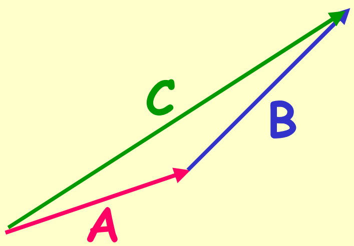
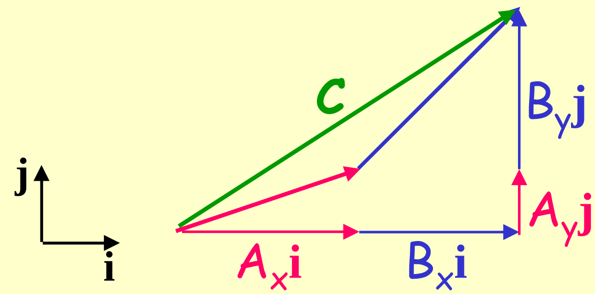
# PHYS 100: Lecture 3

## VECTORS

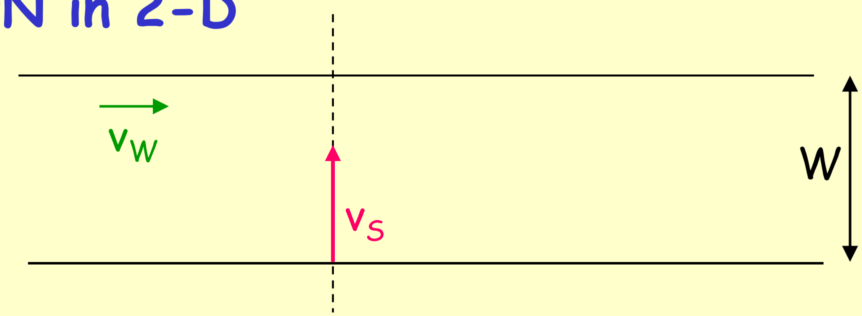


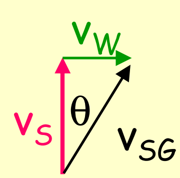

$$\vec{C} = \vec{A} + \vec{B}$$




$$C_x = A_x + B_x$$
$$C_y = A_y + B_y$$

## RELATIVE MOTION in 2-D



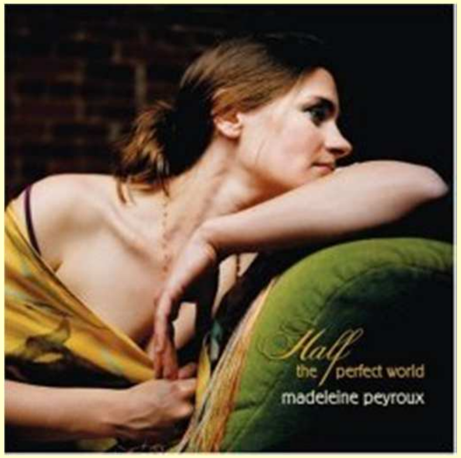

$$\overrightarrow{v_{SG}} = \overrightarrow{v_S} + \overrightarrow{v_W}$$

# Music


BB

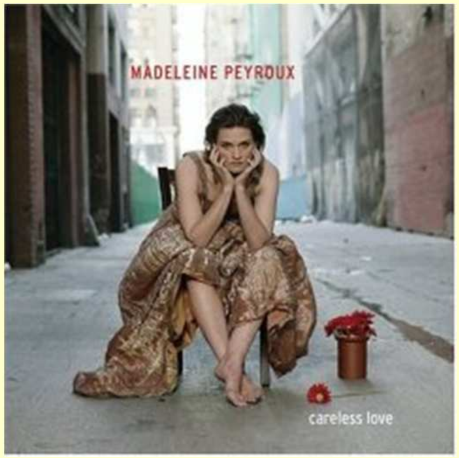
Who is the Artist?

A) Diana Krall
B) Norah Jones
C) kd lang
D) Madeline Peyroux
E) Edith Piaf




Great version of River (joni mitchell) with kd lang

Haunting voice.. Highly Recommended..
Tough to categorize (at New Orleans Jazzfest, they put her in the traditional jazz tent (with Pete Fountain, etc…))

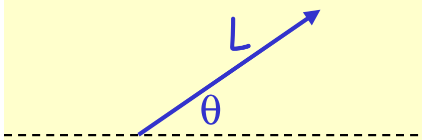




Other great albums

# THE BIG IDEAS

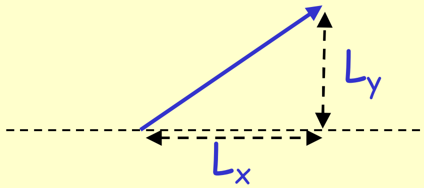
## VECTORS:

### Representations




$$L = \sqrt{L_x^2 + L_y^2}$$

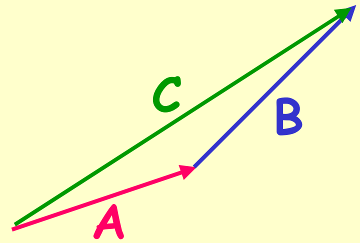
$$\tan\theta = \frac{L_y}{L_x}$$




$$L_x = L\cos\theta$$

$$L_y = L\sin\theta$$

### Vector Addition




$$\vec{C} = \vec{A} + \vec{B}$$

What's the point? What do I expect you to know?

Vectors are "just" math, but math that you NEED to KNOW to do physics !

