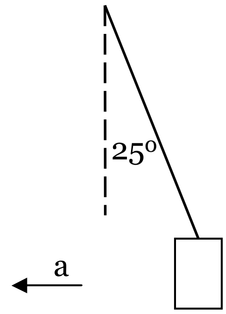
Poor Man's Accelerometer

A driver hangs an air freshener from their rearview mirror with a string. When accelerating onto the highway, the driver notices that the air freshener makes an angle of about 25 degrees with respect to the horizontal. What is the acceleration of the car?

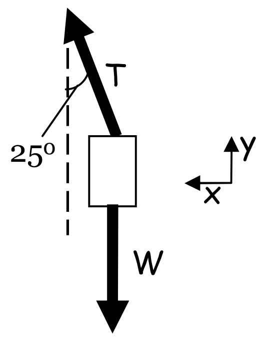


(1) Comprehend the Problem

We've got a hanging air freshener (of unknown mass) accelerating forward. It's hanging from a string at 25 degrees from the vertical. From this angle we're to find the object's acceleration. Since the air freshener appears at rest to the driver, it must be accelerating at the same rate as the car. Since the car's acceleration is all horizontal, the air freshener's acceleration should be entirely horizontal as well.

(2) Express the Problem in Formal Terms (Describe the Physics)

We're looking for the acceleration of the air freshener, so we probably need to find all the forces acting on the air freshener that cause this acceleration. The free body diagram is shown at the right.



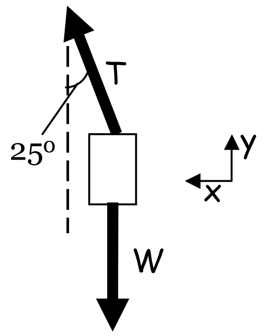
There are two forces on the air freshener, a tension up and to the left and a weight straight down. The acceleration is to the left, so the sum of these two forces must point to the left. We'll choose to the left as the +x direction so the acceleration is purely x (this should make the math simpler later).

The relationship between forces and acceleration is given by Newton's Second Law, $\vec{F}_{net} = m\vec{a}$. To find the net force, we'll need to know the directions of all the forces in the problem. We know the weight points straight down (all y-direction), and the tension in the string has a positive x-component and a positive y-component (i.e. "up and to the left").

(3) Plan a Solution

We'll write Newton's Second Law as two component equations, one in the x-direction and the other in the y-direction. We'll have two unknowns, the magnitude of the tension T and the acceleration in the x-direction a_x. We can solve the two equations for the two unknowns T and a_x.

(4) Execute the Plan



Write Newton's Second Law as two component equations, one in the x-direction and the other in the y-direction.	$\vec{F}_{net} = \sum_i \vec{F}_i = m\vec{a}$ $\begin{cases} F_{net,x} = \sum_i F_{i,x} = ma_x \\ F_{net,y} = \sum_i F_{i,y} = ma_y \end{cases}$
Express the net force as a sum of all the forces acting on the object.	x-direction: $F_{net,x} = ma_x$ $T_x + W_x = ma_x$ $(T\sin 25^\circ) + (0) = ma_x$ $T\sin 25^\circ = ma_x$ y-direction: $F_{net,y} = ma_y$ $T_y + W_y = ma_y$ $(T\cos 25^\circ) + (-mg) = m(0)$ $T\cos 25^\circ = mg$ $T = \frac{mg}{\cos 25^\circ}$
We can insert the tension value from the y-direction equation into the x-direction equation.	$T\sin 25^\circ = ma_x$ $\left(\frac{\cancel{m}g}{\cos 25^\circ}\right)\sin 25^\circ = \cancel{m}a_x$ $a_x = g\tan 25^\circ = \left(9.81\frac{m}{s^2}\right)(0.466)$ $\boxed{a_x = 4.6\frac{m}{s^2}}$

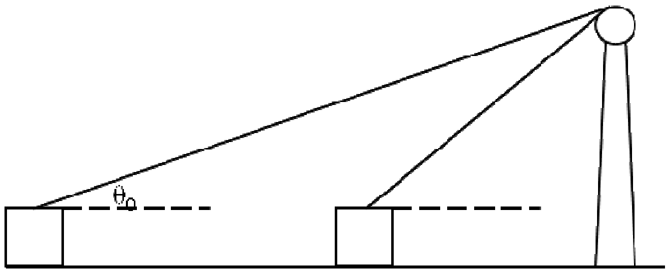
The air freshener (and therefore the car) accelerates at 4.6 m/s².

