Neutron Shielding Design

- Radiological assessment of radiation hazard associated with neutron sources is normally a complex task.
  - Involves both neutron radiation and secondary gamma ray radiation.
  - Few problems can be solved with elementary techniques.

- General treatment involves considerations on the dose delivered by the elastic scattering of fast neutrons and the thermal neutron capture gamma ray radiations.
- Neutron removal cross-section. Under special circumstances, the fast neutron dose after a shielding can be derived with an exponential attenuation factor.

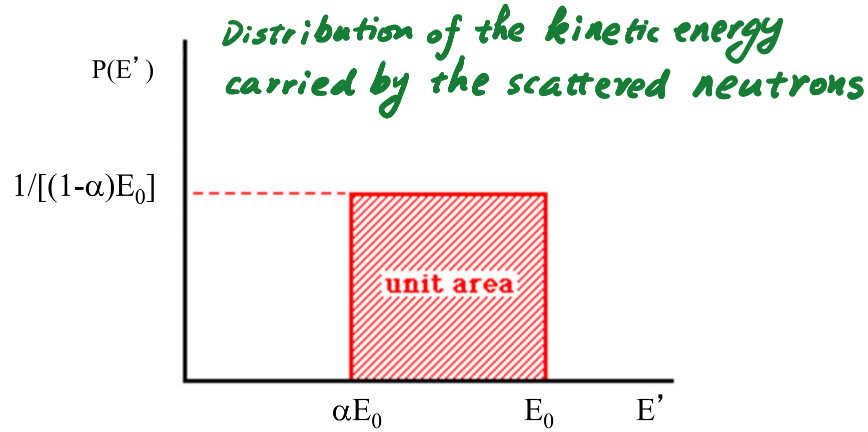
Radiation Dose from Fast Neutrons

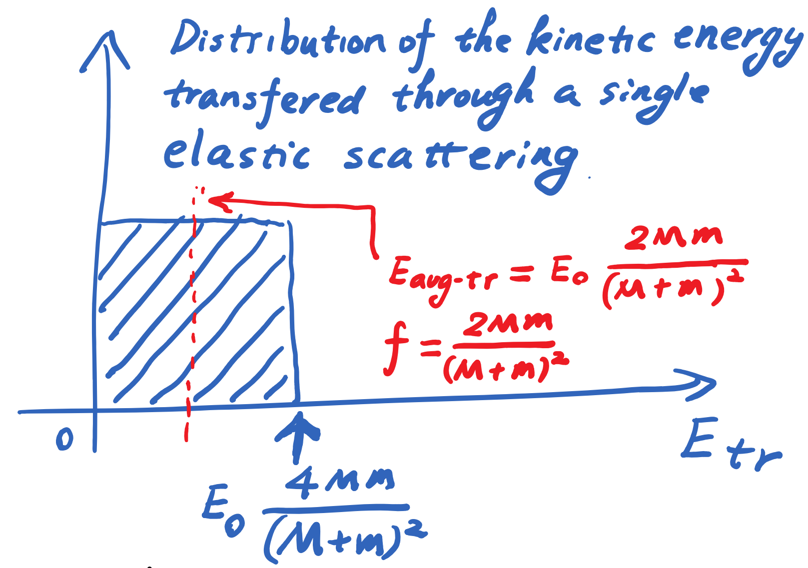
- Neutron dose is deposited through scattering and neutron induced nuclear reactions.
- In cases of elastic scattering, the scattered nuclei dissipate their energy in the immediate vicinity of the primary neutron interaction. The radiation dose absorbed locally in this way is called the first collision dose. The scattered neutron is not considered after this primary interaction.
- For fast neutrons, the first collision dose rate is given by

$$\dot{D}_n(E) = \frac{\phi(E) E \sum_i N_i \sigma_i f}{1 \text{ J/kg} \cdot \text{Gy}},$$

where $\phi(E)$ = flux of neutrons whose energy is E, in neutrons/cm^2. s,
E = neutron energy, in joules,
N_i = atoms per kilogram of the ith element,
σ_i = scattering across section of the ith element for neutrons of energy E, in barns $\times 10^{-24}$ cm^2,
f = mean fractional energy transferred from neutron to scattered atom during collision with neutron.

