can be sued as a dosimetric tool when persons are exposed to relatively high does of neutrons, for example, in a criticality accident.

Neutron Induced Reactions

$$_{0}^{1}n + _{7}^{14}N \rightarrow _{6}^{14}C + _{1}^{1}p$$

- ☞ Cross section for thermal neutron is 1.70 barns.
- ☞ Q=0.626MeV.
- ☞ Since the range of the proton and the ^{14}C nucleus are relatively small, their energy is deposited locally at the site where the neutron was captured.
- ☞ Capture by hydrogen and nitrogen are the only two processes through which neutron deliver a significant does to soft tissue.

Radiation Dose from Thermal Neutrons

- ☞ Two reactions are normally considered, namely $^{14}N(n,p)^{14}C$ and $^{1}H(n,r)^{2}H$ reactions.

- ☞ For the $^{14}N(n,p)^{14}C$ reaction, the dose is given by

$$\dot{D}_{np} = \frac{\phi N_N \sigma_N Q \times 1.6 \times 10^{-13}\ \mathrm{J/MeV}}{1\ \mathrm{J/kg \cdot Gy}},$$

where ϕ = thermal flux, neutrons per cm^2 per second,
N_N = number of nitrogen atoms per kg tissue, 1.49×10^{24},
σ_N = absorption cross section for nitrogen, 1.75×10^{-24} cm^2,
Q = energy released by the reaction = 0.63 MeV.

Neutron Induced Reactions

$$ {}_{0}^{1}n + {}_{1}^{1}H \rightarrow {}_{1}^{2}H + {}_{0}^{0}\gamma $$

- ☞ Neutron absorption followed by the immediate emission of a gamma ray photon.
- ☞ Since the thermal neutron has negligible energy by comparison, the gamma photon has the energy Q=2.22MeV released by the reaction, which represents the binding energy of the deuteron.
- ☞ The capture cross section per atom is 0.33barn.
- ☞ When tissue is exposed to thermal neutrons, this reaction provides a source of gamma rays that delivers dose to the tissue.

Radiation Dose from Thermal Neutrons

- ☞ For the $^{1}H(n, \gamma)^{2}H$ reaction, the dose is deposited by the gamma rays emitted throughout the entire volume. The number of reaction per second per gram is governed by the neutron flux and is given by

$$A = \phi N_H \sigma_H \text{ “Bq”/kg},$$

where ϕ = thermal flux, neutrons per cm^2 per second,

N_H = number of hydrogen atoms per kg tissue = 5.98×10^{25},

σ_H = absorption cross section for hydrogen = 0.33×10^{-24} cm^2.

- ☞ The resulting gamma ray dose is illustrated with the following example.

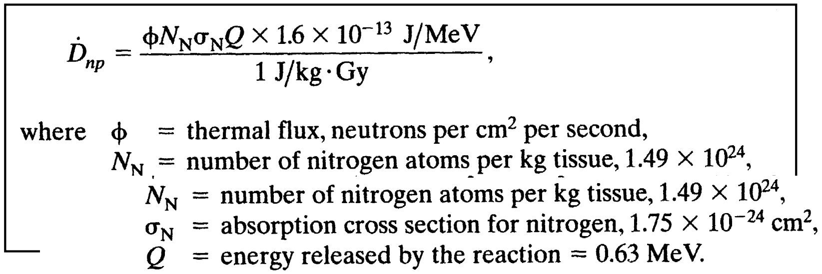
Example 6.17

What is the absorbed dose rate to a 70-kg person from a whole body exposure to a mean thermal flux of 10,000 neutrons per cm^2 per second?

