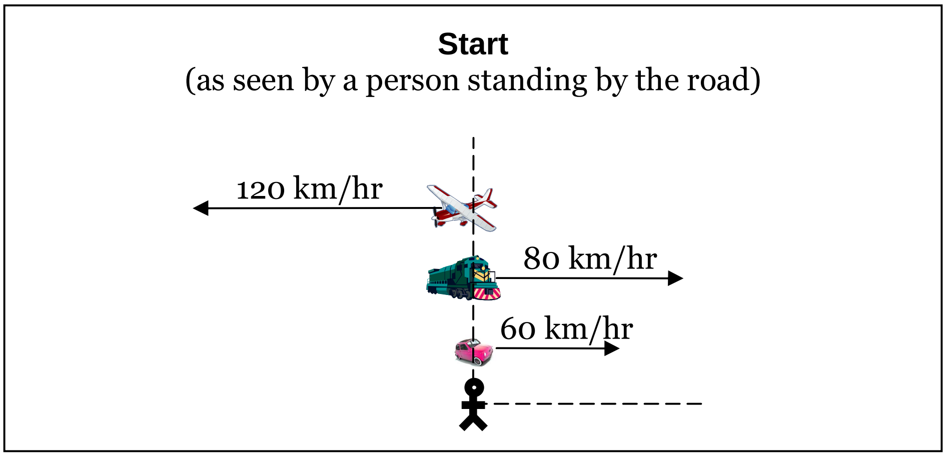
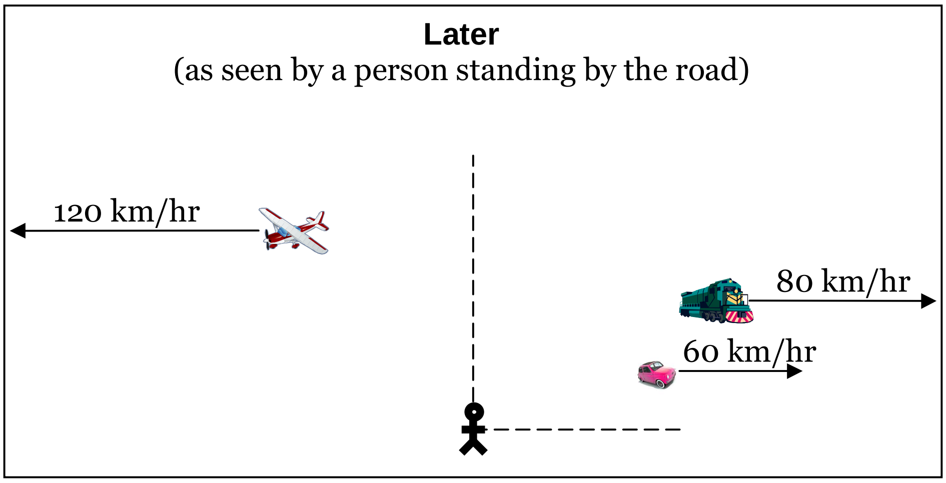
Planes, Trains, and Automobiles

An engineer drives a train traveling 80 km/hr eastward. A car drives eastward at 60 km/hr on a road parallel to the tracks. A plane flies over the road, travelling 120 km/hr westward. What are the velocities of the car, plane, and train as seen by the train engineer?

(1) Comprehend the Problem

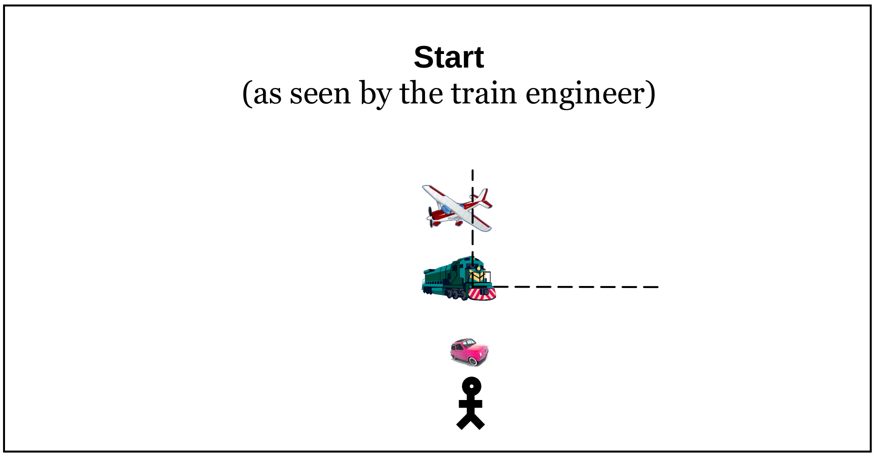
In this problem, we're given the velocities of objects seen from a bystander at rest with respect to the road. We're then asked to find the velocities of the car, train, and plane with respect to the train. Let's start by sketching the situation as seen by a person standing on the road (i.e. with velocities as given in the problem). For convenience, we'll draw all three objects as lining up with the roadside person's origin when we start.

