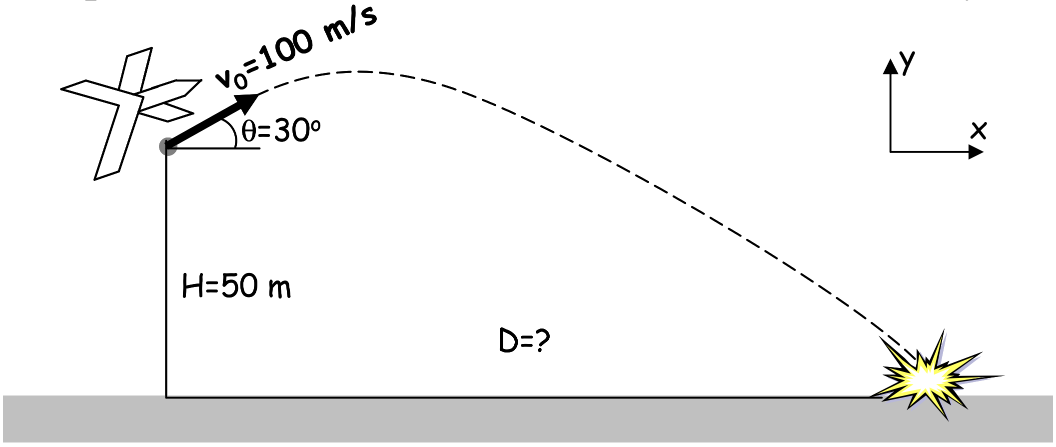
The Empire's at it Again…

The Empire has managed to build yet another monster Death Star, a spherical space station the size of a moon. A Rebel Alliance X-wing fighter is trying to take it out by dropping a bomb down a reactor ventilation shaft. Due to its huge size and heavy-metal construction, the Death Star has an acceleration due to gravity essentially equal to that of Earth. For safe delivery, the fighter must release the bomb while climbing at an angle of 30 degrees to the horizon. If the fighter travels at a speed of 100 m/s and is 50 m above the surface of the Death Star, how far away (along the "ground") from the target should the pilot release the bomb?

(1) Comprehend the Problem

In this situation, a bomb is released from a known elevation with some initial velocity, oriented above the horizontal. The bomb flies through the air until it lands at a target some time later. We want to know the horizontal distance to the target when the bomb was first released.

(2) Represent the Problem in Formal Terms (Describe the Physics)



We know that the acceleration of objects released near the surface of a planet is constant. Here we're told that the gravitational acceleration is that of Earth, namely 9.81 m/s² downward. This acceleration will change the vertical component of the bomb's velocity, but not its horizontal component. In symbols,

$$\vec{a} = \text{acceleration of the bomb} = \left(9.81\tfrac{\text{m}}{\text{s}^2}\right)\left(-\hat{j}\right) = g\left(-\hat{j}\right)$$

where g is defined as 9.81 m/s².

Alternatively, we can just state the acceleration's components $\begin{cases} a_x = 0 \\ a_y = -g \end{cases}$

The initial velocity has both x and y components, as determined from trigonometry:

$$\begin{aligned}\vec{v}_0 &= \text{bomb's initial velocity} \\ &= v_{0x}\hat{i} + v_{0y}\hat{j} \\ &= (v_0 \cos\theta)\hat{i} + (v_0 \cos\theta)\hat{j}\end{aligned}$$

Again, we could also just write the two components of the bomb's initial velocity: $\begin{cases} v_{0x} = v_0 \cos\theta \\ v_{0y} = v_0 \sin\theta \end{cases}$.

The time that the bomb spends in the air will depend on its motion in the y-direction because hitting the ground is something that results from the vertical motion of the bomb. Since we're interested in positions and times, it might be useful to use the general kinematic equation relating position and time:

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

(3) Plan the Solution

We can use the y-component of the position-time kinematic equation to solve for the time the bomb spends in the air (its "flight time"). Then we can plug this time into the x-component of the position-time kinematic equation to find how much horizontal distance the bomb travels in this flight time. This horizontal distance is how far along the ground from the target the X-wing must have started. We'll use the ground directly beneath the initial position of the fighter as our origin (i.e. the bomb starts with height H, horizontal position zero and ends with height zero, horizontal position D).

(4) Execute the Solution

Let's define one more symbol before we start using into all the equations. We'll need to find the flight time of the bomb through the air:

t_{flight} = time the bomb spends in the air

Take the y-component of the distance-time equation from constant acceleration kinematics.	$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ $y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$
To find the flight time, set the final position of the bomb equal to zero and plug in the other starting parameters.	$(0) = (H) + (v_0 \sin\theta)t_{flight} + \frac{1}{2}(-g)(t_{flight})^2$
While we could solve this expression symbolically with the quadratic formula, we'll plug in numbers here to make the solution clearer. We choose the positive time because it's the one AFTER the drop occurs. (drop was at t=0)	$0 = (50\text{ m}) + \left(100\frac{m}{s}\sin(30^\circ)\right)t_{flight} + \frac{1}{2}\left(-9.81\frac{m}{s^2}\right)(t_{flight})$ $0 = (50\text{ m}) + \left(50\frac{m}{s}\right)t_{flight} + \left(-4.905\frac{m}{s^2}\right)(t_{flight})^2$ $t_{flight} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-\left(50\frac{m}{s}\right) \pm \sqrt{\left(50\frac{m}{s}\right)^2 - 4\left(-4.905\frac{m}{s^2}\right)(50\text{ m})}}{2\left(-4.905\frac{m}{s^2}\right)}$ $= \frac{50 \mp 59}{9.81}\text{ s} = -0.917$ seconds OR $\boxed{11.11\text{ seconds}}$
Write the horizontal component of the bomb's position-time equation.	$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ $= (0) + (v_0\cos\theta)t + \frac{1}{2}(0)t^2$
We can insert the flight time (t_{flight}) and find how far the bomb has gone horizontally from the launch point (D).	$D = (v_0\cos\theta)(t_{flight})$ $= \left(100\frac{m}{s}\cos(30^\circ)\right)(11.11\text{ s})$ $= \left(86.6\frac{m}{s}\right)(11.11\text{ s}) = \boxed{962\text{ meters}}$

This means the fighter has to launch the bomb 962 meters away (horizontally) from the target.

(5) Interpret and Evaluate the Solution

This seems like a long way, but the fighter has a very fast horizontal speed and the bomb spends a long time (over ten seconds) in the air. Keeping these two facts in mind, this large distance is more reasonable.