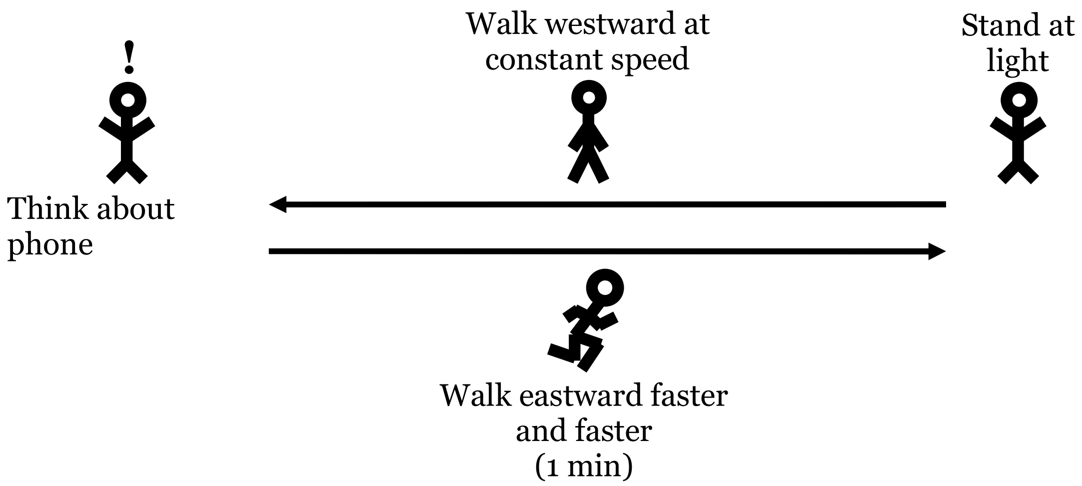
Lost Cell Phone

After physics class, Amélia waits at the light on Goodwin and Green talking to a friend for 1 minute. Once the light turns green, she walks westward at roughly constant speed for 2 minutes until she realizes that she doesn't have her cell phone! She stands still, thinking for 15 seconds, then turns around and walks faster and faster back toward Loomis. She reaches the light at Goodwin and Green again 1 minute later. What does her motion look like on a position vs. time graph?

(1) Comprehend the Problem

Amélia's motion sounds complicated, so let's draw a picture to organize the information:



We've been given information about Amélia's velocity (how fast and which way she's moving) at different times. We want to express her position as a graph versus time. We know that position and velocity are related: the velocity is the slope of position versus time.

(2) Represent the Problem in Formal Terms (Describe the Physics)

Velocity is defined as the change in position over time, i.e. Amelia's velocity ($v_{Amélia}$) is the slope of her position ($x_{Amélia}$) plotted against time (t):

$$v_{Amélia} = \text{slope of } x_{Amélia} \text{ vs. } t$$

(3) Plan a Solution

We can use the information about velocity and position stated in the problem to sketch Amélia's position versus time graph. We'll use the points where her velocity changes as boundaries where the motion changes. In addition, we'll vary the slope of the position vs. time graph according to what her velocity is doing, keeping in mind two ways velocity affects the position vs. time graph:

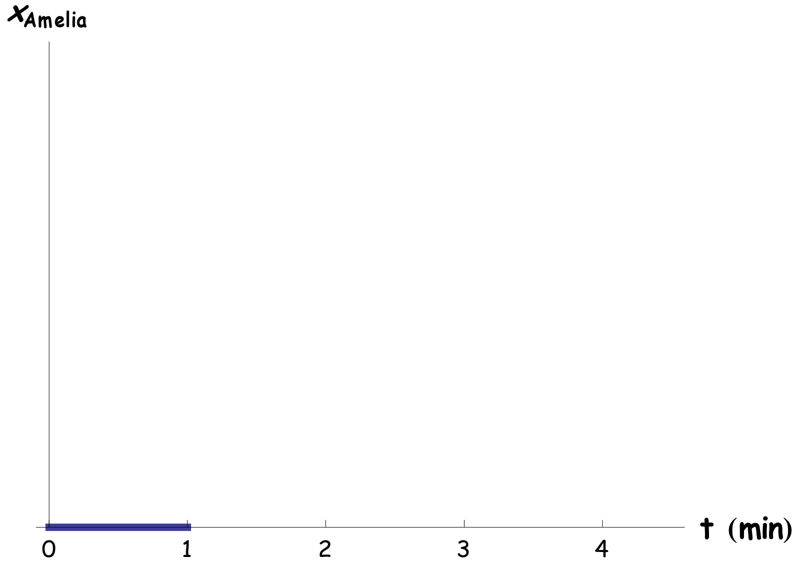
1. the velocity's direction determines whether the postion's slope is positive or negative
2. faster velocity means a steeper slope for position vs. time