The dose rate due to the n, p reaction is calculated from Eq. (6.105)

$$\dot{D}_{np} = 1 \times 10^4 \times 1.49 \times 10^{24} \times 1.75 \times 10^{-24} \times 0.63 \times 1.6 \times 10^{-13}$$

$$= 2.628 \times 10^{-9}\ \text{Gy/s} \quad (2.628 \times 10^{-7}\ \text{rad/s}),$$



$$\dot{D}_{np} = \frac{\phi N_N \sigma_N Q \times 1.6 \times 10^{-13}\ \text{J/MeV}}{1\ \text{J/kg}\cdot\text{Gy}},$$

where ϕ = thermal flux, neutrons per cm^2 per second,
N_N = number of nitrogen atoms per kg tissue, 1.49×10^{24},
N_N = number of nitrogen atoms per kg tissue, 1.49×10^{24},
σ_N = absorption cross section for nitrogen, 1.75×10^{-24} cm^2,
Q = energy released by the reaction = 0.63 MeV.

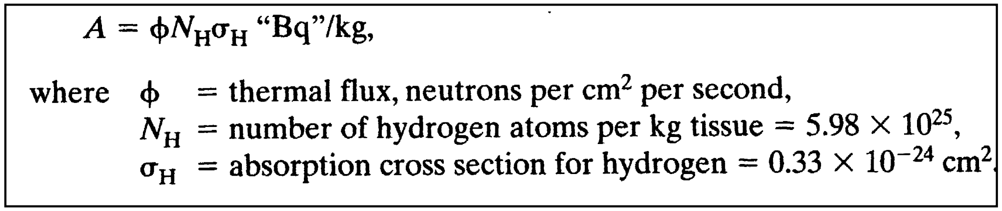
or

$$\dot{D}_{np} = 9.461\ \mu\text{Gy/h} \quad (0.95\ \text{mrad/h}).$$

The autointegral gamma-ray dose rate is calculated with Eq. (6.82). The gamma-ray "activity," from Eq. (6.106) is

$$A = 10^4\ \text{cm}^2\ \text{s}^{-1} \times 5.98 \times 10^{25}\ \text{atoms/kg} \times 3.3 \times 10^{-25}\ \text{cm}^2/\text{atom}$$

$$= 1.973 \times 10^5\ \text{“Bq”/kg}.$$



$$A = \phi N_H \sigma_H\ \text{“Bq”/kg},$$

where ϕ = thermal flux, neutrons per cm^2 per second,
N_H = number of hydrogen atoms per kg tissue = 5.98×10^{25},
σ_H = absorption cross section for hydrogen = 0.33×10^{-24} cm^2,

The dose rate from this uniformly distributed gamma ray activity is calculated from

$$\dot{D}_H = A \cdot E_r \cdot \varphi$$

The absorbed fraction, φ, for the 2.23-MeV gamma ray is found, by interpolating in Table 6.8 between the 2.000- and 4.000-MeV values, to be 0.278, and Δ, the dose rate

