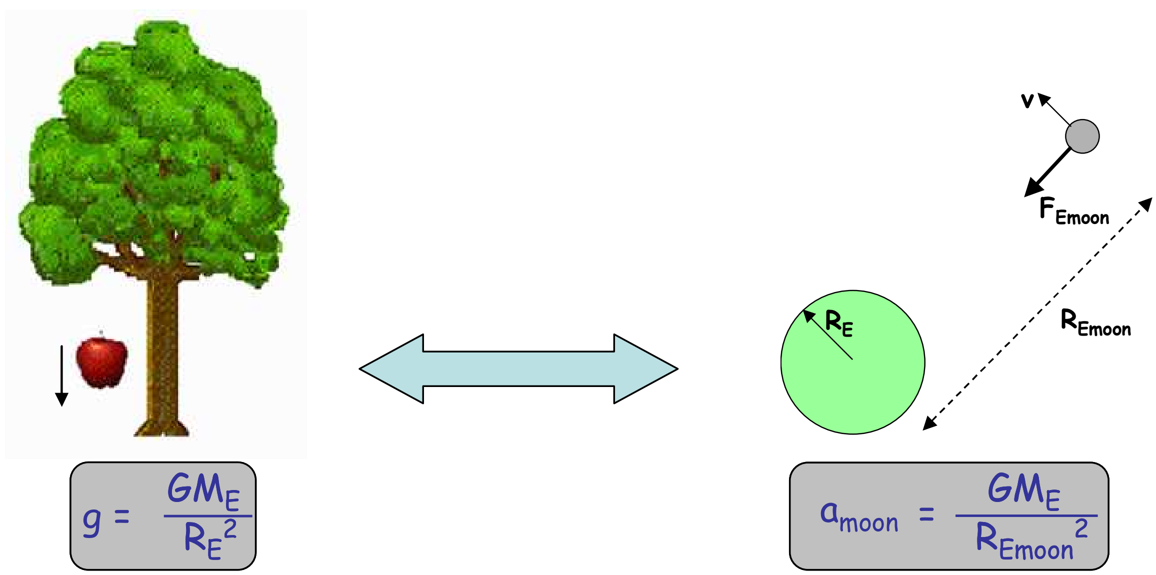
University of Illinois

Physics 100

Thinking about Physics



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Gary Gladding

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How to Use this Book

This book is different from other physics books you may have seen or used. Most physics books try to do everything, but have not proved successful. From our previous surveys, we have learned that the vast majority of our students (about 70%) state that they “rarely” or “never” read the text before class, and that the text was “not very useful” or “useless” in helping them understand the course material.

We see this book as just one component in a coherent structure we have created that will help you learn the physics. Your first engagement with the material will be the “***prelecture***” web-based activity that you do before coming to lecture. This prelecture consists of a set of Flash movies (animation plus narration) that present the material within a player that gives you complete control (pause, rewind, fast-forward, etc) of the presentation. After completing the prelecture, you should do the web-based “***preflight***” assignment. Your answers and explanations to these questions will give us some indication of your understanding of the material “just in time” for the “***lecture***”. The lecture will be highly interactive, informed by your preflight responses and featuring in-class polling using the *i>clicker* system. We have included the preflight questions and some blank pages in this textbook so that you can bring this book to lecture to use as a reference and to take notes. After lecture, you should be prepared to do the ***web-based homework*** assignment. The capstone experience then is the ***discussion section*** in which you will work collaboratively with other students on materials prepared by the PHYS 100 staff. This cycle repeats itself for seven weeks. This book contains an additional unit (*Universal Gravitation and Springs*) that will be covered in the first week of the follow-up course (PHYS 199Z) which will be given in the second half of the semester and will focus on problem solving for those students who need the additional practice. These eight units cover all of the material in the first hour exam of PHYS 211 and hopefully will have prepared you to “think about physics” in a productive way that will serve you well in your later physics courses.
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1. Kinematic Definitions

A) Overview of Course

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The purpose of this course is to prepare you for success in PHYS 211, the initial course in the introductory physics sequence for scientists and engineers. Our plan is to present in this course the material that is covered in the first hour exam of PHYS 211. We feel this choice of content is essential since this initial material will be already familiar to most of the students in PHYS 211.