Reel It In

A crate of mass 100 kg is connected to a winch on the top of a tower by a cable. The reel supplies a constant force of 3000 N to the cable, sliding the crate across a very slick floor. As the crate slides toward the tower across the very slick floor, the angle the cable makes with the horizontal gets larger. Answer the following two questions.

a) If the initial angle the cable makes with the horizontal is 15 degrees, what is the initial acceleration of the crate?
b) At what cable angle does the crate first get lifted off of the floor?

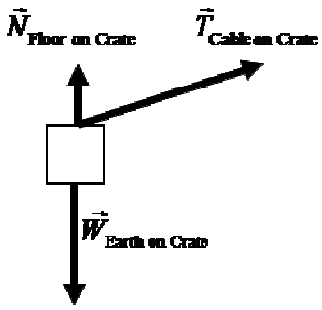
(1) Comprehend the problem



This situation involves a crate being pulled across the floor. We know the tension in the cable remains a constant 3000 N and the mass of the crate is 100 kg. We're also given the initial angle the crate makes with the horizontal is 15 degrees. We need to figure out the acceleration that all the forces on the crate produce at this starting point. After this, we are to find the first angle the cable makes with the horizontal at which the crate accelerates upward (i.e. gets lifted off of the floor).

(2) Express the Problem in Formal Terms (Describe the Physics)

There are two questions we must address. The first involves finding the initial acceleration of the crate. The second involves finding the first angle at which the crate accelerates upward, rather than just horizontally. Since the acceleration of the crate is caused by the net force on the crate, let's begin by identifying all the forces on the crate.



There are three forces on the crate.

1) the tension from the cable on the crate up and to the right: $\vec{T}_{\text{Cable on Crate}}$
2) the weight from the earth attracting the crate downward: $\vec{W}_{\text{Earth on Crate}}$
3) the normal force from the floor on the crate upward: $\vec{N}_{\text{Floor on Crate}}$

So the cable pulls the crate up and to the right, the crate's weight pulls it straight down, and the normal force from the floor pushes straight up to prevent the crate from penetrating the floor.

Since we're asked about motion along the floor and perpendicular to the floor, let's choose our axes for this problem to be horizontal (i.e. parallel to the floor) for x and vertical (i.e. perpendicular to the floor) for y.

For both questions a) and b), we can identify the forces on the object and are asked about the acceleration. The relationship between forces and acceleration is embodied in Newton's Second Law:

$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i = m\vec{a}$$

where the net force is the sum of all the forces acting on the crate. This vector equation stands for two scalar (i.e. "just number") equations, one in the x-direction and one in the y-direction:

$$F_{\text{net},x} = \sum_i F_{i,x} = ma_x$$

$$F_{\text{net},y} = \sum_i F_{i,y} = ma_y$$

where each equation deals only with x- or y-components, respectively.
We know the following parameters:

$$m = \text{mass of the crate} = 100 \text{ kg}$$

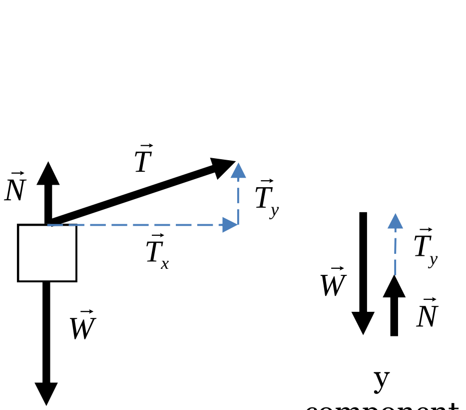
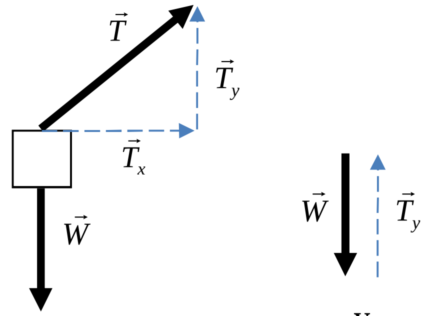
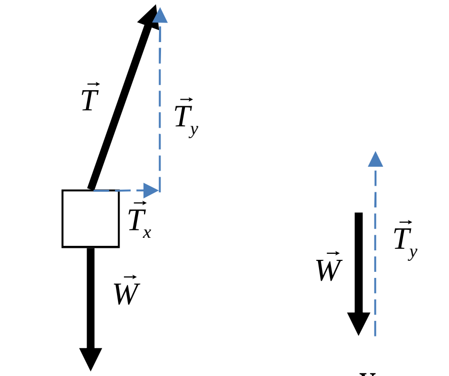
$$T = \text{tension in the cable} = 3000 \text{ N}$$

$$g = \text{acceleration due to gravity near earth} = 9.81 \frac{\text{m}}{\text{s}^2}$$

$$\theta_0 = \text{the initial cable angle with the horizontal} = 15°$$

(3) Plan a Solution

a) We know the magnitude of the tension in the cable and we can find the magnitude of the crate's weight using the relationship between weight and the acceleration due to gravity near earth, $W = mg$. We also know the angle the tension makes with the horizontal, since it points along the cable at 15 degrees. To find the initial acceleration of the crate, we can find the vector sum of all the forces on the crate and divide by the mass. This will give us the initial acceleration.
b) Here we are asked to find the first cable angle that lifts the crate off of the ground. This happens just after the normal force goes to zero.
How do we know this? The normal force is needed initially because the vertical component of the cable tension is not as big as the vertical component of the crate's weight downward. (see the figures on the next page) This means that if there were no normal force, there would be a net force downward on the crate. The crate would therefore accelerate downward and penetrate the floor. Since this doesn't happen, the floor must be pushing upward with a normal force just big enough to make up the difference between the y-component of the tension and the weight. As the crate approaches the tower, the cable points in a more vertical direction. This increases the y-component of the tension. Eventually the tension's y-component will be large enough to cancel the crate's weight on its own, and the normal force won't be needed (i.e. it will be zero). If the y-component of the tension grows past this value, there will be a net force upward on the crate (the normal force can't pull downward unless the floor is "sticky"). A net upward force means that the crate will accelerate upward.

Initial Situation: A normal force is needed to keep the block from accelerating downward	Later: The tension's y-component just cancels the crate's weight	Even Later: The tension's y-component is larger than the crate's weight
 Free Body Diagram y components	 Free Body Diagram y components	 Free Body Diagram y components
$\vec{F}_{net,y} = 0$ (no y-acceleration)	$\vec{F}_{net,y} = 0$ (no y-acceleration)	$\vec{F}_{net,y} > 0$ (accelerates upward)

We should therefore write the equation for the net force on the crate in the y-direction. We will then set the normal force magnitude equal to zero and solve for the angle that makes this happen. That will be the last angle at which the crate remains on the floor. Any larger angle will accelerate the crate upward.

(4) Execute the Plan

a) Find the magnitude of the crate's initial acceleration from Newton's Second Law.