Energy Distributions of Scattered Neutrons





$$\alpha = \frac{(M-m)^2}{(M+m)^2}$$

The fraction of energy carried by the scattered neutron is

$$\frac{E'}{E_0} = \frac{M^2 + m^2 + 2Mm\cos\theta}{(M+m)^2}$$

The distribution of the energy of the scattered neutrons is given by

$$p(E) = \frac{1}{1-\alpha}\frac{1}{E_0}, \ \mathrm{E} \subset [\alpha E_0, E_0].$$

Radiation Dose from Thermal Neutrons

☞ Two reactions are normally considered, namely $^{14}N(n,p)^{14}C$ and $^{1}H(n,r)^{2}H$ reactions.

☞ For the $^{14}N(n,p)^{14}C$ reaction, the dose is given by

$$\dot{D}_{np} = \frac{\phi N_N \sigma_N Q \times 1.6 \times 10^{-13}\ \text{J/MeV}}{1\ \text{J/kg}\cdot\text{Gy}},$$

where ϕ = thermal flux, neutrons per cm^2 per second,
N_N = number of nitrogen atoms per kg tissue, 1.49×10^{24},
σ_N = absorption cross section for nitrogen, 1.75×10^{-24} cm^2,
Q = energy released by the reaction = 0.63 MeV.

Radiation Dose from Thermal Neutrons

- For the $^{1}H(n, \gamma)^{2}H$ reaction, the dose is deposited by the gamma rays emitted throughout the entire volume. The number of reaction per second per gram is governed by the neutron flux and is given by

$$A = \phi N_H \sigma_H \text{ "Bq"/kg},$$

where ϕ = thermal flux, neutrons per cm^2 per second,
N_H = number of hydrogen atoms per kg tissue = 5.98×10^{25},
σ_H = absorption cross section for hydrogen = 0.33×10^{-24} cm^2.

- The resulting gamma ray dose is illustrated with the following example.

Neutron Shielding – An Example

Example 10.11

Design a shield for an 18.5×10^4 MBq (5 Ci) Pu-Be neutron source that emits 5×10^6 neutrons per second, such that the dose rate at the outside surface of the shield will not exceed 15 μSv/h (1.5 mrems/h). The mean energy of the neutrons produced in this source is 4 MeV.

☞ Cember, P452.

Step 1: Shielding for fast neutrons

Assuming that the shielding is made of water.

For the 4 MeV fast neutrons produced by the Pu-Be source, the cross sections of H and O atoms for elastic scattering are 1.9 barns and 1.7 barns, respectively.

So the linear scattering coefficient of water is given by

$$\Sigma = 1.9 \times 10^{-24} \left(\frac{\mathrm{cm}^2}{atom}\right) \times 6.7 \times 10^{22} \left(\frac{atoms}{cm^3}\right) + 1.7 \times 10^{-24} \left(\frac{\mathrm{cm}^2}{atom}\right) \times 3.35 \times 10^{22} \left(\frac{atoms}{cm^3}\right)$$

$$= 0.186\ cm^{-1}$$

which is corresponding to a Half-Valued Layer (HVL) of T=3.71 cm.

Step 1: Shielding for fast neutrons (continued)

Let's arbitrarily allow for a maximum dose from fast neutrons leaking from the shielding to be 10 μSv/h. We could use either the Table (see next page) given above or the equation for fast neutron dose to derive the fast neutron flux that leads to this dose rate would be 3.7 n/s·cm^2.

What is the thickness of a shielding needed to bring the fast neutron flux to this level?

Consider a point source of neutrons, the fast neutron flux after passing through a thickness of nT (cm) could be calculate approximately as

$$\dot{\phi} = \frac{B \cdot S}{4\pi (nT)^2} \frac{1}{2^n} \left(\frac{neutrons}{cm^2 \cdot s} \right).$$

B: the build-up factor, (5 for this case).
T: HVL (3.71 cm).
S: Source strength in neutrons/s, (5×10^{-6} neutrons/s).