Perhaps even more important than introducing you to this specific material will be the development of your ways of thinking about physics, which, in fact, is the title of this course. We stress this approach because most students entering PHYS 211 have a pretty big misunderstanding of what "doing physics" means. In particular, many students see "doing physics" as finding the formula in the book into which to plug the given numbers in order to find the answer to the problem. This approach will not succeed in PHYS 211; the problems there are not "plug and chug". The problems there will require reasoning based on a firm conceptual understanding of the physical situation described. It is our main goal in PHYS 100 to bring you to the point where you naturally approach physics problems in this way.

That said, what will be the content we discuss in order to help you develop your ways of thinking about physics? Well, the first four weeks, we will cover ***kinematics***: the ***description of motion***. To describe motion, we will specify how three quantities, displacement, velocity, and acceleration change as a function of time. The quantities are not independent; the velocity of an object describes how its displacement (position) changes in time, while the acceleration of an object describes how its velocity changes in time. We will find that the language of calculus allows us to relate these quantities in a very natural way.

The final four weeks, we cover ***dynamics***: the ***causes of the motions*** that we learned to describe in the kinematics section. There are two new concepts that we will need to understand. The first is *mass*, the measure of resistance to a change in velocity. The second is *force*, the thing that brings about changes in velocity. These concepts are related by the kinematical concept of acceleration in Newton's Second Law, the foundation of classical mechanics.

B) Zeno's Paradoxes

We begin by making the important point that it is not easy to discuss motion using ordinary language. To support this point, we introduce some arguments made by the Greek philosopher Zeno.

First, he claims that it is impossible to move from some point A to another point B. Why would he make this claim?? He begins with the statement that before we can move to point *B*, we need first to move to point *C* which is halfway between points *A* and *B*. Sounds true enough, however, this argument can be repeated *ad infinitum*. *i.e.*, once at *C*, we would need to move first to point *D* which is halfway between points *C* and *B*. You get the drift, I'm sure. We will need to make an infinite number of moves to get to point *B*.

He makes a similar argument to support the claim that the faster will never catch the slower. Namely, suppose Achilles (the faster) gives the tortoise (the slower) a head start. Then, before Achilles can catch up to the Tortoise, he must certainly reach the Tortoise's starting point. But when he does reach this point, the tortoise will have progressed to a further point. Once again he can repeat this argument *ad infinitum*.

What is my point here? It certainly is not to prove that motion is impossible; we all know that's not true. In fact, the reason that these arguments are called "paradoxes" is that what seems to be a reasonable argument leads to a conclusion that we know is false. Zeno initiated these arguments as ways to investigate the nature of space and time.

How do we resolve these paradoxes? Clearly the problem lies with the notion of infinity. Mathematics can help us. We know, for example, that an infinite series can have a well-defined sum. Our approach will be to carefully define the concepts of displacement, velocity, and acceleration and then use mathematics as a tool to answer questions about motion.

C) Kinematic Concepts

i) Time

Our program begins with careful definitions of the concepts we will need to describe motion. We will talk about "events", things that happen. An individual event occurs at some location in space at a given time. We begin with "time". How can we "define carefully" time? Well, actually it's a pretty difficult job. We all have a sense of time that can be described by the metaphor of "time as a river". Time continually flows forward and any event can be assigned a "place" in this river, a definite value for the time that it happened.

In this course, we will accept this intuitive sense of time. You should know, however, that this picture of time is not our current best understanding of this concept. According to the theory of relativity, for example, if two events are simultaneous according to one observer, they will not be simultaneous according to another observer that is moving relative to the first. We can safely ignore this effect in this course, though, as long as we restrict ourselves to velocities that are small with respect to the speed of light.

ii) Displacement

Accepting our intuitive sense of time, how do we "carefully define" the location in space of a particular event? We will define the location of an event in terms of its displacement from an origin. We represent displacements as vectors. Vectors are mathematical quantities that have magnitude and direction.

