Physics 101: Lecture 02

Forces: Equilibrium Examples

- Today’s lecture will cover Textbook Sections 2.1-2.7

Phys 101 URL:

http://courses.physics.illinois.edu/phys101/

Read the course web page!
Overview

● Last Lecture

→ Newton’s Laws of Motion
  » FIRST LAW: Inertia
  » SECOND LAW: \( F_{\text{net}} = ma \)
  » THIRD LAW: Action/reaction pairs

→ Gravity

\[
W = G \frac{M_{\text{Earth}} m}{r_{\text{Earth}}^2} = m \left( G \frac{M_{\text{Earth}}}{r_{\text{Earth}}^2} \right)
\]

\( \approx mg \) (near Earth’s surface!)

● Today

→ Forces as Vectors

→ Free Body Diagrams to Determine \( F_{\text{net}} \)
  » Draw coordinate axes, each direction is independent.
  » Identify/draw all force vectors

→ Friction: kinetic \( f = \mu_k N \); static \( f \leq \mu_s N \)

→ Contact Forces – Springs and Tension
Forces as Vectors

- Last lecture we calculated the force of gravity on a book (i.e. its WEIGHT):

  Calculate the gravitational force on a 3 kg book held 1 meter above the surface of the earth.

  \[
  W = G \frac{M_{\text{Earth}} \cdot m}{r_{\text{Earth}}^2}
  \]

  \[
  = (6.7 \times 10^{-11} \frac{m^3}{(kg \cdot s^2)}) (6 \times 10^{24} \text{ kg}) (3 \text{ kg})/ (6.4 \times 10^6 + 1)^2 \text{ m}^2
  \]

  \[
  = 29.4 \text{ kg m/s}^2 = 29.4 \text{ N}
  \]

- We missed something: The direction!

  \[\downarrow \text{W} \quad \text{is different than} \quad \uparrow \text{W}\]
Forces as Vectors

- A quantity which has both magnitude and direction is called a VECTOR; FORCES are VECTORS.

- Usually drawn as an arrow pointing in the proper direction, where the length indicates the magnitude.

- This is an example of VECTOR ADDITION: to add vectors, you place them head to tail, and draw the RESULTANT from the start of the first to the end of the last.

\[ \overrightarrow{A} + \overrightarrow{B} = \overrightarrow{C} \]
Another Example of a Force: Tension

- Tension in an Ideal String, $T$:
  - Direction is parallel to string (only pulls)
  - Magnitude of tension is equal everywhere.

Now we are ready to do some physics!

**QUESTION:**

We suspend a mass $m = 5$ kg from the ceiling using a string. What is the tension in the string?
Newton’s 2\textsuperscript{nd} Law and Equilibrium Systems

We suspend a mass \( m = 5 \text{ kg} \) from the ceiling using a string. What is the tension in the string?

- Every single one of these problems is done the same way!

- Step 1: Draw a simple picture (called a Free Body Diagram), and label your axes!

- Step 2: Identify and draw all force vectors

- Step 3: Use your drawing to write down Newton’s 2\textsuperscript{nd} law

\[ F_{\text{Net}} = ma \quad \text{In equilibrium, everything is balanced!} \quad a = 0 \]

\[ T - W = 0 \]

\[ T = W = mg = (5 \text{ kg}) \times (9.8 \text{ m/s}^2) = 49 \text{ N} \]
Checkpoint!

• What does scale 1 read?
  
  • A) 225 N  
  • B) 550 N  
  • C) 1100 N

The magnitude of tension in an ideal string is equal everywhere.
Two boxes are connected by a string over a frictionless pulley. **In equilibrium**, box 2 is lower than box 1. Compare the weight of the two boxes.

A) Box 1 is heavier  
B) Box 2 is heavier  
C) They have the same weight

Step 1 – Draw!  
Step 2 – Forces!  
Step 3 – Newton’s 2\textsuperscript{nd}!  
\[ F_{\text{Net}} = m \ a \]  
1) \[ T - m_1 \ g = 0 \]  
2) \[ T - m_2 \ g = 0 \]
Another Force Example: Springs

- Force exerted by a spring is directly proportional to its displacement $x$ (stretched or compressed).

$$F_{\text{spring}} = -k \cdot x$$

- **Example:** When a 5 kg mass is suspended from a spring, the spring stretches $x_1 = 8$ cm. If it is hung by two identical springs, they will stretch $x_2 =$

A) 4 cm  B) 8 cm  C) 16 cm

<table>
<thead>
<tr>
<th>1 Spring</th>
<th>2 Springs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1 - W = 0$</td>
<td>$S_1 + S_2 - W = 0$</td>
</tr>
<tr>
<td>$S_1 = W$</td>
<td>$kx_2 + kx_2 = 2kx_2 = W = mg$</td>
</tr>
<tr>
<td>$kx_1 = mg$</td>
<td>$x_2 = mg/(2k) = (5 \text{ kg})<em>(9.8 \text{ m/s}^2)/(2</em>612.5 \text{ N/m})$</td>
</tr>
<tr>
<td>$k = mg/x_1 = 612.5 \text{ N/m}$</td>
<td>So: $x_2 = 4$ cm.</td>
</tr>
</tbody>
</table>
Calculate force of hand to keep a book sliding at constant speed (i.e. $a = 0$), if the mass of the book is 1 Kg, $\mu_s = .84$ and $\mu_k = .75$

We do exactly the same thing as before, except in both x and y directions!