Neutron Shielding – An Example

TABLE 9.5. Values of Neutron Fluence Rates Which, in a Period of 40 H, Result in a Maximum Dose Equivalent of 1 mSv

Neutron Energy, MeV	Neutron Fluence Rate, $cm^{-2}s^{-1}$: Adapted from NCRP Report No. 38 (NCRP 1971)[a]	Neutron Fluence Rate, $cm^{-2}s^{-1}$: Adapted from Cross and Ing. 1985[a]
2.5×10^{-8}	270	280
10^{-7}	340	—
10^{-6}	280	280
10^{-5}	280	280
10^{-4}	290	290
10^{-3}	340	280
5×10^{-3}	—	310
10^{-2}	350	300
2×10^{-2}	—	250
5×10^{-2}	—	110
10^{-1}	58	40
3×10^{-1}	—	20
3.8×10^{-1}	—	16
4.4×10^{-1}	—	13
5×10^{-1}	14	16
6×10^{-1}	—	15
8×10^{-1}	—	14
9×10^{-1}	—	13
1.0	10	9.7
1.20	—	12
2.00	—	11
2.30	—	12
2.50	10	11
3.00	—	11
3.50	—	8.5
4.50	—	9.9
5.00	8.0	9.7
6.25	—	9.2
7.00	8.5	9.0
10.0	8.5	8.0
14.0	6.0	6.8
14.7	—	6.5
20	5.5	
40	5.0	
60	5.5	
100	7.0	
200	6.5	
300	5.5	
400	5.0	

[a]The fluence rates presented here have been obtained from the cited references by dividing the respective reference values for thermal neutrons by 2.5 and the respective values for all other energies by 2.0. These adjustments have been made to reflect recommendations of the NCRP (1987) to increase the effective quality factors for thermal neutrons and more energetic neutrons by 2.5 and 2.0. respectively.
SOURCE: From NCRP Report No. 112. By permission.

The fast neutron flux that introduces a dose equivalent rate of 1mSv/40h (25 uSv/h).
So we need ~9/2.5 = 3.7n/s·cm^2 to deliver 10 uSv/h or 1 mRem/h dose

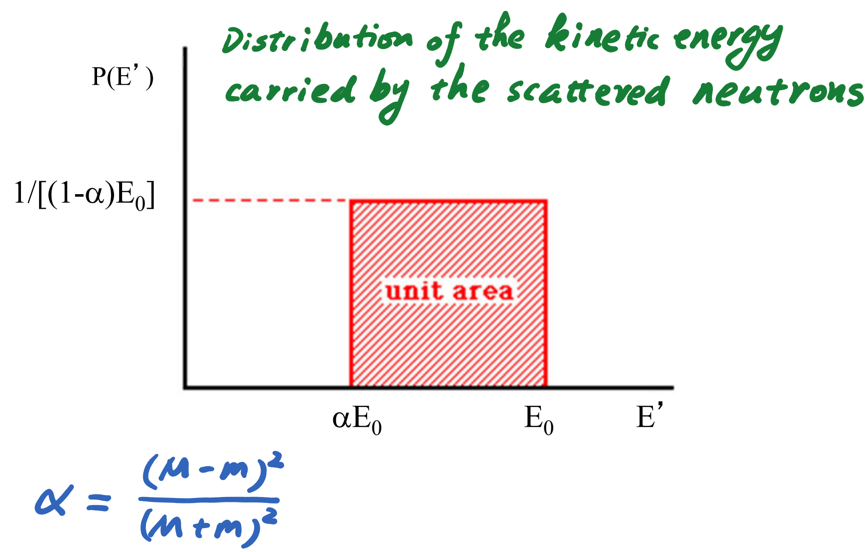
Radiation Dose from Fast Neutrons

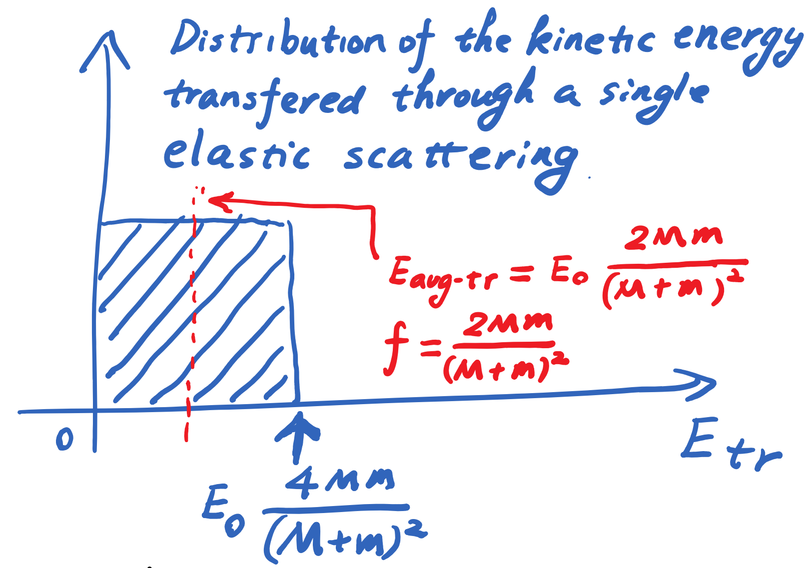
- Neutron dose is deposited through scattering and neutron induced nuclear reactions.
- In cases of elastic scattering, the scattered nuclei dissipate their energy in the immediate vicinity of the primary neutron interaction. The radiation dose absorbed locally in this way is called the first collision dose. The scattered neutron is not considered after this primary interaction.
- For fast neutrons, the first collision dose rate is given by