For example, we can choose our origin to be the center of the universe, Loomis Lab. Well, maybe it only seems like the center of the universe to some of us, but the choice of an origin is totally arbitrary. If our event of interest happens in Engineering Hall, for example, we describe the event with a displacement vector that points from Loomis Lab to Engineering Hall. This displacement vector has a certain magnitude (say 100 meters) and a direction (say due west). Figure 1 shows such a displacement vector.

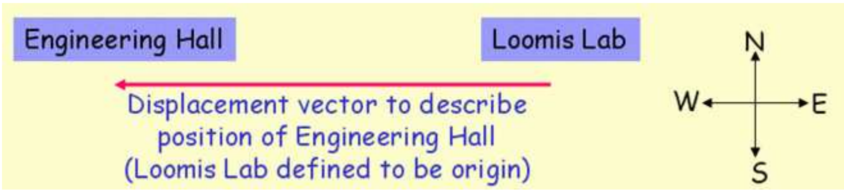


Figure 1.1: The displacement vector that locates Engineering Hall in a co-ordinate system defined with Loomis Lab as the origin.

We should note here that for the first two weeks, we will restrict ourselves to one spatial dimension in order to simplify the mathematics so that we can concentrate on the physics. Displacements will still be vectors, but they will exist in only one dimension; the vector nature then simplifies from specifying angles to simply specifying a sign (positive or negative).

So, an event is located in space and time by two quantities, its time of occurrence and its displacement from an origin. We will now describe the motion of an object in terms of these quantities.

iii) Velocity

We begin our discussion of the motion of objects by defining the velocity of an object as the *change* in its displacement vector per unit time. We represent velocity mathematically as the vector that is defined by the derivative of the displacement vector with respect to time.

$$\vec{v} = \frac{d\vec{x}}{dt}$$

Yes, we just defined a physics quantity, the velocity, in terms of a derivative, a thing used in calculus. How did that happen? Let's look a little closer to see why the use of calculus is "natural" here.

Let's talk about the motion in terms of events. Event *1* corresponds to the location of an object at some time t. We represent this location as the displacement of the object, $\boldsymbol{x}(t)$, from some origin. Event *2* corresponds to the location of the object a small time dt later. We represent this event in terms of the displacement of the object at time $t = t + dt$, which we represent as $\boldsymbol{x}(t + dt)$. The change in displacement of the object during this time interval dt is itself a vector and is just given by the vector difference between these two displacements.

In one dimension, the vector difference is just the arithmetic difference. Figure 1.2 shows an example of this vector subtraction in one dimension. *i.e.*, if the displacement at time t is represented as +4 and the displacement at time $t + dt$ is represented as +3, then the change in displacement is just equal to $\boldsymbol{x}(t + dt) - \boldsymbol{x}(t) = 3 - 4 = -1$.

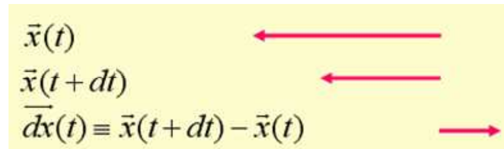


Figure 1.2: Change of displacement $\boldsymbol{dx}$ is given by $\boldsymbol{x}(t+dt) - \boldsymbol{x}(t)$.

In general, vectors are added by placing the tail of the second vector at the head of the first and then drawing a vector from the tail of the first to the head of the second. This procedure works here, as well, of course, since we start with $\boldsymbol{x}(t + dt)$, a vector of length 3 pointing to the left and then adding to it $-\boldsymbol{x}(t)$ which is a vector of length 4 pointing to the right which results in the displacement vector $\boldsymbol{dx}(t)$ which is represented by an arrow of length 1 pointing to the right.

In English, the change in displacement ($\boldsymbol{dx}$) is the vector you have to add to the initial displacement ($\boldsymbol{x}(t)$) to get the final displacement ($\boldsymbol{x}(t + dt)$).