**Step 1** – Draw!

**Step 2** – Forces!

**Step 3** – Newton’s 2nd ($F_{Net} = ma$)!

Treat x and y independently!

\[ F_{Net, y} = N - W = ma_y = 0 \]

\[ F_{Net, x} = H - f = ma_x = 0 \]

This is what we want!
Calculate force of hand to keep the book sliding at a constant speed (i.e. \( a = 0 \)), if the mass of the book is 1 Kg, \( \mu_s = .84 \) and \( \mu_k = .75 \).

\[
F_{Net, y} = N - W = 0
\]
\[
F_{Net, x} = H - f = 0
\]

- Magnitude of frictional force is proportional to the normal force and always opposes motion!

\[ f_{kinetic} = \mu_k N \]
\[ f_{static} \leq \mu_s N \]

\[
H = f = \mu_k N = \mu_k W = \mu_k mg
\]
\[
= (0.75) \times (1 \text{ kg}) \times (9.8 \text{ m/s}^2)
\]
\[
H = 7.35 \text{ N}
\]
Forces in 2 Dimensions: Ramp

Calculate tension in the rope necessary to keep the 5 kg block from sliding down a frictionless incline of 20 degrees.

Step 1 - Draw!

You should draw axes parallel and perpendicular to motion!

Step 2 - Forces!

Weight is not in x or y direction! Need to DECOMPOSE it!
Vector Decomposition

Now: Step 3 – Newton’s 2\textsuperscript{nd}!

Split \( \mathbf{W} \) into COMPONENTS parallel to axes

Using Trig:

\[
\mathbf{W} = \mathbf{W}_y + \mathbf{W}_x
\]

\[
W_x = W \sin \theta
\]

\[
W_y = W \cos \theta
\]
Calculate force necessary to keep the 5 kg block from sliding down a frictionless incline of 20 degrees.

Now: Step 3 – Newton’s 2\textsuperscript{nd}!

**x direction:**

\[ F_{\text{net}, x} = ma_x \]

System is in equilibrium (\( a = 0 \))!

\[ F_{\text{net}, x} = 0 \]

\[ W_x - T = 0 \]

\[ T = W_x = W \sin \theta \]

\[ = mg \sin \theta \]

\[ = (5\text{kg})(9.8\text{m/s}^2) \sin(20^\circ) \]

\[ T = 16.8 \text{ N} \]
Normal Force ACT

What is the normal force of the ramp on the block?
A) $N > mg$  B) $N = mg$  C) $N < mg$

$W_x = W \sin \theta$
$W_y = W \cos \theta$

y direction:
$F_{net, y} = ma_y$
Equilibrium (a = 0)!
$F_{net, y} = 0$
$N - W_y = 0$
Summary

- **Contact Force: Tension**
  - Force parallel to string
  - Always Pulls, tension equal everywhere

- **Contact Force: Spring**
  - Can push or pull, force proportional to displacement
  - $F = kx$

- **Contact Force: Friction**
  - Static and kinetic
  - Magnitude of frictional force is proportional to $N$

- **Two Dimensional Examples**
  - Choose coordinate system; choose wisely!
  - Analyze each direction independently
A person is pushing a 15 kg block across a floor with $\mu_k = 0.4$ at a constant speed. If she is pushing down at an angle of 25 degrees, what is the magnitude of her force on the block?

**x-direction:** $F_{Net, x} = ma_x$

- $P_x - f = P \cos(\theta) - f = 0$
- $P \cos(\theta) - \mu N = 0$
- $N = P \cos(\theta) / \mu$

**y-direction:** $F_{Net, y} = ma_y$

- $N - W - P_y = N - W - P \sin(\theta) = 0$
- $N - mg - P \sin(\theta) = 0$

**Combine:**

- $(P \cos(\theta) / \mu) - mg - P \sin(\theta) = 0$
- $P (\cos(\theta) / \mu - \sin(\theta)) = mg$
- $P = m g / (\cos(\theta)/\mu - \sin(\theta))$

**Result:**

$P = 80 \text{ N}$
Tension Example:

- Determine the force exerted by the hand to suspend the 45 kg mass as shown in the picture.

\[ \text{Net force} = \text{mass} \times \text{acceleration} \]

\[ T + T - W = 0 \]

A) 220 N  B) 440 N  C) 660 N  D) 880 N  E) 1100 N

- Remember the magnitude of the tension is the same everywhere along the rope!
Tension ACT II

Determine the force exerted by the ceiling to suspend pulley holding the 45 kg mass as shown in the picture.

\[ \Sigma F = ma \]
\[ F_c - T - T - T = 0 \]

A) 220 N  B) 440 N  C) 660 N  
D) 880 N  E) 1100 N

• Remember the magnitude of the tension is the same everywhere along the rope!