I expect you to MASTER this concept. It can be done with practice, practice..

## RELATIVE MOTION:

This topic is HARD. Why?

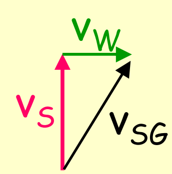
Unfamiliar? Unnatural?

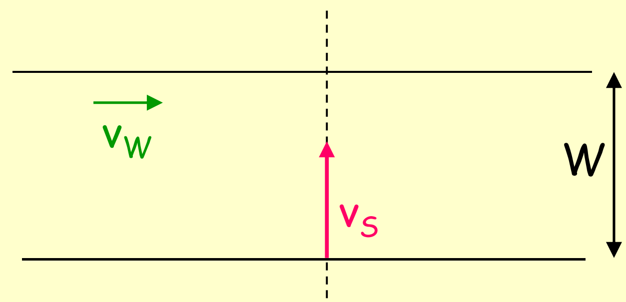
NEED to LEARN to THINK in a different way !!

Good News?

ONLY ONE EQUATION !

$$\overrightarrow{v_{SG}} = \overrightarrow{v_S} + \overrightarrow{v_W}$$

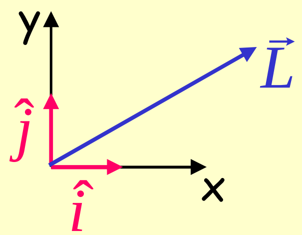


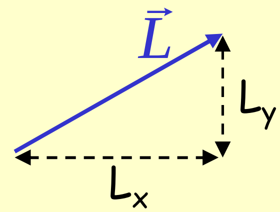



# WHAT DID YOU FIND DIFFICULT?

Also, what are "i-hat, j-hat, and k-hat"?

Just notation to make clear what's a vector and what's a scalar




$$\vec{L} = L_x\,\hat{i} + L_y\,\hat{j}$$

$$\boxed{\vec{L} = L_x\,\hat{i} + L_y\,\hat{j}}$$

$L_x$ and $L_y$ are SCALARS

$\vec{L}$ $\hat{i}$ $\hat{j}$ are VECTORS

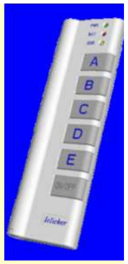
$L_x\,\hat{i}$ is a VECTOR of magnitude $L_x$ in the +x-direction

$L_y\,\hat{j}$ is a VECTOR of magnitude $L_y$ in the +y-direction

The swimming example somewhat made sense, and I understand the relationships of velocities with respect to difference reference frames, but when it comes to actually calculating these I find that i am stumped. The equations confuse me.

You are not alone.. I will try to nail the swimming problem later…

# Preflight 1



BB

The displacement vector $L$ describing the location of an object points in a direction 70° North of West and has magnitude 60 m.
Taking North to be aligned with the positive y-axis and East to be aligned with the positive x-axis, What is the value of $L_x$, the x-component of $L$?

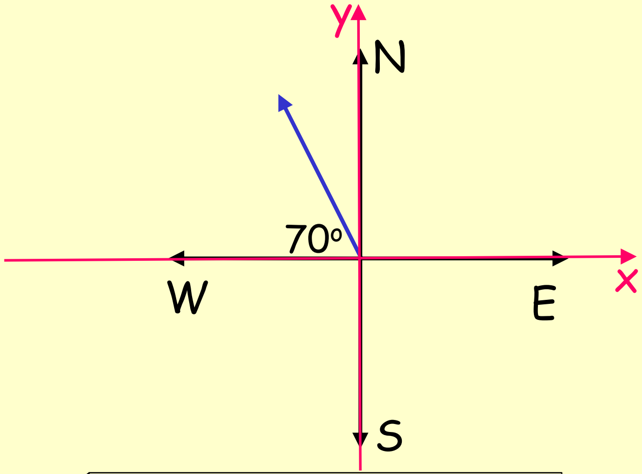
(A) - 60 sin(70°) m

$\boxed{\text{(B) - 60 cos(70°) m}}$

(C) + 60 cos(70°) m

(D) + 60 sin(70°) m

(E) None of the above

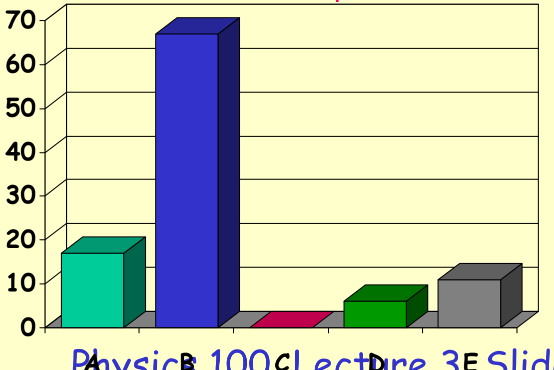



You said:

- Because it is west of north, the x component will be negative. We then see that there is a triangle formed with theta = 70 degrees. We then solve for the x component and get the result of 56m

- Lx = Lcos(theta). Since theta is 70 and L is 60m, Lx has a magnitude of 21 m. The answer is negative because the x component points in the negative x direction.