Break the general statement of Newton's Second Law into x- and y-component equations. We are told that the crate initially slides along the floor, so its y-velocity remains zero. This implies that its y-acceleration is zero. All that's left is to find the x-acceleration.	$\vec{F}_{net} = m\vec{a}$ $\begin{cases} F_{net,x} = ma_x \\ F_{net,y} = ma_y = m(0) = 0 \end{cases}$
Write the net force on the crate as a sum of all the forces acting on the crate (see the free body diagram drawn earlier). We've chosen horizontal and vertical as our x- and y-axes. Up and right are the positive directions. We'll stick with the x-direction since we already know that the y-acceleration is zero.	$F_{net,x} = ma_x$ $N_x + W_x + T_x = ma_x$ $(0) + (0) + (T\cos(\theta_0)) = ma_x$
Solve for the x-component of the acceleration and insert the numbers from the problem.	$T\cos(\theta_0) = ma_x$ $a_x = \frac{T\cos(\theta_0)}{m}$ $= \frac{(3000\ \frac{\text{kg m}}{\text{s}^2})(\cos 15^\circ)}{100\text{ kg}} = \boxed{29.0\ \frac{\text{m}}{\text{s}^2}}$

b) Find the cable angle at which the crate is lifted off of the floor. We'll do this by finding the angle that makes the normal force zero and keeps the acceleration in the y-direction equal to zero. Any angle bigger than this will lift the crate off of the ground.

Break the general statement of Newton's Second Law into x- and y-component equations. We're keeping the y-acceleration zero because we're looking for the angle just before the crate lifts off the ground.	$\vec{F}_{\text{net}} = m\vec{a}$ $\begin{cases} F_{\text{net},x} = ma_x \\ F_{\text{net},y} = ma_y = m(0) = 0 \end{cases}$
Since we're interested in y-motion, let's look at the y-direction first. We write the net force as a sum of all the forces on the crate (see the free body diagram drawn in step (2)) We set the normal force equal to zero and write the other y-components, remembering that we chose "up" as positive.	$F_{\text{net},y} = 0$ $N_y + W_y + T_y = 0$ $(0) + (-mg) + (T\sin(\theta_{\text{last}})) = 0$
Solve for the last angle with zero upward acceleration and insert the numbers from the problem.	$-mg + T\sin(\theta_{\text{last}}) = 0$ $\sin(\theta_{\text{last}}) = \frac{mg}{T}$ $\theta_{\text{last}} = \sin^{-1}\left(\frac{mg}{T}\right)$ $= \sin^{-1}\left(\frac{(100\text{ kg})(9.81\,\frac{\text{m}}{\text{s}^2})}{(3000\text{ N})}\right)$ $\boxed{\theta_{\text{last}} = \sin^{-1}(0.327) = 19^\circ}$

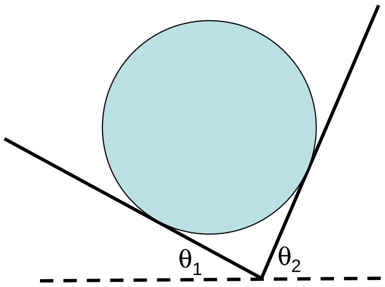
Once the cable reaches 19° the crate will lift off of the ground.

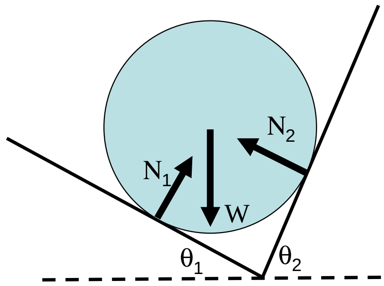
Groovin'

A sphere of mass M is resting in the groove created by two inclined planes meeting at their tips. The planes have known different angles of incline with respect to the horizontal. Find the magnitude of the normal force supplied by each plane.

(1) Comprehend the Problem

We have a sphere resting on two flat surfaces, each with its own known incline with the horizontal. A sketch of the situation looks like this:



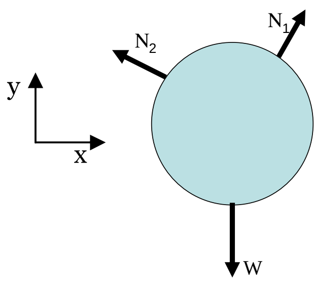


We know that the sphere is resting on the incline. We know the mass of the ball, as well. We are also assumed to know the two angles of the incline, θ_1 and θ_2. We know that there is a gravitational force (the weight) pulling the sphere straight down. Therefore, there must also be a normal force on the ball from each plane to keep the ball from penetrating the surfaces.