In the preceding example, we looked at the change in an object's displacement vector during a small time interval dt. If we take the limit as dt goes to zero, we obtain our definition of velocity as the derivative of the displacement with respect to time. We employ calculus here so that we can define an *instantaneous* velocity of the object at any time t. Namely, at any time t, we can describe an object in terms of its displacement vector $\boldsymbol{x}(t)$ and its a velocity vector $\boldsymbol{v}(t) = d\boldsymbol{x}(t)/dt$. The velocity vector tells us how the location of the object will change in the next instant of time.

This instantaneous velocity describes the motion of the object at any instant of time. Sometimes, we do not need this precision and we speak of an average velocity instead of the instantaneous velocity. The average velocity of an object between times t_1 and t_2 is simply defined as the change in displacement of the object during these times divided by the time interval, $t_2 - t_1$. Clearly, in the limit that this time interval is small, the average velocity becomes the instantaneous velocity.

To give an example, suppose you left Lincoln and Green at noon and arrived at Engineering Hall at 12:20pm. What was your average velocity during noon and 12:20pm?

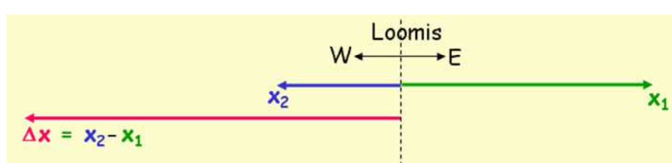


Figure 1.3: The displacement $\boldsymbol{\Delta x}$ between two locations used to calculate the average velocity.

To answer this question, let's draw the displacement vectors and apply the definition of the average velocity as shown in Figure 1.3. Keeping Loomis Lab as our origin, your initial displacement is represented by the vector $\boldsymbol{x_1}$, having magnitude 200 m and pointing to the right (*i.e.*, east), while your final displacement is represented by the vector $\boldsymbol{x_2}$, having magnitude 100m and pointing to the left (*i.e.*, west). To calculate the average velocity, we first determine the vector $\boldsymbol{\Delta x} = \boldsymbol{x_2} - \boldsymbol{x_1}$, which is represented by the vector $\boldsymbol{\Delta x}$, having magnitude 300m and pointing to the left. Consequently, the average velocity is just equal to the change in displacement (300m) divided by the time interval (20 minutes) which is 15 m/minute. The direction of the velocity is to the left (*i.e.*, west).

Before moving on, we want to one point here. Namely, note that the average velocity does not really depend on where we chose our origin. We chose our origin at Loomis here, just so that we could get practice with dealing with signs, positive and negative displacements. The average velocity is defined only in terms of the *change in displacement*. Had we chosen the origin to be Lincoln and Green, for example, $\boldsymbol{x_1}$ would have been 0 and $\boldsymbol{x_2}$ would have been 300 m pointing west, resulting in the same change in displacement (300m to the west) and therefore the same average velocity (15 m/min towards the west).

D) Graphical Representations

We've defined the instantaneous velocity of an object in terms of the change in its displacement from some origin. We now want to solidify this relationship by examining the graphical representation of a particular one dimensional motion.

Figure 1.4 shows a plot of the displacement of an object as a function of time. This object is located at the origin at time t = 0 and then moves to a maximum positive distance from the origin at t = 2 seconds and then returns to the origin at t = 4 seconds.

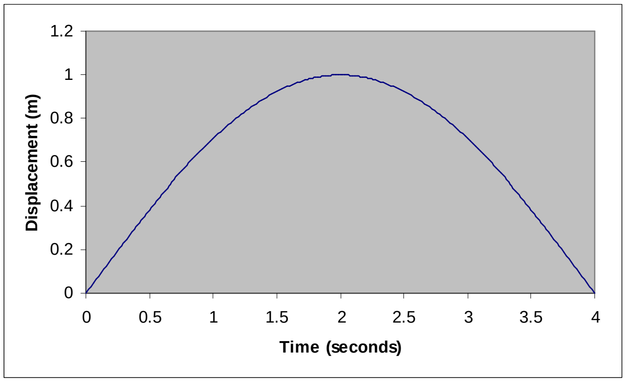


Figure 1.4: Graph of the displacement as a function of time for a specific 1-D motion.