DRAW A PICTURE.. THAT IS THE KEY



# Preflight 1

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(A) $- 60 \sin(70°)$ m

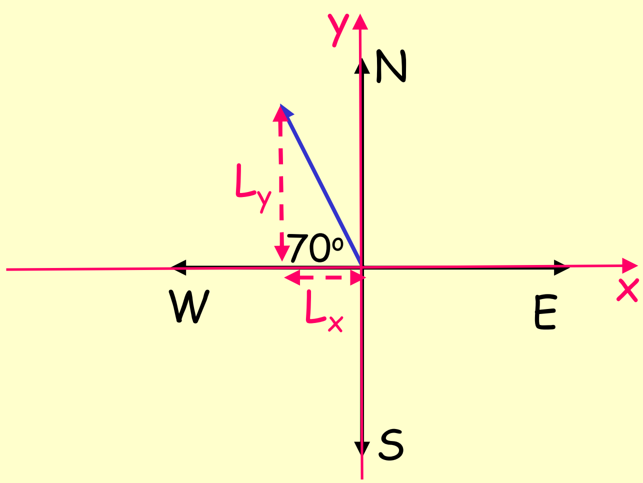
(B) $\boxed{- 60 \cos(70°)}$ m

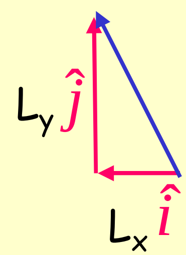
(C) $+ 60 \cos(70°)$ m

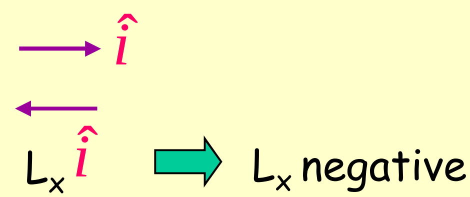
(D) $+ 60 \sin(70°)$ m

(E) None of the above

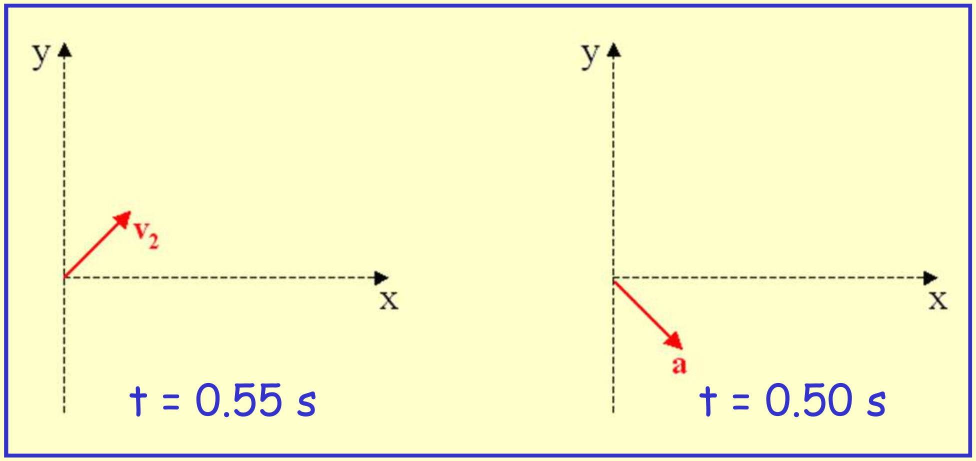
$$\vec{L} = L_x \hat{i} + L_y \hat{j}$$



# Velocity & Acceleration Vectors




What is the vector v(t=0.45 s)?

## TWO ISSUES

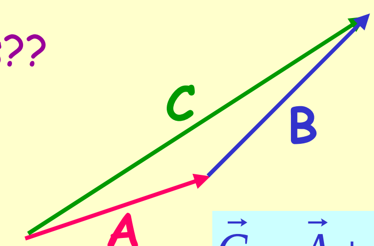
- How are velocity & acceleration related??
  - Definitions are the KEY

DEFINITIONS are ALWAYS TRUE
(by definition !!)

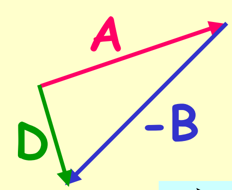
$$\vec{a} \equiv \frac{d\vec{v}}{dt} \quad \Rightarrow \quad \vec{a}(t)\Delta t = \Delta\vec{v}(t) \quad \Rightarrow \quad \underbrace{\vec{a}(t)}_{\text{given}}\Delta t = \underbrace{\vec{v}(t+\tfrac{1}{2}\Delta t)}_{\text{given}} - \underbrace{\vec{v}(t-\tfrac{1}{2}\Delta t)}_{\text{unknown}}$$

- How do you SUBTRACT vectors??