$$\dot{D}_n(E) = \frac{\phi(E) E \sum_i N_i \sigma_i f}{1\ \text{J/kg} \cdot \text{Gy}},$$

where
$\phi(E)$ = flux of neutrons whose energy is E, in neutrons/cm^2. s,
E = neutron energy, in joules,
N_i = atoms per kilogram of the ith element,
σ_i = scattering across section of the ith element for neutrons of energy E, in barns $\times 10^{-24}$ cm^2,
f = mean fractional energy transferred from neutron to scattered atom during collision with neutron.

Energy Distributions of Scattered Neutrons





The fraction of energy carried by the scattered neutron is

$$\frac{E'}{E_0} = \frac{M^2 + m^2 + 2Mm\cos\theta}{(M+m)^2}$$

The distribution of the energy of the scattered neutrons is given by

$$p(E) = \frac{1}{1-\alpha}\frac{1}{E_0},\ \ \mathrm{E} \subset [\alpha E_0, E_0].$$

Step 1: Shielding for fast neutrons (continued)

Consider a point source of neutrons, the fast neutron flux after passing through a thickness of nT (cm) could be calculate approximately as

3.7 n/s·cm² 5 5x10⁶ n/s 3.71 cm

$$\dot{\phi} = \frac{B \cdot S}{4\pi(nT)^2}\frac{1}{2^n}\left(\frac{neutrons}{cm^2 \cdot s}\right).$$

B: the build-up factor, (5 for this case).
T: HVL (3.71 cm).
S: Source strength in neutrons/s, (5×10^{-6} neutrons/s).

Solving the above equation for n, it gives n=9.We would need 9x3.71 =33cm of water to achieve the neutron flux of 3.7 n/s/cm^2, which ensures the fast neutron dose to stay below 10 μSv/h at the surface of the shielding.

Fast- and Thermal-Diffusion Lengths

The **fast-diffusion length**: the average straight-line distance covered by fast neutrons traveling in a given medium.

The **thermal-diffusion length**: the average distance covered by thermalized neutrons before it is absorbed. It is measured by the thickness of a slowing down medium that attenuates the beam of thermal neutrons by a factor of e. Thus the attenuation of a beam of thermal neutrons by a substance of thickness t (cm), whose thermal diffusion length is L (cm) is given by

$$n = n_0 e^{-t/L}$$

TABLE 5.6. Fast and Thermal Diffusion Lenghts of Selected Materials

Substance	Fast Diffusion Length, cm	Thermal Diffusion Length, cm	Diffusion Coefficient, cm
H_2O	5.75	2.88	0.16
D_2O	11	171	0.87
Be	9.9	24	0.50
C (graphite)	17.3	50	0.84

Neutron Induced Reactions

$$^{1}_{0}n + ^{14}_{7}N \rightarrow ^{14}_{6}C + ^{1}_{1}p$$

- Cross section for thermal neutron is 1.70 barns.
- Q=0.626MeV.
- Since the range of the proton and the ^{14}C nucleus are relatively small, their energy is deposited locally at the site where the neutron was captured.
- Capture by hydrogen and nitrogen are the only two processes through which neutron deliver a significant does to soft tissue.

Radiation Dose from Thermal Neutrons

- Two reactions are normally considered, namely $^{14}N(n,p)^{14}C$ and $^{1}H(n,r)^{2}H$ reactions.

- For the $^{14}N(n,p)^{14}C$ reaction, the dose is given by

$$\dot{D}_{np} = \frac{\phi N_N \sigma_N Q \times 1.6 \times 10^{-13}\ \text{J/MeV}}{1\ \text{J/kg} \cdot \text{Gy}},$$

where ϕ = thermal flux, neutrons per cm^2 per second,
N_N = number of nitrogen atoms per kg tissue, 1.49×10^{24},
σ_N = absorption cross section for nitrogen, 1.75×10^{-24} cm^2,
Q = energy released by the reaction = 0.63 MeV.