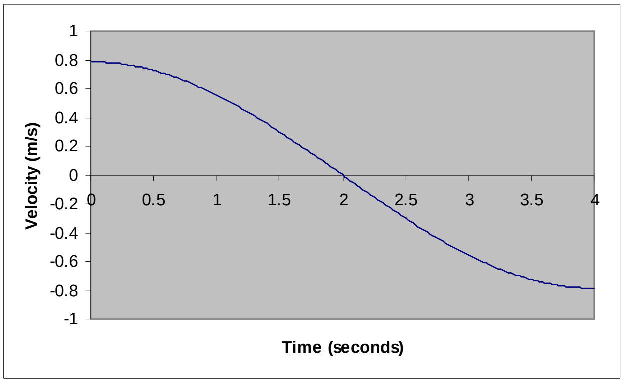


Figure 1.5: Graph of the velocity as a function of time for the 1-D motion shown in Fig 1.4.

What can we say about the velocity of the object? Well, the instantaneous velocity at any time t can be determined from the displacement. Namely, the velocity is defined to be the derivative of the displacement with respect to time. We can determine this derivative graphically by drawing the tangent to the displacement curve at every time t.

Note that initially the slope of the tangent to the curve is positive and in fact, it's as big as it's ever going to be. As time increases, the slope is still positive, but it is less steep, corresponding to a velocity in the same direction (that's determined by the sign of the slope) but with a smaller magnitude. As we reach t = 2 seconds, we see the slope of the tangent is zero. What does this mean? Well, it means that the instantaneous velocity at t = 2 seconds is zero. It is momentarily at rest. It has been slowing down (*i.e*, the magnitude of the velocity has been decreasing) and finally becomes zero at t = 2 seconds.

As we move beyond t = 2 seconds, the slope of the tangent is small and negative. What does this mean? Well, it means that the object has reversed its direction. Its velocity is negative. In one dimension, the sign of the velocity simply indicates the direction in which the object is moving. As time goes on, the slope remains negative, but gets steeper, approaching its maximum (negative) value at t = 4 seconds.

To recap, we have constructed the plot of the velocity as a function of time from the plot of the displacement as a function of time. For our particular example, the object began with a large velocity in the positive direction, slowed down to a momentary stop at t = 2 seconds and then sped up to a large velocity in the negative direction.

We'll now formalize this talk of slowing down and speeding up by defining the acceleration of the object.

D) Acceleration

We have defined the velocity of an object as the change of its displacement per unit time. In an exactly analogous way, we now define the acceleration of an object as the change in its velocity per unit time. *i.e.*, we represent acceleration mathematically as the vector that is defined by the derivative of the velocity vector with respect to time.

$$\vec{a} = \frac{d\vec{v}}{dt}$$

We can repeat the exercise we initially did for velocity now for acceleration. Only the names have been changed. In particular, Figure 1.6 shows a vector pointing to the left that now represents the velocity of the object at time t. After a time interval dt, the new velocity has changed as shown, It still points to the left, but its magnitude has decreased. We can therefore determine the change in velocity, ***dv****(t)* during this time interval by subtracting ***v****(t)* from ***v****(t + dt)* to obtain the vector shown that now points to the right.

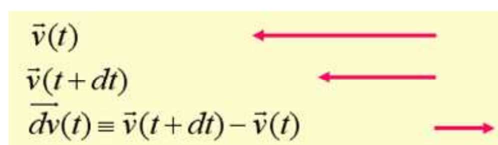


Figure 1.6: Change of velocity ***dv*** is given by $v(t+dt) - \mathbf{v}(t)$.

Once again, in English, we say that the change in velocity (***dv***) is the vector you have to add to the initial velocity (***v****(t)*) to get the final velocity (***v****(t + dt)*).