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

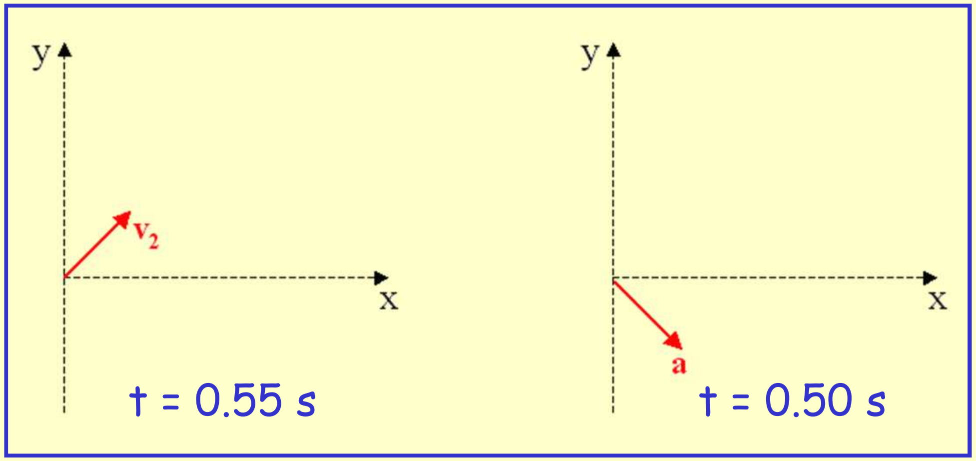



$\vec{C} = \vec{A} + \vec{B}$

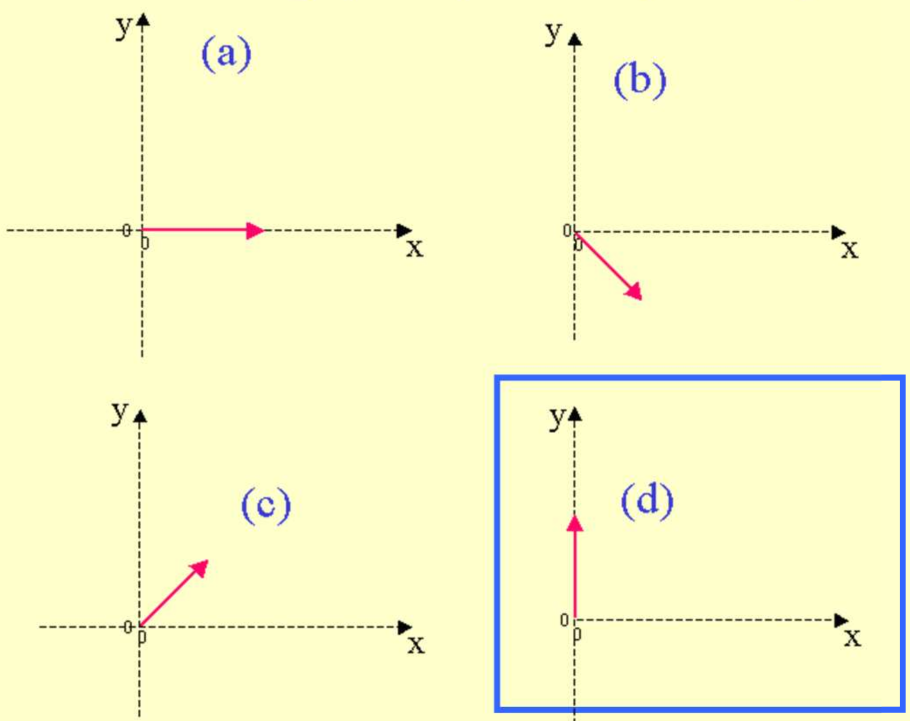


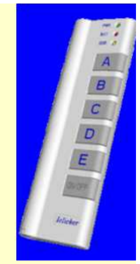

$\vec{D} = \vec{A} - \vec{B}$

# Preflight 3




Which of these graphs represents v(t=0.45 s)?



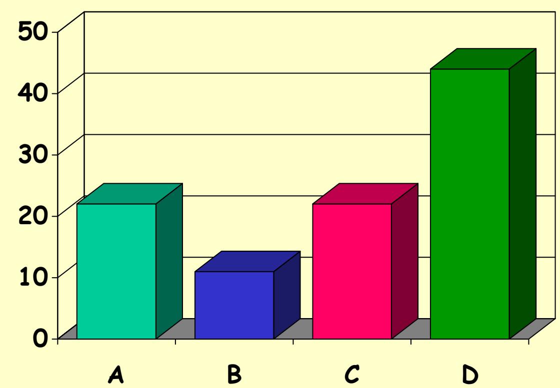




BB

You said:

- Since v1+v2=a and the tail and head of v1 and v2 have to be touching, the graph on b is the only one that would fit this.

- I obtain (d), being my V1 vector, since I subtracted V2-a to get V1. I attached the head of 'a' to the head of V2.Therefore being able to draw the V1 vector from the tail of V2 to the tail of 'a'.



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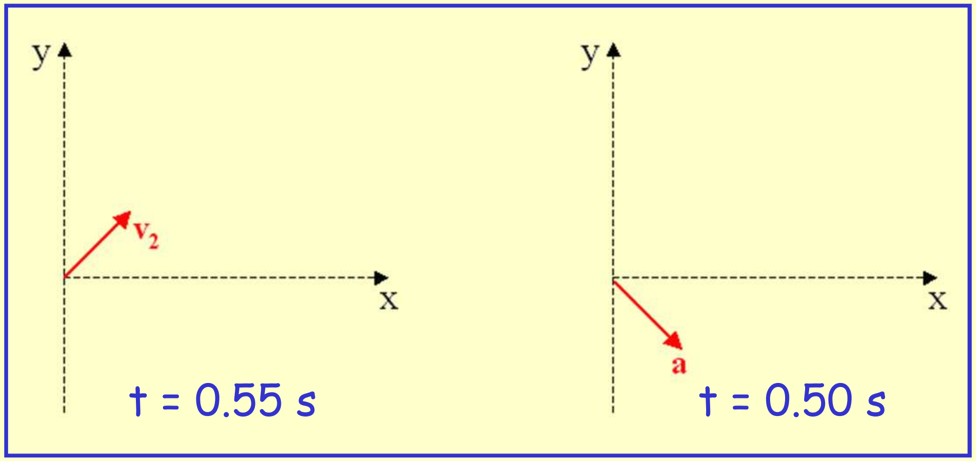