Radiation Dose from Neutrons as a Function of Depth

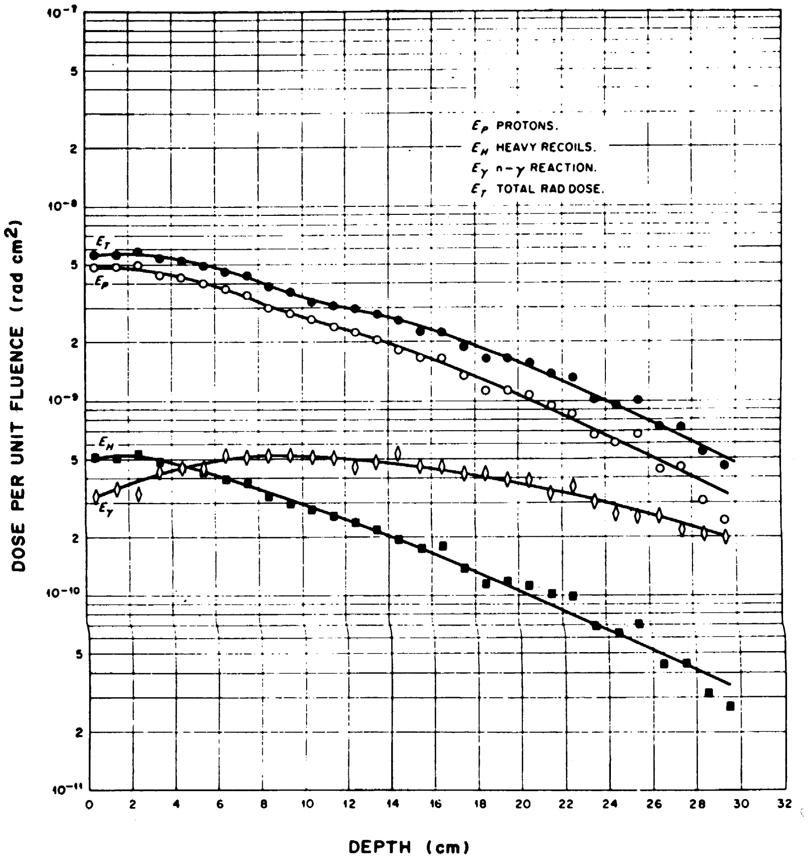


FIGURE 12.10. Depth–dose curves for a broad beam of 5-MeV neutrons incident normally on a soft-tissue slab. Ordinate gives dose per unit fluence at different depths shown by the abscissa. [From "Protection Against Neutron Radiation Up to 30 Million Electron Volts," in *National Bureau of Standards Handbook 63*, p. 44, Washington, D.C. (1957).]

Tuner, p373.

Radiation Dose from Neutrons as a Function of Depth

Figure 12.10 shows the results of Monte Carlo calculations carried out for 5-MeV neutrons incident normally on a 30-cm soft-tissue slab, approximating the thickness of the body. (The geometry is identical to that shown for the charged particles in Fig. 12.9.) The curve labeled E_T is the total dose, E_p is the dose due to H recoil nuclei (protons), E_γ is the dose from gamma rays from the $^1H(n,\gamma)^2H$ slow-neutron capture reaction, and E_H is the dose from the heavy (O, C, N) recoil nuclei. The total dose builds up somewhat in the first few cm of depth and then decreases as the beam becomes degraded in energy and neutrons are absorbed. The proton and heavy-recoil curves, E_p and E_H, show a similar pattern. As the neutrons penetrate, they are moderated and approach thermal energies. This is reflected in the rise of the gamma-dose curve, E_γ, which has a broad maximum over the region from about 6 cm to 14 cm. Note

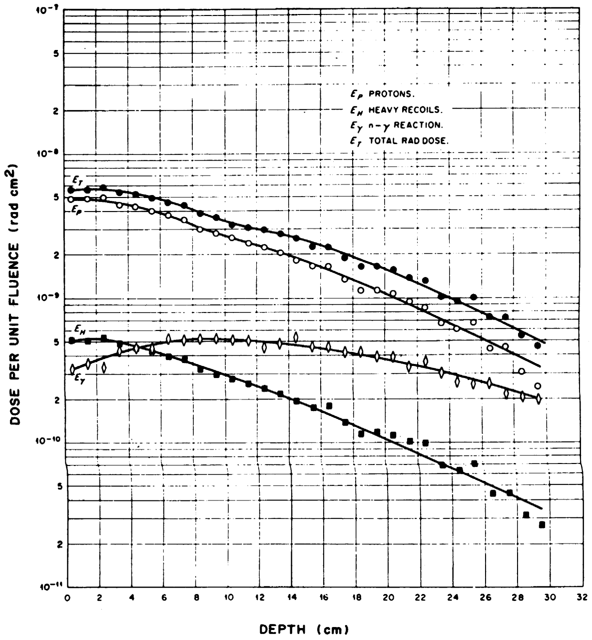


FIGURE 12.10. Depth–dose curves for a broad beam of 5-MeV neutrons incident normally on a soft-tissue slab. Ordinate gives dose per unit fluence at different depths shown by the abscissa. [From "Protection Against Neutron Radiation Up to 30 Million Electron Volts," in *National Bureau of Standards Handbook 63*, p. 44, Washington, D.C. (1957).]