Note that in this example, the change in velocity vector points to the right (*i.e.*, east), while both velocity vectors point to the left (west). What is the significance of this change in sign? Well, this change in sign tells us that the object is "slowing down". *i.e.*, it is still going in the same direction (west), but the magnitude of its velocity is getting smaller. It is slowing down.

If we had chosen a different example in which $\boldsymbol{v}(t + dt)$ was still in the same direction as $\boldsymbol{v}(t)$, but had a larger magnitude than $\boldsymbol{v}(t)$, the change in velocity vector would then have the same direction as each of the velocity vectors. In this case, we would say that the object is speeding up.

Consequently, the direction of the acceleration, relative to the velocity, determines whether the object is speeding up or slowing down. If the acceleration and velocity have the same direction (*i.e.,* the same signs for the one-dimensional case), then the object is speeding up. If the acceleration and velocity have opposite directions (*i.e.,* the opposite signs for the one-dimensional case), then the object is slowing down.

The acceleration we have just defined, namely $\boldsymbol{a} = d\mathbf{v}/dt$, is the instantaneous acceleration. It is defined at specific time t. Just as for velocity, we can also define an average acceleration during a time interval Δt, as the change in velocity ($\boldsymbol{\Delta v}$ which is equal to $\boldsymbol{v_2} - \boldsymbol{v_1}$) divided by the time interval, Δt, which is equal to $t_2 - t_1$.

The average acceleration then is defined for a time interval and therefore gives a coarser representation of the motion than does the instantaneous acceleration which is defined at every instant in time.

To this point, we have defined the velocity as the change in displacement per unit time and the acceleration as the change in velocity per unit time. The mathematics used to represent velocity and acceleration are identical. Physically, though, there is a very important difference between velocity and acceleration. Namely, *velocity* is a *relative* concept, while *acceleration* is an *absolute* concept.

Velocity must always be described relative to some reference frame. For example, if you are sitting in an airplane are you at rest or are you moving at 400 mph? There is no correct answer to this question. You can say is that your velocity with respect to the airplane is zero. You can also say that your velocity with respect to the Earth (or maybe the air) is 400 mph. You must always specify your velocity with respect to some reference frame; your velocity at any time is not an absolute quantity.

While your velocity at any time does depend on the reference frame, the change in your velocity does not! If you are accelerating, you know it. For example, if you are travelling in a plane and it encounters turbulence, you will experience an acceleration. Your velocity (in either the plane's frame or the earth's frame) will change, in fact, by the same amount. The acceleration at a given time is a property of the object; it is an absolute quantity.

You will see later that since acceleration is an absolute quantity, it (and not velocity) will be important in dynamics. A change in velocity (*i.e.,* acceleration) needs to be explained (*e.g.,* by forces) whereas a constant velocity does not.

E) More Graphs

We've defined the instantaneous acceleration of an object in terms of the change in its velocity. We now want to solidify this relationship by examining the graphical representation of the one dimensional motion we introduced earlier.

We have reproduced below Figure 1.5 which shows the plot of the velocity of an object as a function of time that we constructed from its displacement as a function of time (Fig 1.4). This object starts out with a large velocity in the positive direction, slows down to a momentary stop at t = 2 seconds and then speeds up to a large velocity in the negative direction.

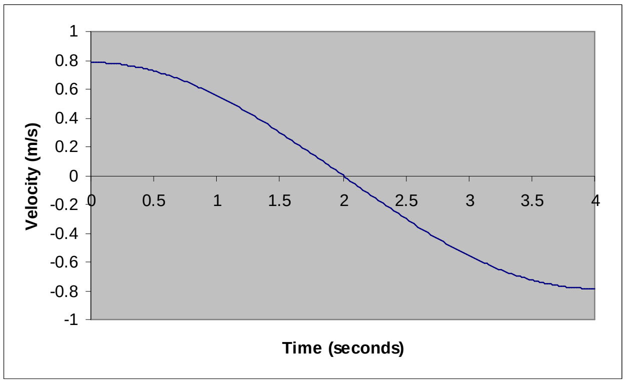


Figure 1.5: Graph of the velocity as a function of time for the 1-D motion shown in Fig 1.4.