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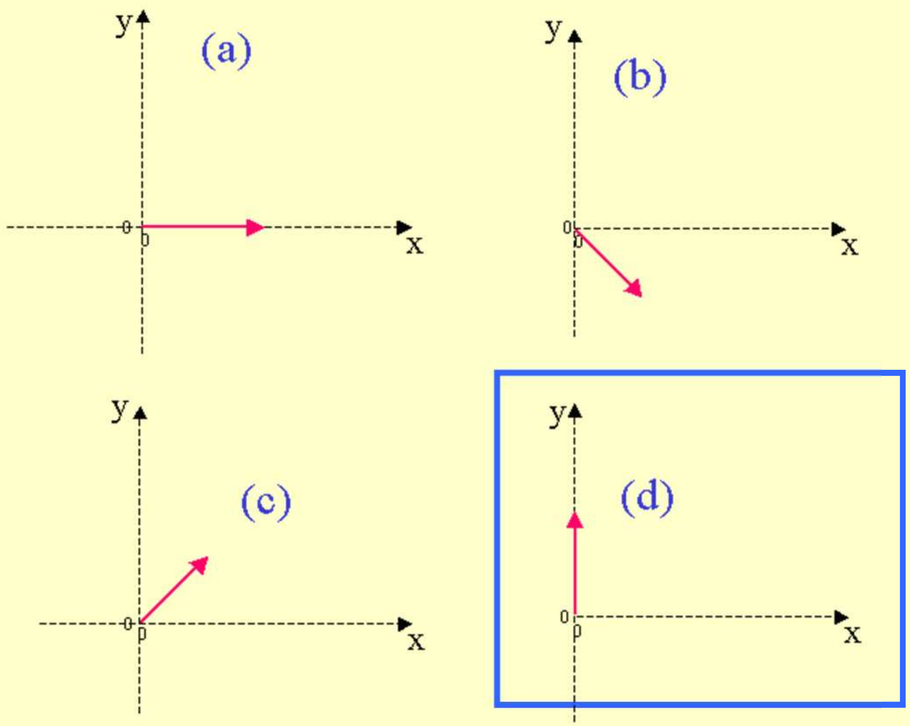
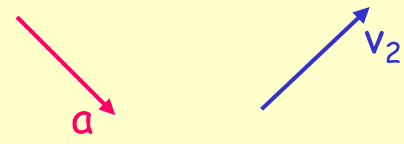
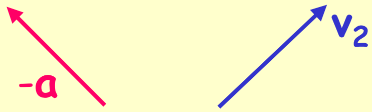


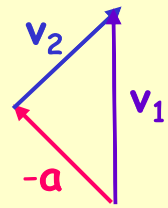

$$\vec{a}(t)\Delta t = \vec{v}(t + \tfrac{1}{2}\Delta t) - \vec{v}(t - \tfrac{1}{2}\Delta t)$$

given given unknown

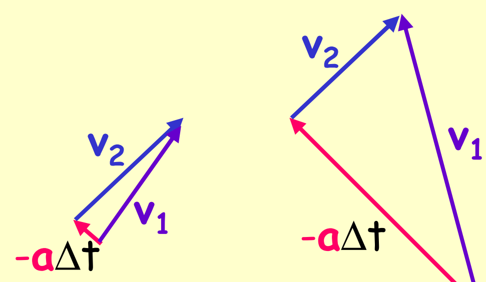



$$-\vec{a}(t)\Delta t + \vec{v}(t + \tfrac{1}{2}\Delta t) = \vec{v}(t - \tfrac{1}{2}\Delta t)$$



