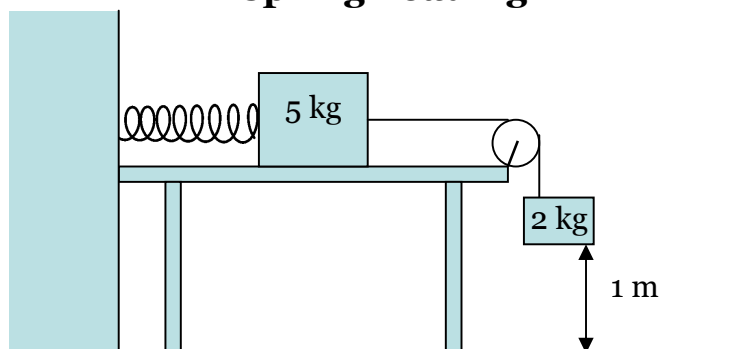


Spring Loading

A hanging block of mass 2 kg is connected by a string threaded through a pulley to a larger block of mass 5 kg sitting on a table. The larger mass is connected to a wall by a relaxed spring, while the 2 kg mass hangs 1 m above the floor. What is the minimum value for the spring constant that will allow the 2 kg mass to hang at rest without touching the floor?

(1) Comprehend the Problem

In this problem, we have two masses connected to a spring. Since both masses move together, the most that the spring can stretch is 1 m. The spring force is proportional to the amount a spring is stretched or compressed. If we could find how much force that the spring must supply to hold the masses stationary, we could use this maximum allowable stretch to find the minimum spring constant we would need.

(2) Represent the Problem in Formal Terms (Describe the Physics)

We know the relationship between a spring's extension and the force that it provides:

$$\vec{F}_s = \text{spring force} = \begin{cases} \text{magnitude} = k|\vec{x} - \vec{x}_0| \\ \text{direction} = \text{toward relaxed length} \end{cases}$$

k = spring constant

\vec{x}_0 = equilibrium (relaxed) position of the spring's end

\vec{x} = current spring position of the spring's end

Since we're dealing with forces from springs and tensions, we'll probably need Newton's Second Law, as well:

$$\vec{F}_{\text{net}} = m\vec{a}$$

We'll use symbols for the two masses and the tension in the string so we don't have to carry so many numbers around.

m_2 = mass of the small block = 2 kg

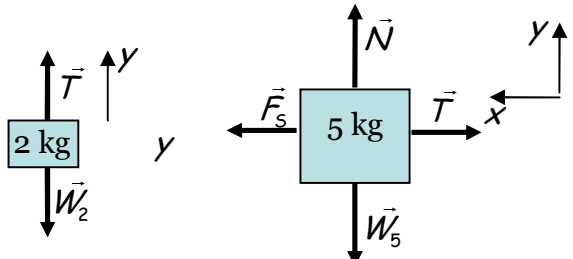
m_5 = mass of the large block = 5 kg

T = magnitude of the tension in the string

(3) Plan the Solution

We know that the acceleration of the system will be zero if the mass is just hanging in place. We also know that the spring is allowed to stretch at most 1 m to allow this to happen. We can use Newton's Second Law on each of the masses to find how much force the spring must supply to hold up the small mass. Once we know this, we can use the formula for the spring force magnitude ($F_s = k|\vec{x} - \vec{x}_0|$) to find the necessary spring constant.

(4) Execute the Solution

<p>Draw free body diagrams for the blocks.</p> <p>We choose "up" as the y-axis for the 2 kg mass so all the forces on it have only y-components. To be consistent, we should therefore choose to the left as positive for the 5 kg mass, because lifting the 2 kg mass up would move the 5 kg mass left.</p>	
<p>Since we're interested in the spring force, start by writing Newton's Second Law for the 5 kg mass. We know it has zero acceleration because its velocity never changes.</p>	$F_{\text{net},x} = m_5 a_x$ $F_{s,x} + N_x + W_{5x} + T_x = m_5 (0)$ $F_s + 0 + 0 + (-T) = 0$ $F_s = T$
<p>We need to find the tension in the string to get the spring force. We'll therefore write Newton's Second Law for the 2 kg block. We know it also has zero acceleration because its velocity never changes.</p>	$F_{\text{net},y} = m_2 a_y$ $T_y + W_{2y} = m_2 (0)$ $T + (-m_2 g) = 0$ $T = m_2 g$
<p>Combining these results, we can find the spring force in terms of parameters that we know</p>	$F_s = T$ $F_s = m_2 g$
<p>Now we can use the formula for the magnitude of the spring force to find the spring constant. We'll call the relaxed length of the spring L. The end of the spring is at position $-(L+d)$, where d is the stretch of the spring.</p> <p>(the positions are negative because we've chose the wall as the origin and left as positive... the relaxed position of the spring is therefore negative)</p>	$F_s = k \vec{x} - \vec{x}_0 $ $m_2 g = k (-(L+d) - (-L)) $ $m_2 g = k -d $ $m_2 g = k(d)$ $k = \frac{m_2 g}{d}$
<p>The minimum required spring constant coincides with the maximum allowable stretch of the spring, or 1 meter. We can plug in the numerical values for this problem here.</p>	$k_{\text{min}} = \frac{m_2 g}{d_{\text{max}}}$ $= \frac{(2 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})}{(1 \text{ m})} = \boxed{19.6 \frac{\text{N}}{\text{m}}}$