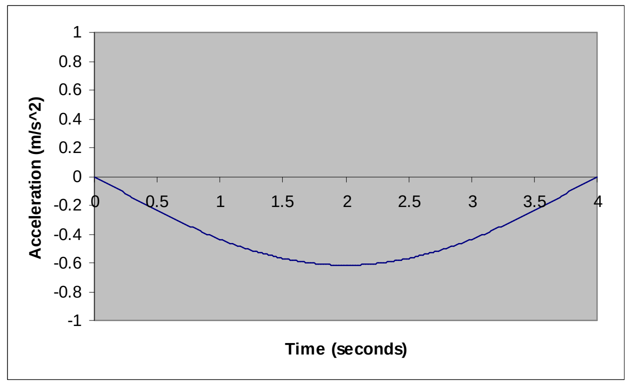


Figure 1.8: Graph of the acceleration as a function of time for the 1-D motion shown in Fig 1.4.

What can we say about the acceleration of the object? Well, just as in the previous case, we can determine the instantaneous acceleration at any time t from the velocity. Namely, the acceleration is defined to be the derivative of the velocity with respect to time. We can determine this derivative graphically by drawing the tangent to the velocity curve at every time t.

Note that initially the slope of the tangent to the curve is zero. Consequently, the initial acceleration is zero. As time increases, the slope is negative and getting steeper, corresponding to an acceleration in the opposite direction of the velocity (*i.e.*, the velocity is positive, but the slope is negative) with an increasing magnitude. What's going on here? We have to be a little careful. Since the magnitude of the acceleration is increasing, it may be tempting to think that the object is speeding up. In fact, we know this is not right; the object is clearly slowing down since the acceleration is in the opposite direction of the velocity. The increase in the magnitude of the acceleration indicates simply that the rate at which it is slowing down is increasing.

As we reach t = 2 seconds, we see the slope of the tangent reaches its maximum negative value, corresponding to the maximum negative acceleration. As we move beyond t = 2 seconds, the slope of the tangent remains negative, but becomes less steep. Once again, we have to be careful. The object is definitely speeding up since the velocity and the acceleration are in the same direction. The rate at which the object is speeding up is decreasing however. As time goes on, the slope remains negative, but decreases in magnitude, approaching a value of zero at t = 4 seconds.

To recap, we have constructed the plot of the acceleration as a function of time from the plot of the velocity as a function of time. For our particular example, the acceleration of the object started at zero and increased in magnitude in the negative direction until it reached its maximum magnitude at t = 2 seconds and then decreased again to a value of zero at t = 4 seconds..

F) Summary

We conclude with a brief discussion of the main points of this unit.

First, we introduced the ***displacement*** vector to define the location of an object at any give time. This displacement vector was defined with respect to an arbitrary origin.

Second, we defined the ***velocity*** of the object at any time to be equal to the derivative of its displacement vector with respect to time. The velocity vector does not depend on the choice of the origin within a particular reference frame, but it does depend on the choice of a reference frame. Velocities must always be defined with respect to a particular reference frame.

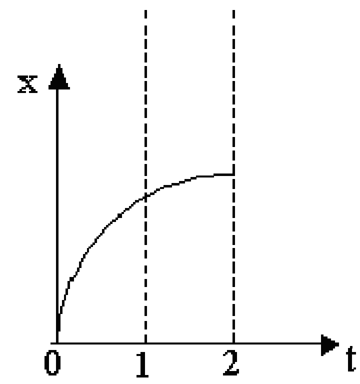
Third, we defined the ***acceleration*** of the object at any time to be equal to the derivative of the velocity vector with respect to time. The acceleration vector does not depend on the choice of a reference frame. The acceleration of an object has the same value in all reference frames. If the acceleration vector and the velocity vector are in the same direction, the object is speeding up. If the acceleration vector and the velocity vector are in opposite directions, the object is slowing down.