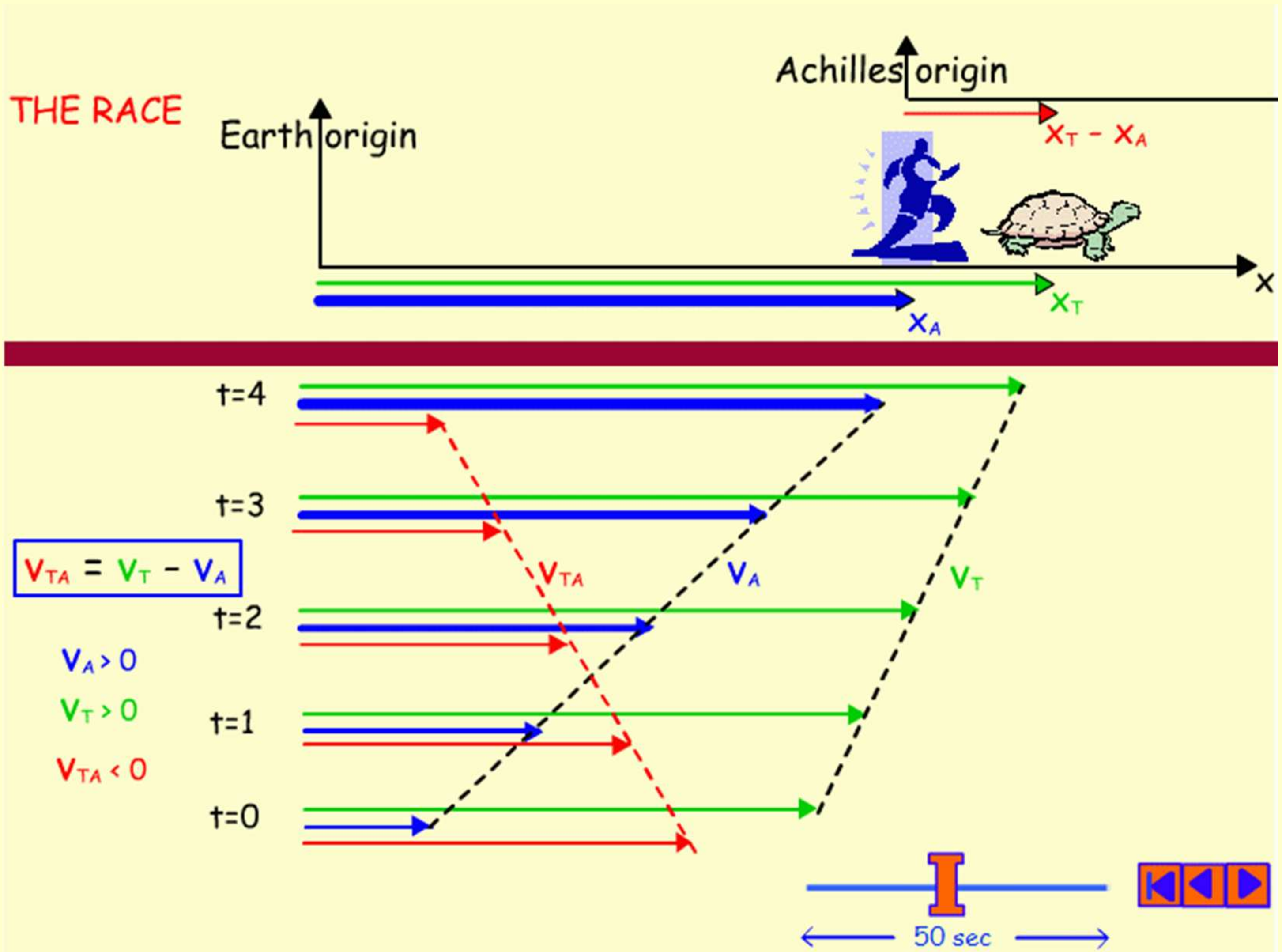

Actually. a should be a$\Delta$t




small $\Delta$t large $\Delta$t

# Relative Motion: Reference Frames

- There is only one equation: $$\boxed{v_{TA} = v_T - v_A}$$



# Preflight 5


BB

**A boat is trying to cross a flowing river.**

**Which direction should the boat point in order to reach the other side of the river in the least amount of time?**

(A) At some angle upstream, against the flow

(B) Straight across the river

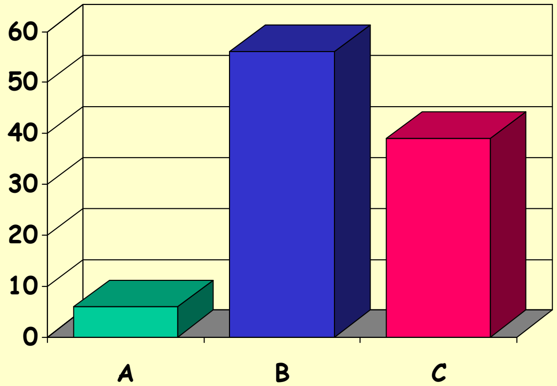
(C) At some angle downstream, with the flow

You said:

- If he points at some angle upstream, he will offset the flow of the river and take the most direct path across the stream. It will be the shortest distance and therefore the shortest amount of time.

- Since the flow of the river is perpendicular to the shortest path across the river it doesn't affect your ability to cross.

- river flows downstream so if the boat points with the flow of water instead of against it, he will travel faster.



# A Simpler (?) Question

A boat is trying to cross a ~~flowing~~ CALM river.

Which direction should the boat point in order to reach the other side of the river in the least amount of time?

(A) At some angle "upstream"

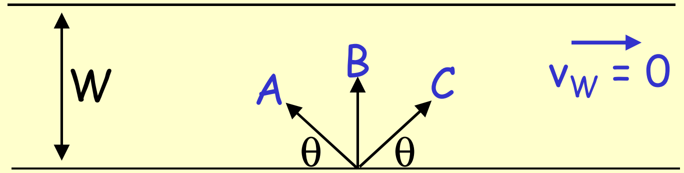
(B) Straight across the river

BB

(C) At some angle "downstream"

$$t_A = t_C = \frac{W/\sin\theta}{v_{boat}}$$

$$t_B = \frac{W}{v_{boat}}$$




WHY DID I ASK THIS QUESTION??

BECAUSE IT'S THE **SAME** QUESTION !!

It's just posed in a different reference frame !!

