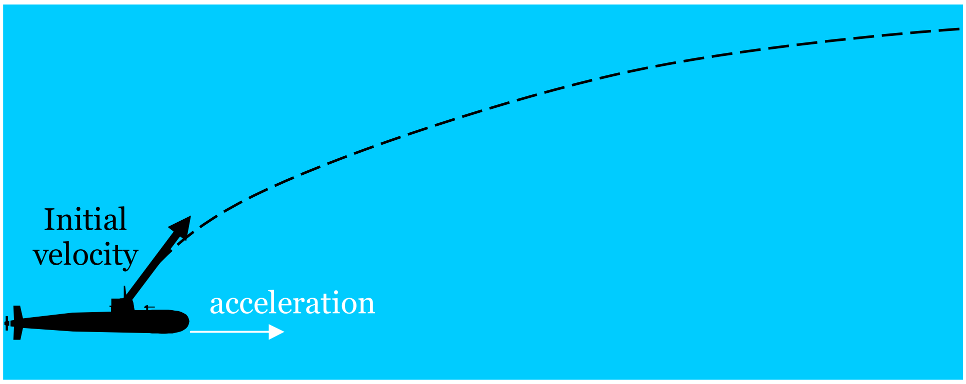
Submarine Antics

A submarine is travelling due East (course 090) at 10 m/s (20 knots). The sub is at a depth of 200 meters (about 660 ft). In an unusual maneuver, the captain orders the ballast tanks emptied to provide 5 m/s (5 knots) of upward velocity while increasing the horizontal speed gradually to 15 m/s (30 knots). If the speed increase takes 30 seconds, what is the sub's displacement from its original position?

(1) Comprehend the Problem

Our submarine is accelerating in the horizontal direction and moving at constant velocity in the vertical direction. We're given information about how the horizontal velocity changes and the constant vertical velocity. We want to find the final displacement of the submarine.



(2) Represent the Problem in Formal Terms (Describe the Physics)

We'll use East as the positive x-direction and "up" as the positive y-direction. We know the initial velocity of the submarine. It has an x-component of 10 m/s and a y-component of 5 m/s.

$$\vec{v}_{0,x} = \text{initial velocity of the submarine} = v_{0,x}\hat{i} + v_{0,y}\hat{j} = \left(10\tfrac{m}{s}\right)\hat{i} + \left(5\tfrac{m}{s}\right)\hat{j}$$

The submarine is also accelerating in the x-direction with an as-yet undetermined magnitude.

$$\vec{a} = \text{acceleration of the submarine} = a_x\hat{i}$$

We also know the final horizontal velocity and how long the acceleration is applied.

$$v_{Final,x} = \text{final horitonal component of the sub's velocity} = 15\tfrac{m}{s}$$

$$t = \text{ the time the maneuver lasts} = 30 \text{ s}$$

Since we're looking for the displacement of the sub after a given time, the displacement equation will probably come in handy:

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

In addition, we'll probably use the two horizontal velocities and the maneuver time to find the sub's acceleration. We'll probably need the relationship between the velocities, acceleration, and time:

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

(3) Plan the Solution

We'll need to find the acceleration in the x-direction using the velocity-time relation. Then we can write the components of the sub's position as functions of time. Then we can insert the time to get the final displacement.

(4) Execute the Solution

Use the velocity time relation to find the acceleration:

$$v_{\text{Final},x} = v_{0,x} + a_x t$$

$$a_x = \frac{v_{\text{Final},x} - v_{0,x}}{t} = \frac{\left(15\,\tfrac{\text{m}}{\text{s}}\right) - \left(10\,\tfrac{\text{m}}{\text{s}}\right)}{30\ \text{s}}$$

$$= 0.167\,\tfrac{\text{m}}{\text{s}^2}$$

Write the position components as functions of time.

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

x-components

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

$$= (0\ \text{m}) + \left(10\,\tfrac{\text{m}}{\text{s}}\right)(30\ \text{s}) + \tfrac{1}{2}\left(0.167\,\tfrac{\text{m}}{\text{s}^2}\right)(30\ \text{s})^2$$

$$= +375\ \text{m} = \boxed{375\ \text{m East}}$$

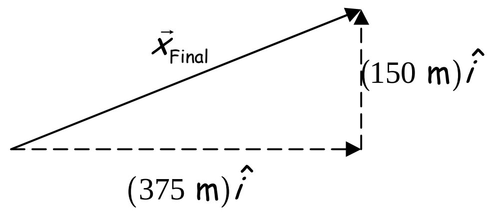
y-components

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

$$= (0\ \text{m}) + \left(5\,\tfrac{\text{m}}{\text{s}}\right)(30\ \text{s}) + \tfrac{1}{2}\left(0\,\tfrac{\text{m}}{\text{s}^2}\right)(30\ \text{s})^2$$

$$= +150\ \text{m} = \boxed{150\ \text{m Up}}$$

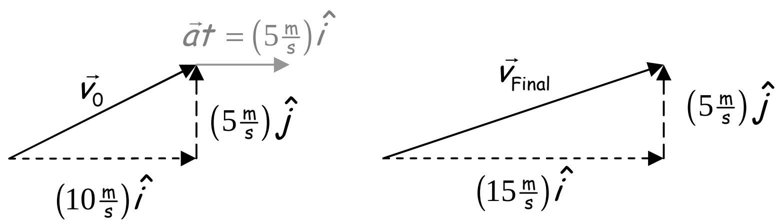
The submarine therefore ends up 375 m Eastward and 150 m Upward from its original position.



(5) Interpret and Evaluate the Solution

Our solution says the submarine ends up moving farther eastward than it did upward. This makes sense because its eastward velocities are always greater than its upward velocities (and keep growing with time).

The initial and final velocity vectors for the motion look something like this.



The upward velocity is unchanged since the acceleration has only a horizontal component. The horizontal velocity increases to the final 15 m/s the captain called for.