We are to find the magnitude of each of these normal forces. Since we know the ball remains at rest, its acceleration must be zero. We therefore know its acceleration must be zero (velocity never changes from zero). Since we know the acceleration and are asked about forces, we'll probably use Newton's Second Law to determine the normal forces.

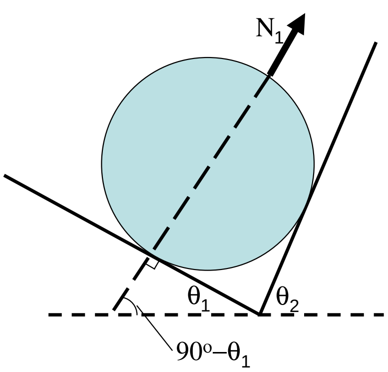
(2) Represent the Problem in Formal Terms (Describe the Physics)

We can organize our qualitative idea about what's going on with a free body diagram of the ball. This means that we draw the ball by itself (i.e. "free" of the rest of the objects) with only the forces ON the ball.



We know we have three forces to deal with. We'll probably need to know their directions. We can choose our x- and y-axes any way we like, but since none of these forces are either perpendicular or parallel to each other we might as well just choose up as +y and right as +x. This at least makes the weight force point entirely in the y-direction.

To find the direction of the normal forces, we'll need to use some geometry. Let's look at N_1 first:



The normal force is always directed perpendicular (i.e. "normal") to the surface that provides it. If we extend N1's direction back to the surface that provides it, we meet that surface at a right angle, as shown.

Extending this line all the way to the x-axis makes a right triangle underneath the left-hand surface. All the angles of a triangle add up to 180°, so the angle we're interested in should be:

$$180^o - 90^o - \theta_1 = (90^o - \theta_1)$$

Similarly, the angle between N_2 and the horizontal is $(90^o - \theta_2)$. Note that N_2 points in the negative x-direction while N_1 points in the positive x-direction.

We therefore have the following forces:

$$\vec{N}_1 = \text{normal force from the left surface } = \begin{cases} \text{magnitude } N_1 \\ \text{angle } \left(90^\circ - \theta_1\right) \text{ from the horiztonal} \end{cases}$$

$$\vec{N}_2 = \text{normal force from the right surface} = \begin{cases} \text{magnitude } N_2 \\ \text{angle } \left(90^\circ - \theta_2\right) \text{ from the horiztonal} \end{cases}$$

$$\vec{W} = \text{weight of the sphere} = \begin{cases} \text{magnitude mg} \\ \text{angle -90}^\circ \text{ from the horiztonal} \end{cases}$$

We can relate these forces by using Newton's Second Law:

$$\vec{F}_{net} = \sum_i \vec{F}_i = m\vec{a} \ .$$

(3) Plan a Solution

We will break Newton's Second Law into x- and y-component equations. Since we know the acceleration is zero (sphere remains at rest), we can solve these two equations for the two unknowns N_1 and N_2.