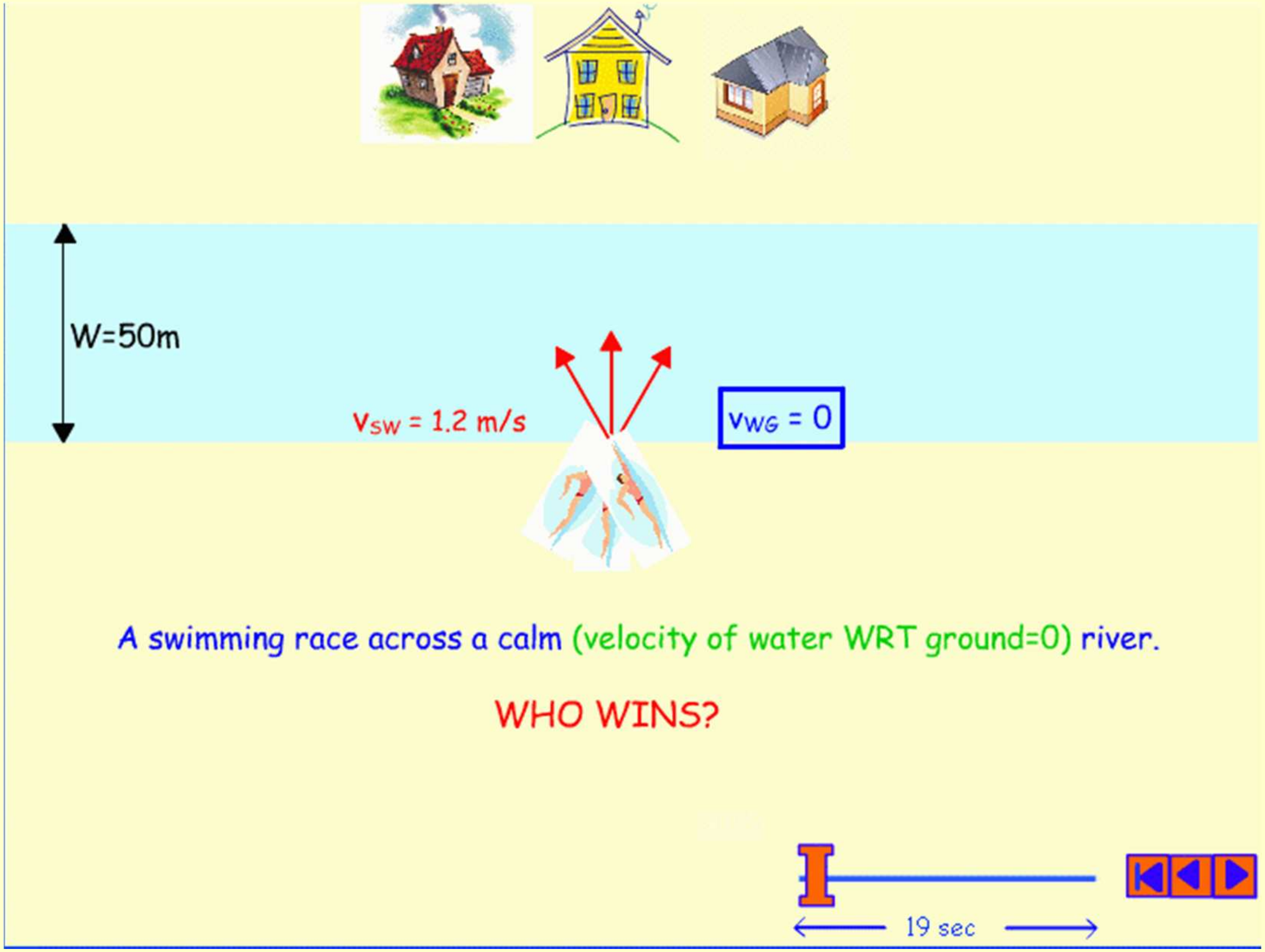
If $v_W = 0$, then we ARE in the reference frame of the water

How does the water know if it is "flowing"?

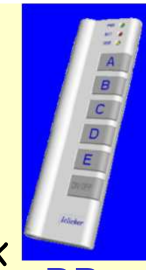
If it is "flowing", then the houses on shore are moving past it.

If it is not "flowing", then the houses on shore are not moving.

# The Race viewed from the Water

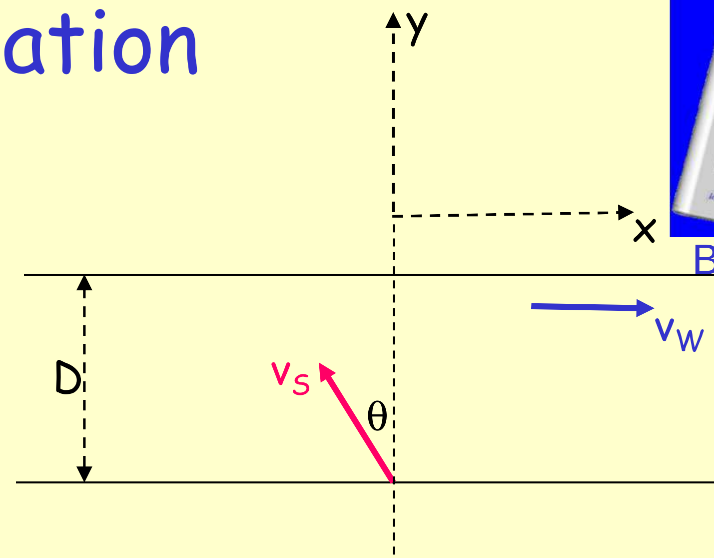


# Relative Motion Calculation




**A swimmer, who can maintain a constant speed of $v_S$ = 1.2 m/s in calm water, heads upstream at angle $\theta$ = 30°. The stream measures D = 50m across and the current flows at a speed of $v_W$ = 0.3 m/s with respect to the shore.**

**We want to determine how far downstream the swimmer will be when she reaches the other side.**




How far, in the frame of the water, does the swimmer swim to get to the other side?