Preflight 1

1. The graph shows the displacement *x* as a function of time *t* for an object in motion in one dimension.

Which one of the following statements is correct?

A) The instantaneous velocity at $t = 1$ is less than the average velocity from $t = 0$ to $t = 2$.
B) The instantaneous velocity at $t = 1$ is equal to the average velocity from $t = 0$ to $t = 2$.
C) The instantaneous velocity at $t = 1$ is greater than the average velocity from $t = 0$ to $t = 2$.

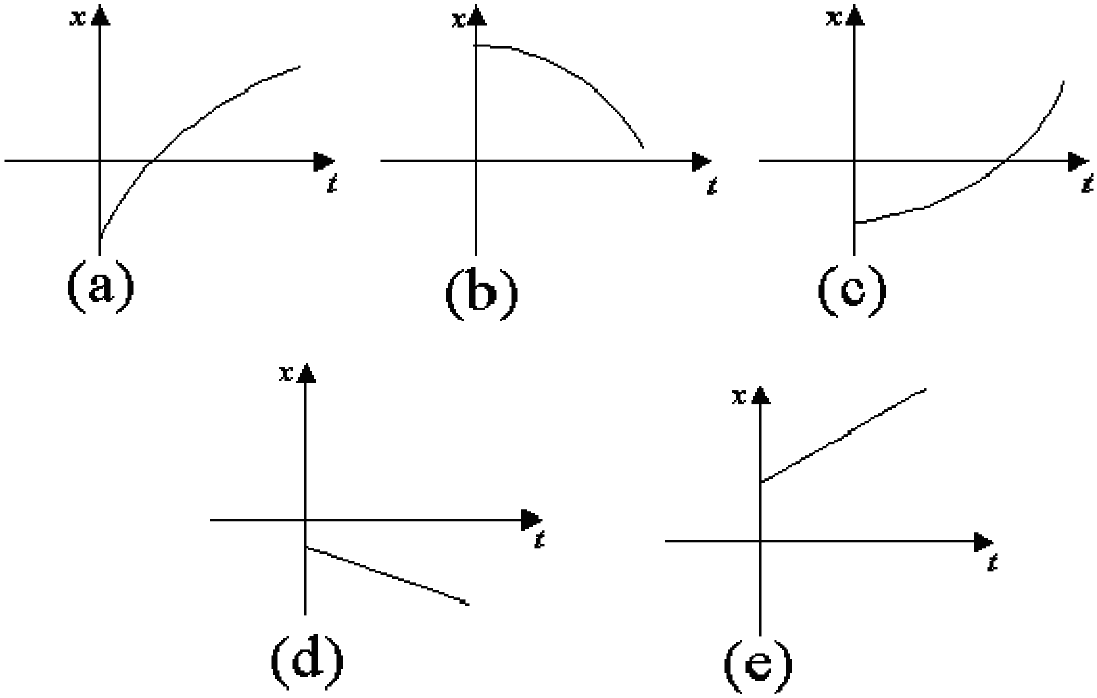


3. An object experiences a positive acceleration, thus its speed must always increase.

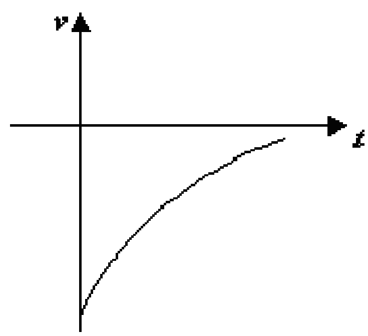
A) True
B) False

5. An object in one-dimensional motion moves in the positive direction and is slowing down.

Which one of the five displacement (x) vs. time (t) graphs below represents this motion?



7. The graph shows the velocity (v) vs. time (t) for a particle in one-dimensional motion.



Which one of the following statements represents the correct verbal description of this motion?

A) The acceleration of the particle is always negative and it is always speeding up.
B) The acceleration of the particle is always positive and it is always speeding up.
C) The acceleration of the particle is always negative and it is always slowing down.
D) The acceleration of the particle is always positive and it is always slowing down.

Notes

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