(4) Execute the Plan

First, we break Newton's Second Law down into two component equations. We know that the sphere remains at rest, so the acceleration along both the x- and y-directions is zero.	$\vec{F}_{net} = \sum_i \vec{F}_i = m\vec{a}$ $\begin{cases} F_{net,x} = \sum_i F_{i,x} = ma_x \\ F_{net,y} = \sum_i F_{i,y} = ma_y \end{cases}$ $\begin{cases} F_{net,x} = \sum_i F_{i,x} = m(0) = 0 \\ F_{net,y} = \sum_i F_{i,y} = m(0) = 0 \end{cases}$
Now we write the net force in each direction as the sum of each force's components. (see the free body diagram in step (2)) We are also using the trigonometric identities $\sin\left(90^\circ - \theta\right) = \cos(\theta)$ $\cos\left(90^\circ - \theta\right) = \sin(\theta)$	x-direction $F_{net,x} = 0$ $N_{1x} + N_{2x} + W_x = 0$ $\left(N_1 \cos\left(90^\circ - \theta_1\right)\right) + \left(-N_2 \cos\left(90^\circ - \theta\right)\right) + (0) = 0$ $N_1 \sin(\theta_1) - N_2 \sin(\theta_2) = 0$ y-direction $F_{net,y} = 0$ $N_{1y} + N_{2y} + W_y = 0$ $\left(N_1 \sin\left(90^\circ - \theta_1\right)\right) + \left(N_2 \sin\left(90^\circ - \theta\right)\right) + (-mg) = 0$ $N_1 \cos(\theta_1) + N_2 \cos(\theta_2) = 0$

We now have a system of two equations (one from each dimension) with two unknowns N_1 and N_2.	$\begin{cases} N_1\sin(\theta_1)-N_2\sin(\theta_2)=0 \\ N_1\cos(\theta_1)+N_2\cos(\theta_2)-mg=0 \end{cases}$
We can solve the first equation for N_1...	$N_1\sin(\theta_1)-N_2\sin(\theta_2)=0$ $N_1=\frac{\sin(\theta_2)}{\sin(\theta_1)}N_2$
...and substitute it into the second. Now we can determine N2	$N_1\cos(\theta_1)+N_2\cos(\theta_2)-mg=0$ $\left(\frac{\sin(\theta_2)}{\sin(\theta_1)}N_2\right)\cos(\theta_1)+N_2\cos(\theta_2)-mg=0$ $N_2\left(\frac{\sin(\theta_2)}{\sin(\theta_1)}\cos(\theta_1)+\cos(\theta_2)\right)=mg$ $\frac{mg}{\frac{\sin(\theta_2)}{\sin(\theta_1)}\cos(\theta_1)+\cos(\theta_2)}=N_2$
Here we have multiplied the top and bottom by $\sin(\theta_1)$ to get rid of the annoying fraction in the denominator.	$\frac{mg\sin(\theta_1)}{\sin(\theta_2)\cos(\theta_1)+\sin(\theta_1)\cos(\theta_2)}=N_2$
Before we plug this back into the first equation to get N_1, let's simplify it using the trigonometric addition identity: $\sin(\theta_1)\cos(\theta_2)+\sin(\theta_2)\cos(\theta_1)=\sin(\theta_1+\theta_2)$	$N_2=mg\frac{\sin(\theta_1)}{\sin(\theta_1+\theta_2)}$
Now we can plug this simpler expression for N_2 into the first equation (i.e. the equation from the x-direction).	$N_1=\frac{\sin(\theta_2)}{\sin(\theta_1)}N_2$ $=\frac{\sin(\theta_2)}{\cancel{\sin(\theta_1)}}\left(mg\frac{\cancel{\sin(\theta_1)}}{\sin(\theta_1+\theta_2)}\right)$ $N_1=mg\frac{\sin(\theta_2)}{\sin(\theta_1+\theta_2)}$
We now have each normal force's magnitude in terms of the known mass, acceleration due to gravity, and angles of the inclines.	$\boxed{N_1=mg\frac{\sin(\theta_2)}{\sin(\theta_1+\theta_2)} \quad N_2=mg\frac{\sin(\theta_1)}{\sin(\theta_1+\theta_2)}}$

(5) Interpret and Evaluate the Solution

Let's see if these two expressions make sense by checking some limits. If the left incline were flat, then θ_1 would be zero. The ball at rest would then rest entirely on the flat incline one. Plugging $\theta_1=0$ into the expressions gives $N_1=mg$ and $N_2=0$. This checks, because the left (flat) incline cancels the entire weight mg and the right incline doesn't supply any normal force at all.