(A) D

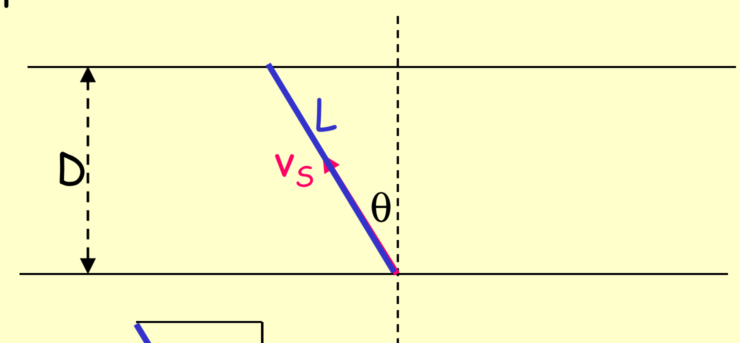
(B) D/sinθ

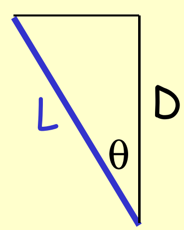
(C) D/cosθ

(D) Dsinθ

(E) Dcosθ

What does the path of the swimmer look like in the water's frame?




$$\cos\theta = \frac{D}{L} \quad \Rightarrow \quad \boxed{L = \frac{D}{\cos\theta}}$$

# Relative Motion Calculation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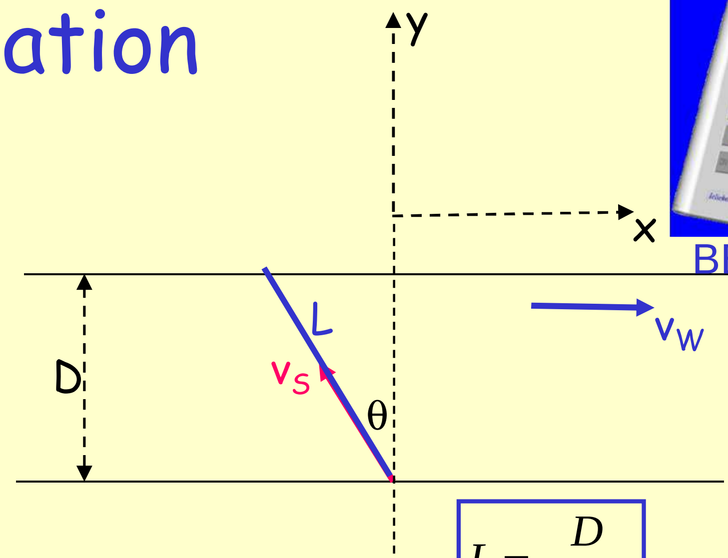



$$L = \frac{D}{\cos\theta}$$

How long does it take for the swimmer swim to get to the other side?

(A) $D/v_S$

(B) $D/(v_S\cos\theta)$

(C) $D/(v_S + v_W)$

(D) $D/((v_S + v_W)\cos\theta)$

Calculate in water frame:
Time = distance/speed

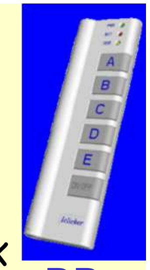
$$t = \frac{L}{v_S} = \frac{D/\cos\theta}{v_S}$$

Calculate in land frame:
Time = y-distance/y-speed

$$t = \frac{D}{v_{S\,y}} = \frac{D}{v_S\cos\theta}$$

Times are the same, as they must be!

# Relative Motion Calculation



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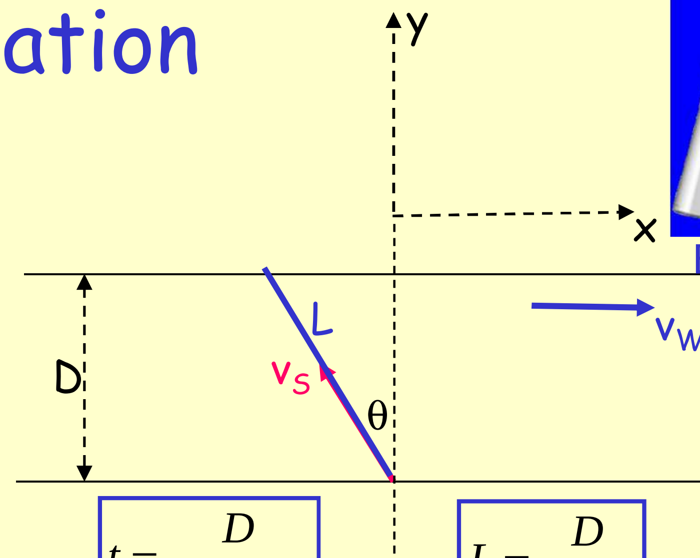



$$t = \frac{D}{v_S \cos\theta}$$

$$L = \frac{D}{\cos\theta}$$

What is the x-component of the swimmer's velocity in the land frame?

(A) $-v_S\sin\theta$

(B) $v_W + v_S\sin\theta$

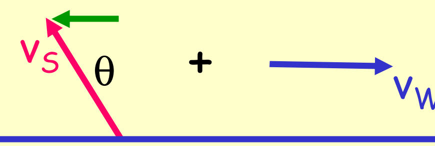
(C) $v_S\sin\theta$

(D) $v_W - v_S\sin\theta$

$$\vec{v}_{SG} = \vec{v}_{SW} + \vec{v}_{WG}$$

$$(v_{SG})_x = (v_{SW})_x + (v_{WG})_x$$

$$(v_{SG})_x = -v_S \sin\theta + v_W$$




Knowing this velocity and the time it takes to cross the stream, we can find how far downstream the swimmer ends up.

$$x = (v_{SG})_x t = (v_W - v_S \sin\theta)\frac{D}{v_S \cos\theta}$$