(4) Execute the Plan

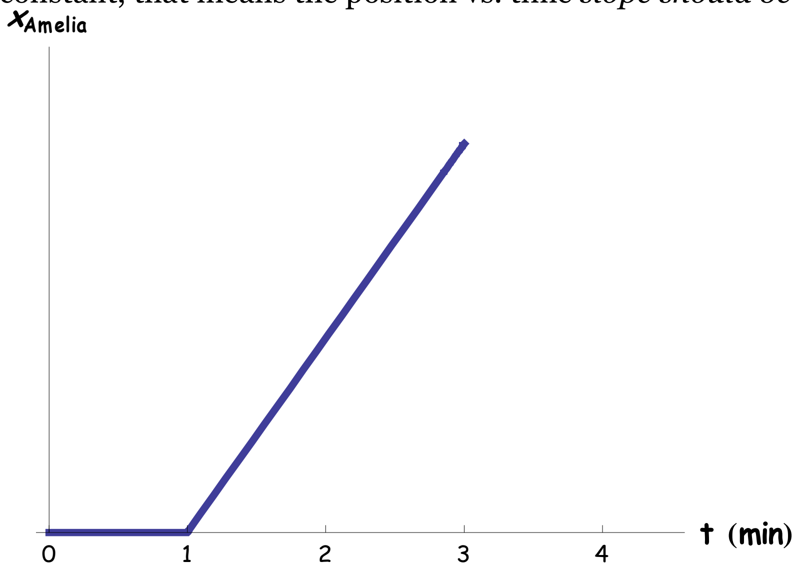
Whenever we deal with motion, we need to keep track of which way things are moving. In one dimension (1-D), this amounts to choosing which direction is positive (and therefore which direction is negative). Since Amélia walks west first, let's choose west as the positive direction (and therefore east is negative).

We also have to choose where we want to measure positions from. The simplest choice is to use Amélia's starting point (the light near Loomis) as zero position (this is also called "choosing the light as our origin").

Amélia begins by standing still for 1 min. This means her position does not change over this time. For this whole minute, she is found with zero displacement, so her graph should start like this:

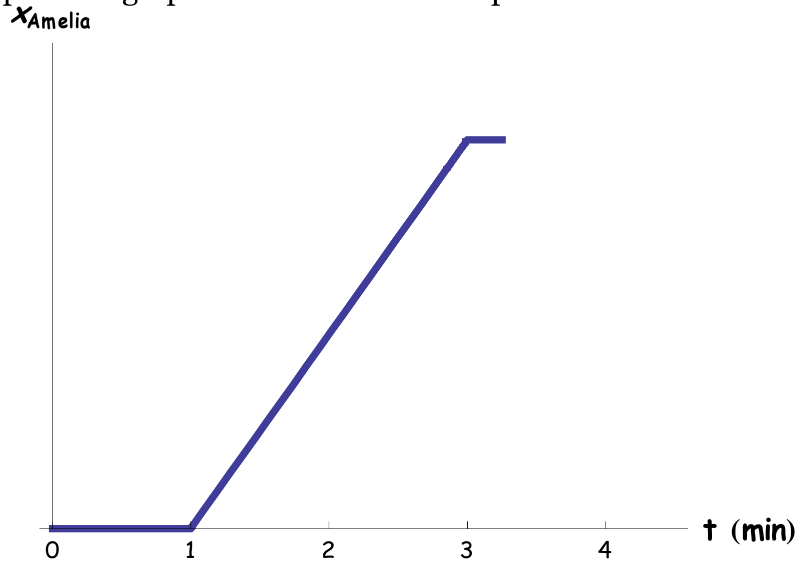


For the next 2 minutes, she walks *westward* at *constant velocity*. We've defined westward as positive, so the position vs. time graph should have positive slope. Since the velocity is constant, that means the position vs. time *slope should be constant* for the next two minutes:



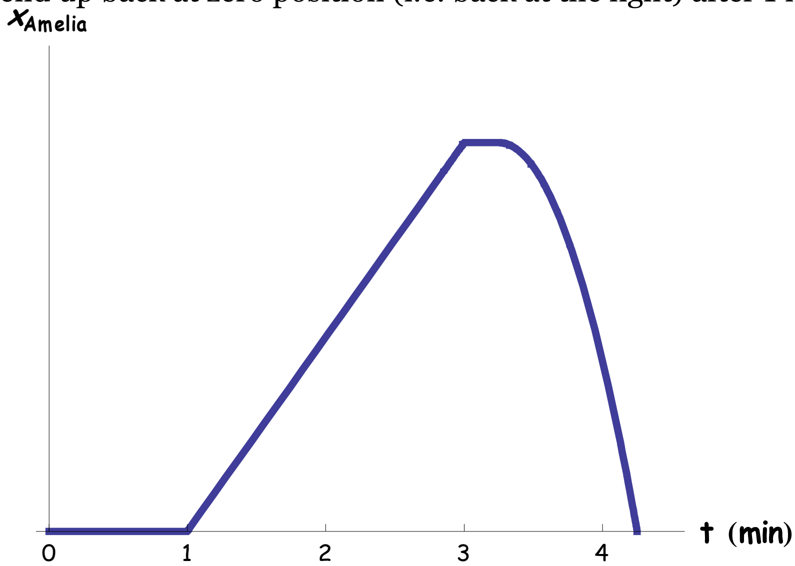
Note that a straight line has a slope that doesn't change.