Neutron Induced Reactions

$$ {}_{0}^{1}n + {}_{1}^{1}H \rightarrow {}_{1}^{2}H + {}_{0}^{0}\gamma $$

- Neutron absorption followed by the immediate emission of a gamma ray photon.
- Since the thermal neutron has negligible energy by comparison, the gamma photon has the energy Q=2.22MeV released by the reaction, which represents the binding energy of the deuteron.
- The capture cross section per atom is 0.33barn.
- When tissue is exposed to thermal neutrons, this reaction provides a source of gamma rays that delivers dose to the tissue.

Radiation Dose from Thermal Neutrons

- For the $^1H(n, \gamma)^2H$ reaction, the dose is deposited by the gamma rays emitted throughout the entire volume. The number of reaction per second per gram is governed by the neutron flux and is given by

$$A = \phi N_H \sigma_H \text{ "Bq"/kg},$$

where ϕ = thermal flux, neutrons per cm^2 per second,
N_H = number of hydrogen atoms per kg tissue = 5.98×10^{25},
σ_H = absorption cross section for hydrogen = 0.33×10^{-24} cm^2.

- The resulting gamma ray dose is illustrated with the following example.

Step 2: Shielding for thermal neutron radiation

Considering that the radius of the water filled spherical shielding is 34 cm, which is corresponding to 9 times of the HVL. We could assume that most of the fast neutrons will be attenuated and thermalized at close to the center. So we could assume the source-shielding volume as a point-source of thermal neutrons placed at the center of a spherical shielding volume with approximately 34 cm radius.

The thermal neutrons escaping from the surface of the spherical shielding volume could be calculated as

$5\times10^6\ n/s$ (S), 34 cm (R), 0.16 cm (D), 2.88 cm (L)

$$\dot{\phi} = \frac{S}{4\pi RD} e^{-R/L} \left(\frac{neutrons}{cm^2 \cdot s}\right) = 0.55 \frac{n}{s \cdot cm^2}$$

⇒ *Dose negligible*

S: Strength of the thermal neutron source.
R: Radius of the spherical shielding volume.
L: Thermal diffusion length.
D: Diffusion coefficient.

Step 3: Shielding for gamma rays

- In this case, the gamma ray dose is coming from the $^1H(n,r)^2H$ reaction.
- For the $^1H(n,\gamma)^2H$ reaction, the dose is deposited by the gamma rays emitted throughout the entire volume. The number of reaction per second per gram is governed by the neutron flux and is given by

$$A = \phi N_H \sigma_H \text{ "Bq"/kg},$$

where ϕ = thermal flux, neutrons per cm^2 per second,
N_H = number of hydrogen atoms per kg tissue = 5.98×10^{25},
σ_H = absorption cross section for hydrogen = 0.33×10^{-24} cm^2.

Step 3: Shielding for gamma rays (continued)

- The energy of each gamma ray is 2.26 MeV.
- With the spherical water volume of 34 cm radius, there will be 3.7 n/s/cm^2, or $5 \times 10^4 \, {}^{n}/_{s}$ escaping from the surface.
- So we assume that the remaining neutrons will eventually be absorbed by hydrogen atoms, giving rise to the 2.26 MeV photons. Considering the apparent gamma ray activity is uniformly distributed across the spherical volume, then the gamma ray activity is

$$A = \frac{5\times10^6 ({}^{n}/_{s}) - 5.4\times10^4 ({}^{n}/_{s})}{\frac{4}{3}\pi\cdot(34cm)^3} = 30 \; (Bq/cm^3)$$

Step 3: Shielding for gamma rays (continued)

- Now consider a spherical volume filled with uniform radioactivity of A (Bq/cm^3), the dose rate at the surface of the sphere is given by

$$\dot{D} = \frac{1}{2} \cdot C \cdot \Gamma \cdot \frac{4\pi}{\mu} \cdot (1 - e^{-\mu r}) = 9 \times 10^{-3} (\, m\, G_y / h)$$

30 Bq/cm^3

0.046 cm^{-1}

34 cm

2.7 mGy·cm^2·MBq^{-1}·h^{-1}

- A spherical shielding of 34 cm radius filled with water would lead to a fast neutron dose of 10 μGy/h, a negligible thermal neutron dose, and gamma ray dose of 9 μGy/h.
- The total is 19 μGy, which is still greater than the 15 μGy/h target. What should we do now?

