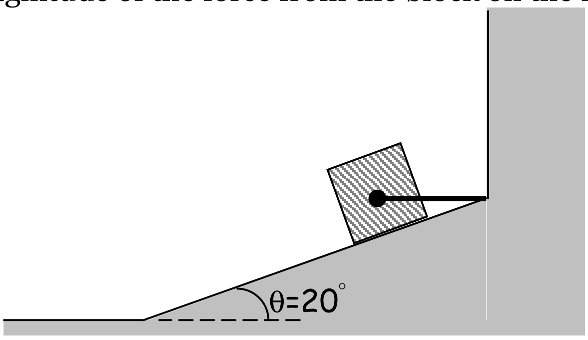
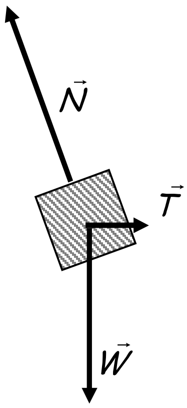
# Tied Up Block

A block of mass 2 kg sits on a frictionless ramp and is tied to the wall with a string as shown. The string is horizontal and tied to the center of the block. If the ramp is inclined at 20 degrees, what is the magnitude of the force from the block on the ramp?




**(1) Comprehend the Problem**
We have a block sitting on a ramp without any friction. A horizontal string is tied to the ramp. The tension in the string, coupled with the normal force from the ramp on the block, keeps the block from sliding down the ramp. We're asked to find the magnitude (strength) of the force with which the block is pushing down on the ramp. The block's free body diagram has three forces on it: the block's weight, the tension in the rope, and a normal force from the ramp keeping the block from penetrating into the ramp.




Since the ramp is connected to the wall and the ground, its free body diagram would probably be hard to draw. It will probably be easier to look at the free body diagram of the block and find the force the ramp pushes on the block with. We could then use Newton's 3rd Law to relate this force to the force with which the block pushes on the ramp.

**(2) Represent the Problem in Formal Terms (Describe the Physics)**

We need to keep track of which forces are applied to which objects. Let's use the following notation:

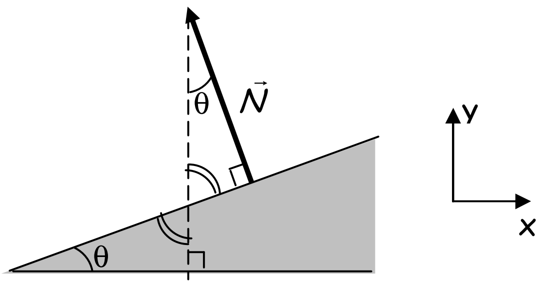
$\vec{T}_{\text{Rope on Block}}$ = Tension by the Rope on the Block

$\vec{W}_{\text{Earth on Block}}$ = Weight by the Earth on the Block

$\vec{N}_{\text{Ramp on Block}}$ = Normal force by the Ramp on the Block

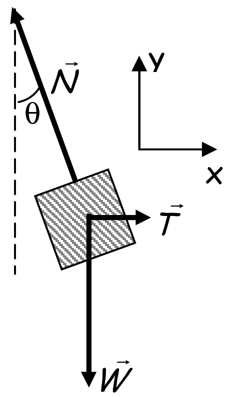
$\vec{N}_{\text{Block on Ramp}}$ = Normal force by the Block on the Ramp

We know that this block remains at rest, so it has zero acceleration (i.e. its velocity never changes). Since there is no acceleration to help us choose a preferred direction, let's set up our x axis horizontally to the right and our y axis vertically upward. This makes the tension and weight force components easy to calculate (they're all x or all y); we will only have to use trigonometry on one force now (the normal).




The angle marked with a double arc has the value 90°-θ. The Normal force therefore makes an angle of θ with the y-axis (i.e. the vertical).

The free body diagram for the block therefore looks like this:




To avoid carrying the numbers and units along, let's use the following symbols for the quantities given in the problem.

$m$ = mass of the block = 2 kg

$\theta$ = angle of the incline = $20^\circ$

$g$ = gravitational acceleration = $9.81\frac{m}{s^2}$

We also know the following relationships:

$$W_{\text{Earth on Block}} = mg \qquad \text{(Weight near earth's surface)}$$

$$\vec{N}_{\text{Ramp on Block}} = -\vec{N}_{\text{Block on Ramp}} \qquad \text{(Newton's Third Law)}$$

$$\vec{F}_{\text{net on Block}} = m_{\text{Block}}\vec{a}_{\text{Block}} \qquad \text{(Newton's Second Law)}$$

### (3) Plan the Solution

We know the acceleration of the block is zero. From this known acceleration we can use Newton's Second Law (in the y-direction) to find the unknown normal force from the ramp on the block. Once we know the force from the ramp on the block, we can use Newton's Third Law to find the force from the block on the ramp (same magnitude, opposite direction).

### (4) Execute the Solution

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|---|---|
| Write Newton's Second Law, breaking it into two component equations. | $\vec{F}_{\text{net on Block}} = m_{\text{Block}}\vec{a}_{\text{Block}}$ <br> $\begin{cases} F_{\text{net},x} = \sum_i F_{i,x} = m_{\text{Block}} a_x \\ F_{\text{net},y} = \sum_i F_{i,y} = m_{\text{Block}} a_y \end{cases}$ |
| Since the normal force on the block points in the y-direction, we'll focus on that direction first. We also know that the acceleration (in either direction) is zero. | $F_{\text{net},y} = m_{\text{Block}}(0)$ <br> $W_y + N_y + T_y = 0$ <br> $(-mg) + (N\cos\theta) + (0) = 0$ <br> $N\cos\theta = mg$ <br> $N = \dfrac{mg}{\cos\theta} = N_{\text{Ramp on Block}}$ |
| We can now use Newton's 3rd Law, which says that the force from the block on the ramp has opposite direction and equal magnitude to the force from the ramp on the block. | $\vec{N}_{\text{Block on Ramp}} = -\vec{N}_{\text{Ramp on Block}}$ <br> $\lvert\vec{N}_{\text{Block on Ramp}}\rvert = \lvert\vec{N}_{\text{Ramp on Block}}\rvert$ <br> $N_{\text{Block on Ramp}} = N_{\text{Ramp on Block}}$ <br> $N_{\text{Block on Ramp}} = \boxed{\dfrac{mg}{\cos\theta}}$ |
| Insert the numerical values from the problem. | $N_{\text{Block on Ramp}} = \dfrac{mg}{\cos\theta}$ <br> $= \dfrac{(2\text{ kg})(9.81\frac{\text{m}}{\text{s}^2})}{\cos(20^\circ)} = \boxed{20.9\text{ N}}$ |

### (5) Interpret and Evaluate the Solution

If we set our angle θ to 0 degrees, the incline would be flat and the string horizontal. This our formula predicts mg/cos(0) = mg for the normal force on the block. Since there wouldn't be any tension force along the horizontal ground, the normal force would then balance only the weight of the object. This agrees with our result of mg.