“g” on Jupiter

The planet Jupiter has 318 times the earth’s mass and a radius 11 times as big. What is the acceleration due to gravity on Jupiter?

(1) Comprehend the Problem

We’re given the dimensions of Jupiter in terms of Earth’s dimensions and are asked to find g_J , the acceleration due to gravity on Jupiter’s surface. We know that on Earth, g_E has the value 9.81 m/s^2 . The attraction to a planet’s surface is governed by the Law of Universal Gravitation, so we’ll probably need to use this to relate the acceleration due to gravity on Earth and Jupiter.

(2) Represent the Problem in Formal Terms (Describe the Physics)

We know that for two objects with mass, the attraction between them is given by

$$F = G \frac{Mm}{r^2} \text{ where}$$

G = the universal gravitational constant = $6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$

M = the mass of one object

m = the mass of the other object

r = the center-to-center distance between the objects

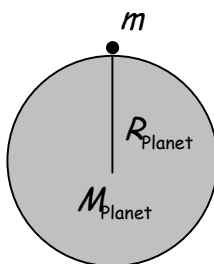
We also know that close to a planet’s surface, the weight force appears to be independent of height and is given by $W = mg$ where

W = the weight of the object

m = the mass of the object

g = the acceleration due to gravity

For this problem, we have an object near the surface of a planet, so the situation looks like this.



The separation between the planet and the object is very nearly equal to the planet’s radius. The two objects we’re looking at are the planet itself and an object that we would watch fall.

(3) Plan the Solution

We could find an expression for the acceleration due to gravity at the surface of any planet by combining the weight definition with the general gravitational force expression. Once we know g on any planet, we could use a ratio to compare Jupiter and Earth. This ratio would cancel out any constants and would produce an expression with ratios of distances and masses, both of which are given in the problem.

(4) Execute the Solution

<p>The weight of an object on a planet is just the gravitational force of attraction on it from the planet.</p> <p>Note that the acceleration due to gravity near a planet's surface does not depend on the object's mass</p>	$W = F_g$ $mg_{\text{Planet}} = G \frac{M_{\text{Planet}} m}{(R_{\text{Planet}})^2}$ $g_{\text{Planet}} = G \frac{M_{\text{Planet}}}{(R_{\text{Planet}})^2}$
<p>We can use a ratio to compare the g values on Jupiter and Earth. The ratio gets rid of any constants and leaves us with mass and distance ratios.</p>	$\frac{g_{\text{Jupiter}}}{g_{\text{Earth}}} = \frac{\cancel{G} \frac{M_{\text{Jupiter}}}{(R_{\text{Jupiter}})^2}}{\cancel{G} \frac{M_{\text{Earth}}}{(R_{\text{Earth}})^2}}$ $= \frac{M_{\text{Jupiter}}}{(R_{\text{Jupiter}})^2} \times \frac{(R_{\text{Earth}})^2}{M_{\text{Earth}}} \times \frac{(R_{\text{Jupiter}})^2}{(R_{\text{Jupiter}})^2}$ $= \frac{M_{\text{Jupiter}}}{M_{\text{Earth}}} \left(\frac{R_{\text{Earth}}}{R_{\text{Jupiter}}} \right)^2$
<p>We can use the ratios given in the problem to plug into this answer.</p>	$\frac{g_{\text{Jupiter}}}{g_{\text{Earth}}} = \left(\frac{M_{\text{Jupiter}}}{M_{\text{Earth}}} \right) \frac{1}{\left(\frac{R_{\text{Jupiter}}}{R_{\text{Earth}}} \right)^2}$ $\frac{g_{\text{Jupiter}}}{g_{\text{Earth}}} = (318) \frac{1}{(11)^2} = 2.628$
<p>We can use the value we know for Earth to get the numerical value for the acceleration due to gravity on Jupiter.</p>	$g_{\text{Jupiter}} = (2.628) g_{\text{Earth}}$ $= (2.628) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)$ $= \boxed{25.8 \frac{\text{m}}{\text{s}^2}}$