Thermal Neutron Capture by Boron

$$^{10}B + {}^{1}n \rightarrow {}^{7}Li + {}^{4}He + \gamma \ (0.48 \text{ MeV})$$

- ☞ Capture cross section: 755 barn.
- ☞ The 0.48 MeV gamma ray is emitted in 93% of the capture.

Step 4: Improve the shielding efficiency by adding boric acid (H_3BO_3) in water

- Consider that we add can add boric acid in water, whose formula weight is 61.84 and solubility is 63 g/L, and the thermal absorption coefficient is 775 Barns.
- If we add the maximum soluble concentration of boric acid in water, then the concentration of boron atoms is

$$\frac{63.2(g/L) \times 10^{-3}(L/mL) \cdot 6.02 \times 10^{23}(molecules/mole)}{61.84(g/mole)}$$

$$= 6.17 \times 10^{20}\ (atoms/mL)$$

- Compute the ratio of linear thermal absorption coefficients due to boron and hydrogen atoms,

$$\frac{\Sigma_H}{\Sigma_B} = \frac{1.9 \times 10^{-24}(cm^2/atom) \cdot 6.7 \times 10^{22}(atoms/cm^3)}{775 \times 10^{-24}(cm^3/atom) \cdot 6.17 \times 10^{20}(atoms/cm^3)} = 0.31$$

- Therefore, for every 1 thermal neutron captured by a hydrogen atom, there will be about 3.23 thermal neutrons each captured by a boron atom.

Step 4: Improve the shielding efficiency by adding boric acid (H_3BO_3) in water

- Therefore, for every 1 thermal neutron captured by a hydrogen atom, there will be about 3.23 thermal neutrons each captured by a boron atom.
- If we assume that all thermal neutrons are captured by either hydrogen or boron atoms, then there will be $[5 \times 10^6 (^n/_s) - 5.4 \times 10^4 (^n/_s)] \times \frac{1}{1+3.23}$ thermal neutrons being captured by hydrogen atoms per second.
- The gamma ray dose due to thermal neutrons captured by hydrogen will be reduced by a factor of $\frac{1}{1+3.23}$ to 2.1 μGy/h. (from 9 μ Gy/h previously with pure water).

Step 4: Improve the shielding efficiency by adding boric acid (H_3BO_3) in water

Gamma ray dose due to thermal neutron captured by boron atoms.

- ☞ Therefore, for every 1 thermal neutron captured by a hydrogen atom, there will be about 3.23 thermal neutrons each captured by a boron atom.
- ☞ If we assume that all thermal neutrons are captured by either hydrogen or boron atoms, then there will be $[5\times10^6({}^n/_s) - 5.4\times10^4({}^n/_s)]\times\frac{3.23}{1+3.23}$ thermal neutrons being captured by boron atoms per second, leading to an apparent gamma ray activity of

$$\frac{[5\times10^6({}^n/_s)-5.4\times10^4({}^n/_s)]\times\frac{3.23}{1+3.23}}{\frac{4}{3}\cdot\pi\cdot(34cm)^3} = 22\ \left({}^{Bq}/_{cm^3}\right)$$

☞Therefore the dose rate at the surface of the spherical volume due to thermal neutron capture by boron is

22 Bq/cm³ 0.097 cm⁻¹ 34 cm

$$\dot{D} = \frac{1}{2}\cdot C\cdot\Gamma\cdot\frac{4\pi}{\mu}\cdot(1-\mathrm{e}^{-\mu r})=0.8\times10^{-3}(m\,G_y/h)$$

0.62 mGy · cm² · MBq⁻¹ · h⁻¹

Step 4: Improve the shielding efficiency by adding boric acid (H_3BO_3) in water

- A spherical shielding of 34 cm radius filled with water would lead to a fast neutron dose of 10 μGy/h, a negligible thermal neutron dose, a gamma ray dose of 2.1 μGy/h from thermal neutron capture by hydrogen, and a gamma ray dose of 0.8 μGy/h from thermal neutron capture by boron.
- The total is 12.9 μGy, which is smaller than the 15 μGy/h target.