Now Amélia stands still for ¼ minute while thinking about where her phone could be. Since she's standing still, her position isn't changing for this time (i.e. her velocity is zero). The position graph should therefore keep the value she had at the end of her walking:



During the last minute, Amélia moves east back toward the light on Green and Goodwin. Since we've chosen east as negative, her velocity is therefore negative. This means the slope of her position vs. time graph should be *negative* over this time period. In addition, she walks faster and faster, meaning the slope should become steeper as time goes on (i.e. "velocity becomes more negative").

A line won't work for this portion of the motion because a line's slope doesn't change. We need a curve whose slope is negative and gets steeper as time goes on. The curve should end up back at zero position (i.e. back at the light) after 1 minute:



(5) Interpret and Evaluate the Solution
Let's check this graph to see if it makes sense.

? Amélia is supposed to end up back where she started

- ✓ The final position value matches the initial position value (i.e. the graph starts and ends at the same values on the vertical axis)

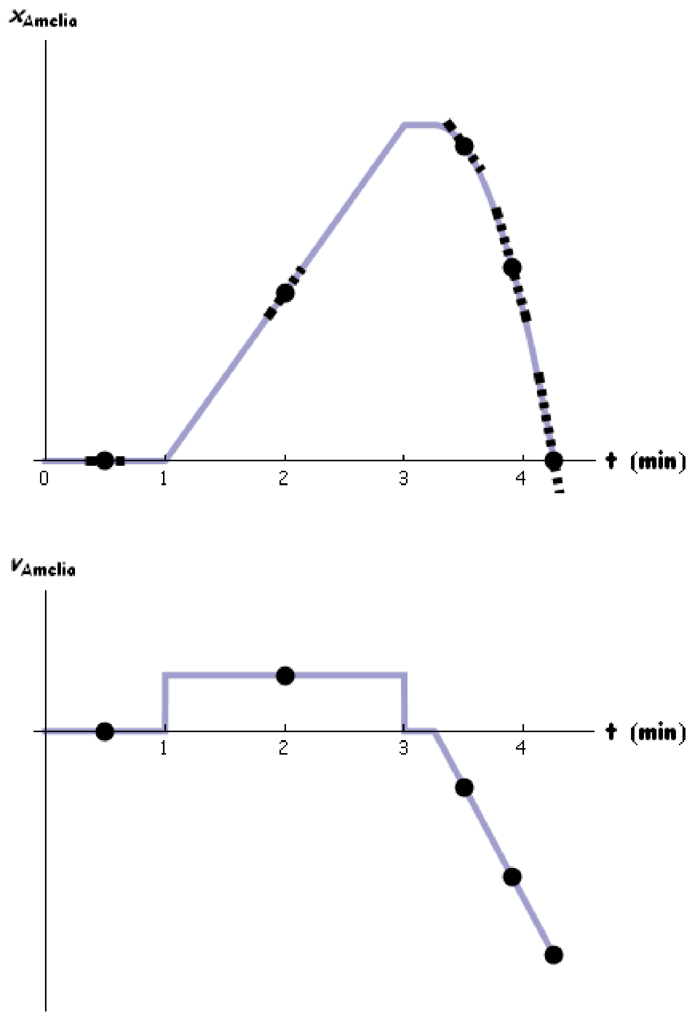
? Amélia is supposed to take less time returning to the light than walking away from it

- ✓ It takes (3 min – 1 min) = 2 minutes to for her to walk away, but only (4.25 min – 3.25 min) = 1 min for her to walk back. 1 min is shorter than 2 min.

? Amélia's velocity is supposed to be constant walking westward, but change walking eastward

- ✓ The walk westward shows a line with constant slope, while the curve representing the eastward motion has a slope that changes.

To make this last point clearer, let's generate the velocity vs. time graph from our position vs. time graph:



The slopes (dotted lines) are shown at 5 points on the position vs. time graph. The corresponding points on the velocity vs. time plot are also shown. When the velocity doesn't change (i.e. position vs. time is a straight line), the velocity is constant.

In contrast, look at the position vs. time portion between 3.25 and 4.25 minutes. The (negative) slope gets steeper and steeper over this time. Looking at the corresponding velocity graph, the velocity gets more and more negative over this time. Note that we've drawn this velocity dependence as a straight line for simplicity, but any sketch that gets more and more negative would be consistent with the problem statement of "faster and faster" eastward.