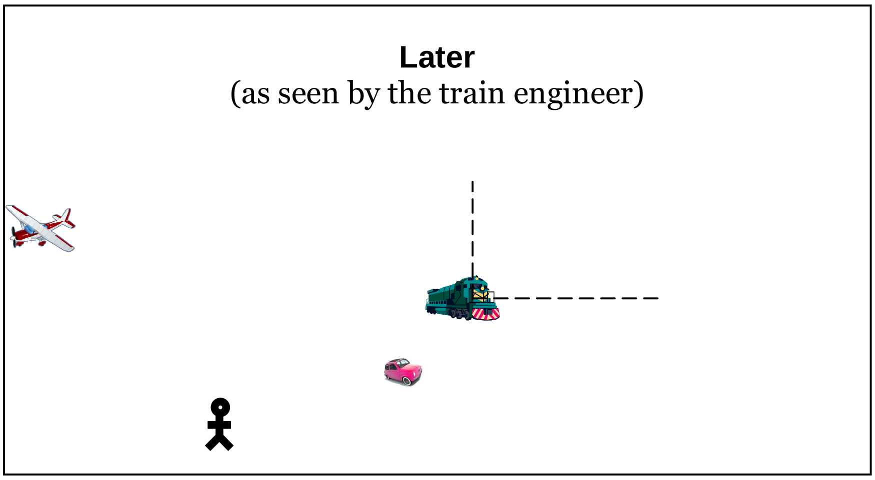




The objects move along at constant velocity, with bigger displacements for bigger velocities as you'd expect.

Now let's see what this situation would look like from the train's perspective. To an engineer sitting on the train, ***the train never moves***! In other words, the engineer's seat starts at the origin of his coordinates and STAYS at the origin of his coordinates. We don't know the velocities in this frame yet, but we can use our picture from the roadside perspective to get an idea of what the situation will look like later.





As we said earlier, the train is always at the train's origin. If this is true, then the plane ends up far behind the train, the car a little behind, and our roadside person is in between. Since all these objects appear westward of their initial positions, we therefore expect that the plane, car, and person should all have *westward* velocities in the train's frame of reference. Since the train always stays at the train's origin, we expect that the train's velocity should be *zero* (as seen by the train engineer).

(2) Represent the Problem in Formal Terms (Describe the Physics)

We wish to find the velocities of the train, plane, and car as seen by the train engineer. Since there are a lot of different velocities in this problem, we'll use subscripts to keep them straight. As is customary in relative motion problems, the first subscript states which object we're talking about and the second subscript states which reference frame (perspective) the velocity is measured in. We're given:

$$\vec{v}_{Train,Person} = \text{velocity of the Train measured by the Person}$$
$$\vec{v}_{Car,Person} = \text{velocity of the Car measured by the Person}$$
$$\vec{v}_{Plane,Person} = \text{velocity of the Plane measured by the Person}$$

We're asked to find:

$$\vec{v}_{Train,Train} = \text{velocity of the Train measured by the Train}$$
$$\vec{v}_{Car,Train} = \text{velocity of the Car measured by the Train}$$
$$\vec{v}_{Plane,Train} = \text{velocity of the Plane measured by the Train}$$

To go from the person's frame to the train's frame, we need to subtract off the motion of the train. That is, we must subtract the train's velocity in the person's frame to get an object's velocity in the train's frame:

$$\vec{v}_{Object,Train} = \vec{v}_{Object,Person} - \vec{v}_{Train,Person} \quad (1)$$

where the "Object" is either the Train, Car, or Plane.

(3) Plan the Solution

We can subtract the train's velocity in the person's frame from the other velocities in the person's frame to get velocities in the train's frame. To do this, we'll need to choose a direction for positive velocities. Since most of the objects in this problem are moving eastward in the person's frame, we'll choose east as positive (and therefore west as negative).

(4) Execute the Solution

Keeping in mind that east is positive, we'll substitute our object velocities into Equation (1) above.

$$\begin{aligned}\vec{v}_{Car,Train} &= \vec{v}_{Car,Person} - \vec{v}_{Train,Person} \\ &= \left(+60\,\tfrac{\text{km}}{\text{hr}}\right) - \left(+80\,\tfrac{\text{km}}{\text{hr}}\right) \\ &= -20\,\tfrac{\text{km}}{\text{hr}} \\ &= \boxed{20\,\tfrac{\text{km}}{\text{hr}}\ \text{westward}}\end{aligned}$$

The train engineer sees the car traveling at 20 km/hr westward.

$$\begin{aligned}\vec{v}_{Plane,Train} &= \vec{v}_{Plane,Person} - \vec{v}_{Train,Person} \\ &= \left(-120\,\tfrac{\text{km}}{\text{hr}}\right) - \left(+80\,\tfrac{\text{km}}{\text{hr}}\right) \\ &= -200\,\tfrac{\text{km}}{\text{hr}} \\ &= \boxed{200\,\tfrac{\text{km}}{\text{hr}}\ \text{westward}}\end{aligned}$$

The train engineer sees the plane travelling at 200 km/hr westward.

$$\begin{aligned}\vec{v}_{Train,Train} &= \vec{v}_{Train,Person} - \vec{v}_{Train,Person} \\ &= \left(+80\,\tfrac{\text{km}}{\text{hr}}\right) - \left(+80\,\tfrac{\text{km}}{\text{hr}}\right) \\ &= \boxed{0\,\tfrac{\text{km}}{\text{hr}}}\end{aligned}$$

As expected, the train engineer sees himself at rest.

(5) Interpret and Evaluate the Solution

The train sees both the plane and the car traveling westwards (negative direction). This agrees with our intuitive prediction from drawing out an initial and final snapshot of the motion. The train also appears at rest with respect to itself. This agrees with our idea that the train is always located at the train's origin.

The plane's velocity is faster in the train's frame than in the roadside person's frame. This makes sense because in one hour, the train travels 80 km east (in the person's frame) and the plane travels 120 km west (also as seen by the roadside person). The distance between them (200 km) therefore grows faster than either the plane's or train's speed alone would predict. The train should see the plane's velocity as faster than the 120 km/hr the person sees.

The car's velocity is slower in the train's frame than in the roadside person's frame. In one hour, the roadside person sees the train go 80 km east and the car 60 km east. The distance between the train and the car would then be 20 km. Since the car and train are moving in the same direction (according to the roadside person), their separation changes more slowly than either of their speeds alone would predict. The train should therefore see the car's velocity as slower than the 60 km/hr the person sees.